Gage & Co's.

Educational Series.

ELEMENTARY ARITHMETIC

Kirkland & Scott.

W. J. Gage & Co's Mathematical Series.

ELEMENTARY ARITHMETIC

ON THE

UNITARY SYSTEM,

Intended as an Introductory Text-Book to Hamblin Smith's Arithmetic.

BY

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PREFACE.

The importance of Arithmetic as a branch of instruction is universally admitted; but, until a comparatively recent period, the results of teaching it were very unsatisfactory, and not at all commensurate with the time usually devoted to it in our schools. This was not owing to any inherent difficulty in the subject itself, but to the method of teaching it. The rule was stated first, an example illustrating the rule followed, and the reason of it came last. Now exactly the reverse of this is adopted by all good teachers. The examples and illustrations precede and lead up to the enunciation of the rule, whenever a rule is considered necessary. But while the method of teaching Arithmetic has undergone a complete change no corresponding change has taken place in our elementary text-books. To remedy this defect the following pages have been written.

We would call attention to the general features of the work:

1. THE UNITARY SYSTEM.—In all our best schools this system has already superseded the cumbrous and illogical methods of our ordinary text-books. Its advantages are so great that it must soon become universal. It has been defined as a method of solving arithmetical problems independently of rules by reasoning out each step of the solution from some previous one, until by a series of deductions, the result sought is obtained. This system trains the pupils to habits of neatness, exactness, and to logical habits of thought; but its chief advantage is its extreme simplicity, dispensing with set rules, and enabling the pupil to solve problems in Simple and Compound Proportion, Simple and Compound Interest, Percentages, Profit and Loss, Partnership, &c., by one uniform, elegant, and simple process.

2. ARRANGEMENT.—The different subjects have been arranged with reference to their importance and their simplicity; the less difficult and more practical first, and the more intricate and less important afterwards. Thus, problems in Canadian Money, Bills, &c., have been introduced immediately after Division, as being of greater importance than any other subject, within the range of the pupils' ability, at that stage of their progress.

3. ORAL EXERCISES.—Each subject has been elucidated by Oral Exercises leading up to written work. This arrangement will assist the pupil in arriving at the reasons for the methods employed, and, to a certain extent, make him the author of his own definitions and rules.

4. RULES.—The rule is given as a convenient summary of the methods employed in the solutions of the examples which precede it. The aim has been to lead the pupil to derive his own methods of operation.

5. EXERCISES.—Special care has been taken in framing and selecting the exercises for the different sections in order to obtain such as will not only evolve thought on the part of the pupil, but more especially prepare him for the business relations of life.

Toronto, May, 1873.

In the present revised Edition a few alterations and additions have been made, due mainly to suggestions from eminent teachers. The Sections on Multiplication and Division of Fractions have been rewritten and it is hoped simplified. At the end of the chapter on Vulgar Fractions a page has been added illustrating the usual mode of eliminating the signs +, —, ×, ÷, and "of." Such minor changes in the wording of definitions, examples, &c., have been made as a careful revision suggested.

Toronto, April, 1880.

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ELEMENTARY ARITHMETIC

ON THE

UNITARY SYSTEM.

CHAPTER I.

Section I.—Definitions; Notation and Numeration.

1. **Arithmetic** is the science of numbers and the art of computing by them.

2. A **Unit** is a single thing regarded as a whole; as *one*, one boy, one dollar, one cent.

3. A **Number** is a unit or a collection of units; *one* dollar is a *unit*; *five* dollars is a collection of units.

4. In common arithmetic, all numbers are expressed by means of the significant figures,

1 2 3 4 5 6 7 8 9

called *one, two, three, four, five, six, seven, eight, nine*, and the figure 0, which is called a *cipher* or *naught*, and which has no value in itself.

5. **Numbers** are considered as being either **Abstract** or **Concrete.**

A **Concrete Number** is one applied to a particular unit; as 5 *men*, 6 *horses*, 9 *dollars*.

An **Abstract Number** is one not applied to any particular unit; as 3, 6, 8.

6. **Similar Numbers** are such as have the same unit: as 6 *boys*, 8 *boys*, 10 *boys*.

Exercise i.

1. How many units in 5? In 5 books? In 9 pencils?
2. What is the unit of 5? Of 5 books? Of 3 balls?
3. State which are abstract and which concrete of the following numbers:

6, 7, 8 books, 9 men, 3, 4, 5 apples, 2, 1 cent.

4. What is the unit of 8 miles? 9 miles? 7? 6 cents?
5. Which are the similar numbers in the following:—3 apples, 7 apples, 4 boys, 7, 6 apples, 9 boys, 2 cents, 4 girls, 5 cents, 9, 8, 5 girls?

NOTATION AND NUMERATION.

The Arabic System.

7. Notation is the art of writing in figures any number expressed in words.

8. Numeration is the art of reading in words any number expressed in figures.

9. All Numbers can be expressed in figures by means of the nine *significant figures* and *naught*, as follows:

1. **All whole numbers under ten are expressed by means of the nine significant figures.**
2. **The value of any figure is increased ten-fold by writing a figure on the right of it.**

It follows that

1. **Ten** may be expressed by writing 1 and 0 on its right, thus, 10; for the value of the 1 is increased ten-fold by the naught which follows it.

Similarly,

Twenty, *Thirty*, *Forty*, *Fifty*, *Sixty*, *Seventy*, *Eighty*, *Ninety*, may be expressed thus:

20, 30, 40, 50, 60, 70, 80, 90, respectively.

2. **Numbers between ten and twenty, between twenty and thirty, &c.,** may be expressed by an application of the preceding statements, thus:

Ex. *Express in figures the number seventy-eight.*

We have first to express the number seventy and as *seventy* is *ten times seven*, we write down 7 and then make it *seventy* by writing some other figure after it. Now, we might use a cipher for this purpose, but since we have to express *eight* besides *seventy*, we write an 8 after the 7, and then read the figures 78 as *seventy-eight*.

Exercise ii.

Write in figures:

1. Seven; nine; four; two.
2. Thirty-six; eighty-four; twenty; sixty-nine.
3. Forty-four; seventy, ninety six; sixteen.
4. Fourteen; twelve; thirty-nine; fifty-six.
5. Write as one number
   four tens and eight units; nine tens and seven units; three tens and six units; six tens

6. Write in words the numbers expressed by the following figures:

6.	7,	11,	15,	19,	50,	84,	96,	98.
7.	71,	12,	28,	91,	44,	17,	25,	34.
8.	20,	37,	48,	76,	99,	69,	70,	87.
9.	14,	35,	89,	78,	54,	49,	50,	13.
10.	90,	80,	39,	28,	11,	19,	27,	31.

10. A Hundred may be expressed by writing *ten*, 10, and then placing a 0 after it thus, 100; for the value of the number 10 is increased ten-fold by writing a figure after it.

Similarly,

Numbers between one hundred and two hundred, between two hundred and three hundred, &c., may be expressed by an application of the statements in Art. 9 thus:

Ex. *Let it be required to write Eight Hundred and Seventy-eight.*

Since Eight Hundred may be expressed by 800, and

Seventy-eight by 78, we are able to express Eight Hundred and Seventy Eight by 878, *i.e.*, by substituting the figures 7, 8, in place of the two ciphers in 800.

Exercise iii.

Write in figures the following numbers:

1. One hundred and forty-nine; three hundred and eight; nine hundred and seventy-four.
2. Two hundred; four hundred and twenty; six hundred and ninety-four.
3. Five hundred and sixty; nine hundred and eight; four hundred and forty-four.
4. 7 hundreds, 3 tens and 5 units; 9 hundreds and 6 tens; 4 hundreds and 6 units.
5. 3 hundreds and nine units; 8 tens, 6 hundreds and 7 units; 2 units, 7 tens and 5 hundreds.

Write in words the numbers expressed by the following figures:

6.	207,	371,	185,	199,	363.
7.	570,	472,	807,	909,	990.
8.	363,	584,	760,	321,	999.
9.	394,	786,	475,	782,	700.
10.	506,	300,	407,	740,	397.

11. Numbers which consist of more than three figures are divided into *periods*, or groups of *three* figures, counting always from the *right* hand side.

12. The names of the periods commencing at the right are *Units, Thousands, Millions, Billions, Trillions*, &c.

13. The places in any period have the same name as in the *units* period, and each place must be filled with a cipher, if not occupied by a significant figure.

Ex. 1. *Write in figures seventy-eight thousand and sixty-four.*

Thousands	Units.
78	064

Ex. 2. *Write in figures twenty millions, six hundred thousand and seven.*

Millions	Thousands	Units.
20	600	007

Exercise iv.

Express in figures the following numbers:

1. Six thousand and six; four thousand three hundred; nine thousand and eighty.
2. Three thousand seven hundred; seven thousand nine hundred and six; three thousand and eighty-four.
3. Sixty-four thousand and nine; eight hundred and seven thousand and sixty-eight; seven hundred thousand, three hundred and sixteen.
4. Four millions, thirty thousand and ninety-seven; eight hundred and nine millions, seven thousand and thirty-nine; five hundred and eighty-six millions and seven.
5. Eight billions; sixty-four billions, seven millions and twenty-four; four billions, four millions and four.
6. Four hundred and eight millions, three thousand and nine; seventy-four billions, seventy-four thousand and four; five hundred billions and five hundred.
7. Eighty billions and seventy millions, eight hundred millions and eight; three hundred billions, three hundred thousand and ninety.
8. Fifty-seven billions, seven hundred millions and eighty; eleven millions and eleven; nineteen billions and fourteen thousand.
9. Seven trillions and seventy; four hundred millions and one; six hundred trillions six hundred billions and six hundred.
10. Ninety-nine trillions and eight; seven hundred billions, seventy millions and seven thousand; sixteen trillions, sixteen billions and sixteen.

Write in words the numbers expressed by the following figures:

11. 7077,850~9,56950,473628.
12. 56418,784006,400507,360094.
13. 309071,901007,720009,182010.
14. 3140006,50000600,3600010070.
15. 51636207640,70003000100,920070070070.

THE ROMAN NOTATION.

14. The system of Notation described above is the one in general use at the present time, and is called the "ARABIC NOTATION" because it was introduced into Europe by the Arabs, who had obtained it from

the Hindoos. Another method was in use among the Romans but is now only employed to denote the chapters and sections of books, etc. The following is a brief description of this notation:

1st. Instead of *figures* being used to express numbers the following *letters* are employed, viz.:

I, V, X, L, C, D, M, of which the simple values are respectively:

1, 5, 10, 50, 100, 500, 1000.

2nd. *If two characters of the same value are placed side by side, or if a character is followed by one of less value than itself,* the number denoted by the expression is the *sum* of their simple values, thus, XX represents 20; XI denotes 11.

3rd. *If a character is followed by one of greater value than itself,* the number denoted by the expression is the *difference* of their simple values, thus, IX represents 9; XL represents 40.

To write any number in Roman Numerals. Resolve the number into its different parts and always write down one part before proceeding to another, beginning at the left hand side.

Ex. Express 1877 in Roman numerals.

1877=1000, 800, 70 and 7.
1000=M
800=DCCC
70=LXX
7=VII

Hence 1877=MDCCCLXXVII.

Exercise v.

Write in Roman numerals—

1. 19, 24, 49, 84, 99.
2. 187, 208, 781, 962, 999.
3. 1301, 1390, 1684, 1815, 1878.

Write in figures—

4. XLIV, LXIX, XCIV, LXXI.
5. XCIX, CXXIX, CLXXVII.
6. DLV, MDCIV, MDCCCXIX, MXC.

Review Exercise.

1. Define unit and number, and distinguish between abstract and concrete numbers. Give examples.

2. Whence was the ordinary system of notation derived? What methods did the Romans adopt to represent numbers?

3. Express in figures the first hundred numbers with their respective names.

4. Write the smallest and largest number possible with the following five characters. 0, 1, 2, 3, 4, and express them in words.

5. Write the different ways in which each of the nine digits can be made up of two less numbers.

6. The number 27 is composed of 16 and 11. Write all the other two numbers which can make up the number 27.

7. How many tens, how many hundreds, how many thousands, and how many ten thousands are there in a million of units?

Section II.—Addition.

James had 3 marbles and John gave him 2 more; how many has James now?

2. How many are 5 apples and 4 apples?

3. How many are 2 books and 3 books and 4 books?

4. John has 8 cents, his father gives him 5 cents more; how much money has he now?

5. How many are 2 balls and 7 balls and 5 balls?

6. Mary is 5 years old; how old will she be seven years hence?

7. James bought 2 books; for one he gave 9 cents and for the other 8 cents; how much did he give for both books?

8. Jane spent 5 cents on candy, 9 cents on a slate, and then had 6 cents left; how much had she at first?

ADDITION TABLE.

1	0	1	2	3	4	5	6	7	8	9
	1	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9	10
2	0	1	2	3	4	5	6	7	8	9
	2	2	2	2	2	2	2	2	2	2
	2	3	4	5	6	7	8	9	10	11
3	0	1	2	3	4	5	6	7	8	9
	3	3	3	3	3	3	3	3	3	3
	3	4	5	6	7	8	9	10	11	12
4	0	1	2	3	4	5	6	7	8	9
	4	4	4	4	4	4	4	4	4	4
	4	5	6	7	8	9	10	11	12	13
5	0	1	2	3	4	5	6	7	8	9
	5	5	5	5	5	5	5	5	5	5
	5	6	7		9	10	11	12	13	14
6	0	1	2	3	4	5	6	7	8	9
	6	6	6	6	6	6	6	6	6	6
	6	7	8	9	10	11	12	13	14	15
7	0	1	2	3	4	5	6	7	8	9
	7	7	7	7	7	7	7	7	7	7
	7	8	9	10	11	12	13	14	15	16
8	0	1	2	3	4	5	6	7	8	9
	8	8	8	8	8	8	8	8	8	8
	8	9	10	11	12	13	14	15	16	17
9	0	1	2	3	4	5	6	7	8	9
	9	9	9	9	9	9	9	9	9	9
	9	10	11	12	13	14	15	16	17	18
10	0	1	2	3	4	5	6	7	8	9
	10	10	10	10	10	10	10	10	10	10
	10	11	12	13	14	15	16	17	18	19

Oral Exercises.

1. How many are 5 and 9? 15 and 9?
2. How many are 6 and 7? 16 and 7? 46 and 7?
3. How many are 4 and 8? 24 and 8? 84 and 8?
4. Count by 2's as far as 30.
5. Count by 4's from 3 to 51.
6. Count by 6's from 4 to 76
7. Count by 7's from 4 to 95.
8. How many are 17 and 7? 27 and 7? 57 and 7?
9. How many are 2 and 3 and 4 and 5 and 6 and 7 and 8 and 9?
10. How many are 6 and 3 and 9 and 4 and 6 and 7 and 8?
11. A farmer sold some oats for 7 dollars, and a ton of hay for 9 dollars; how many dollars did he receive for both?
12. Paid 8 cents for raisins, and 9 cents for cloves; how many cents did both cost?
13. There are 9 boys in one class, and 7 in another; how many in both classes?
14. If you work 8 examples in arithmetic to-day, and 7 to-morrow, how many will you work in both days?
15. There are 9 birds on one tree, and 10 on another; how many birds on both trees?
16. A lady sold 10 pounds of butter at one time, 12 pounds at another, and 3 pounds at another; how many pounds did she sell in all?

15. Combining two or more numbers of the same kind, so as to make *one* number, is called **Addition.**

16. The number found by adding two or more numbers together is called the **Sum.**

17. The numbers which are added together are called **Addends.**

18. The *sign* of Addition, +, is called *Plus*, and when placed between two numbers shows that they are to be added.

NOTE.—The following is a convenient mode of giving a class practice in addition: Write the nine digits on the Black Board. Point to a digit, then to another, etc., the pupils adding the digits as they are pointed to. When the sum is sufficiently large let the pupils write it on their slates. In this way one sum on the Board will serve for many examples.

19. The sign, =, is called the *Sign of Equality*, and when placed between two numbers shows that they are equal. Thus, 2 + 3 = 5, and is read, 2 plus 3 equals 5.

20. Principle. — Only similar numbers can be added. Thus, 4 cents and 7 cents can be added together, but not 4 cents and 7 marbles.

21. Addition may be divided into two cases:

1. *Addition of numbers in which the sum of any column is less than ten.*
2. *Addition of numbers in which the sum of any column exceeds nine units of that column.*

Case I.

22. To add any column of Figures whose sum does not exceed nine.

Ex. 1. How many are 21 cents, 15 cents, and 12 cents?

21 cents.
15 "
12 "
—
48 cents.

Write the numbers as above, placing *units* under *units*, and *tens* under *tens*, and begin at the right to add. Thus, 2 and 5 are 7, and 1 are 8, which we write in the place of units; adding the tens we have 1 and 1 are 2, and 2 are 4, which we write in the tens' place. Hence the entire sum is 48 cents.

After a little practice the pupil should proceed as follows: 2, 7, 8; set the 8 in the units' column. Next, 1, 2, 4; set the 4 in the tens' column.

Exercise vi.

(1)	(2)	(3)
16 horses.	18 boys.	12 girls.
21 "	20 "	14 "
10 "	60 "	13 "
—		—

(4)	(5)	(6)	(7)
421	312	241	4 5
132	231	134	103
425	413	523	121
(8).	(9)	(10)	(11)
342	216	143	351
406	305	322	204
131	461	232	243
(12)	(13)	(14)	(15)
240	650	513	408
401	122	106	871
357	126	260	110
(16)	(17)	(18)	(19)
2341	3213	4021	1056
3214	2340	1045	6131
3034	4326	3923	2802
(20)	(21)	(22)	(23)
23241	31042	12304	21304
31402	24535	35242	30562
44235	32411	41452	28122
(24)	(25)	(26)	(27)
123402	213456	413215	325231
341256	435230	234344	253608
333240	120303	142130	410150

Exercise vii.

Practical Problems.

1. A boy spent 23 cents for a melon, 32 cents for peaches, and 24 cents for pears; how many cents did he spend?

2. Of the trees in an orchard, 23 are peach trees, 10 are plum trees, 12 are pear trees, and 43 are apple trees; how many trees are there in the orchard?

3. A farmer has 323 acres in cotton, 421 acres in corn, 123 acres in wheat, and 101 in oats; how many acres has he in cultivation?

4. A gentleman paid 225 dollars for a buggy, 231 dollars for a horse, 300 dollars for a carriage, and 40 dollars for harness; what did he pay for all?

5. A man travelled on the cars for four days as follows: the first day he went 313 miles, the second day 242 miles, the third day 220 miles, and the fourth day 214 miles; how many miles did he travel in the four days?

6. A merchant bought four bales of cloth; the first bale measured 305 yards, the second 213 yards, the third 240 yards, the fourth 211 yards; how many yards did he buy?

7. Four merchants loaded a freight train with cotton; the first put on 213 bales, the second 232 bales, the third 312 bales, and the fourth 121 bales; how many bales were put on?

8. A merchant went to Montreal and invested 5213 dollars in dry goods, 2431 dollars in groceries, 1000 dollars in hardware, and 345 dollars in confectionery; how much did he invest?

9. Four men built a factory; the first furnished 13214 dollars, the second 31423 dollars, the third 43230 dollars, and the fourth 10122 dollars; what was the cost of the factory?

10. The population of 4 cities is as follows: the first contains 321213 persons, the second 213310 persons, the third 145245 persons, and the fourth 220130 persons; what is the total population?

Case II.

23. To add when the sum of any column exceeds Nine Units of that column.

Ex. 2. Find the sum of 358 dollars, 369 dollars, 4008 dollars, 328 dollars, and 9 dollars.

358 dollars.
369 "
4008 "
328 "
9 "

5072 dollars.

For convenience in adding, write the numbers, placing units under units, tens under tens, etc. Begin at the column of the lowest order; thus, 9, 17, 25, 34, 42: 42

units = 4 tens and 2 units. Write the 2 under the units' column and add the 4 tens with the column of tens; thus, 6, 12, 17: 17 tens = 1 hundred and 7 tens. Write 7 under the column of tens and add 1 with the column of hundreds: thus, 4, 7, 10: 10 hundreds = 1 thousand and 0 hundreds. Write 0 under the column of hundreds and add 1 with the thousands' column; thus 5. Write the 5 under the thousands' column, making the sum 5072.

24. *PROOF.*—Begin at the top of the units' column and add the several columns downwards; if the two results agree the work may be presumed to be correct.

Exercise viii.

(1)	(2)	(3)	(4)
42 dollars	18 cents.	55 boys.	48 girls.
28 "	16 "	13 "	25 "
43 "	44 "	84 "	72 "

(5)	(6)	(7)	(8)	(9)	(10)
45	84	16	46	84	95
60	72	61	64	46	50
32	91	85	51	87	68

(11)	(12)	(13)	(14)	(15)	(16)
642	272	615	465	956	925
347	447	421	641	508	575
812	638	879	848	467	259

(17)	(18)	(19)	(20)	(21)	(22)
752	342	253	897	156	851
423	426	541	111	481	318
709	151	422	343	423	805
820	737	735	625	762	167

(23)	(24)	(25)	(26)	(27)	(28)
4813	1122	2291	3574	4149	1357
5914	7914	5723	3333	2575	2468
6115	1234	2102	4680	4404	5555
7036	8024	6838	3391	3685	6666

(29)	(30)	(31)	(32)	(33)	(35)
5788	3455	2729	4944	3282	1185
2693	6521	8272	5260	6341	5073
1112	6817	3228	3788	3161	9962
6762	7773	9561	5473	2827	9467
8104	6839	5587	2667	7214	3478

(35)	(36)	(37)	(38)
45474	73422	77823	13536
38242	75638	21684	71882
67891	18208	18516	81385
84870	32678	33902	89246
22171	27225	14656	91257

(39)	(40)
4638827	28515534
563725	47224456
434958	31821745
367624	18714924
233647	75584627

Find the sum

41. Of 6472+8736+4633+4854.
42. Of 2762+8756+9783+4578.
43. Of 1617+8743+7284+9621
44. Of 2659+4962+8705+9030.
45. Of 5005+6067+7583+4783.
46. Of 27845+67832+74281+68432.
47. Of 47823+68421+70070+60504
48. Of 127+6434+7805+66782.
49. Of 10+8756+405+66782.
50. Of 7560+804+7854+87400.
51. Of 1525+960+820+16+37830.

Exercise ix.

Practical Problems.

1. A gave 27 dollars for a cow, 45 dollars for an ox, and 150 dollars for a horse what did they all cost?

2. A has 120 acres of land, B has 310 acres. C has 516 acres, and D has 715 acres, how many acres have they altogether?

3. There are 31 days in January, 28 in February, 31 in March, and 30 in April: how many days are there in these four months?

4. A man travelled 215 miles one week, 195 the next, 273 the next, and 378 the next; how far did he travel?

5. A weighs 127 pounds, B 215 pounds, C 176 pounds, D 184 pounds, and E 234 pounds; what is the sum of their weights?

6. A farmer raised 576 bushels of corn, 918 bushels of oats, 349 bushels of wheat, and 2785 bushels of rye; how many bushels did he raise in all?

7. A owns 214 acres of land, B owns 719 acres, C owns 2136 acres, and D owns 372 acres; how many acres do they own altogether?

8. A bought a horse for 168 dollars, and a carriage for 376 dollars, and sold them so as to gain 89 dollars; how much did he receive for them?

9. In one book there are 725 pages, in another book there are 327 pages, and in another book there are as many as in both the former; how many pages in all?

10. A merchant bought cloth for 756 dollars, silk for 859 dollars, muslin for 367 dollars, and calico for 255 dollars; how much did they all cost?

11. A paid 325 dollars for a span of horses, and 248 dollars more than this for a carriage; for how much must he sell them both to gain 275 dollars?

12. A gains in one year 465 dollars, B gains 136 dollars more than A, and C gains as much as A and B together; how much did B gain? how much did C gain? how much did they all gain?

Section III.—Subtraction.

1. John had 5 cents, and bought an orange for 2 cents; how many cents has he left?

2. Mary had 6 cups, but broke 3; how many has she remaining?

3. A man, earning 10 dollars a week, spent 6 dollars for provisions; how many dollars has he left?

4. If a merchant has 12 barrels of flour, and he sells 7 of them, how many barrels has he left?

5. If you have 27 dollars, and spend 12 dollars, how much will you have remaining?

6. How many are 6 apples less 3 apples?

7. How much is 6 less 3? 6 less 4?

SUBTRACTION TABLE.

1	1	2	3	4	5	6	7	8	9	10
	1	1	1	1	1	1	1	1	1	1
	0	1	2	3	4	5	6	7	8	9
2	2	3	4	5	6	7	8	9	10	11
	2	2	2	2	2	2	2	2	2	2
	0	1	2	3	4	5	6	7	8	9
3	3	4	5	6	7	8	9	10	11	12
	3	3	3	3	3	3	3	3	3	3
	0	1	2	3	4	5	6	7	8	9
4	4	5	6	7	8	9	10	11	12	13
	4	4	4	4	4	4	4	4	4	4
	0	1	2	3	4	5	6	7	8	9
5	5	6	7	8	9	10	11	12	13	14
	5	5	5	5	5	5	5	5	5	5
	0	1	2	3	4	5	6	7	8	9
6	6	7	8	9	10	11	12	13	14	15
	6	6	6	6	6	6	6	6	6	6
	0	1	2	3	4	5	6	7	8	9
7	7	8	9	10	11	12	13	14	15	16
	7	7	7	7	7	7	7	7	7	7
	0	1	2	3	4	5	6	7	8	9
8	8	9	10	11	12	13	14	15	16	17
	8	8	8	8	8	8	8	8	8	8
	0	1	2	3	4	5	6	7	8	9
9	9	10	11	12	13	14	15	16	17	18
	9	9	9	9	9	9	9	9	9	9
	0	1	2	3	4	5	6	7	8	9

Oral Exercises.

1. Subtract by 2's from 100 to 2; thus, 2 from 100 leaves 98, 2 from 98 leaves 96, and so on.
2. Subtract by 3's from 100 to 1; by 4's from 100 to 0.
3. Subtract by 4's from 95 to 3; by 5's from 100 to 0.
4. Subtract by 6's from 100 to 4; by 7's from 100 to 2.
5. Subtract by 7's from 99 to 1; by 8's from 100 to 4.

6. Subtract by 9's from 100 to 1 ; by 9's from 99 to 0.
7. Count by 4's from 3 to 39, and back again to 19.
8. Count by 5's from 6 to 66, and back again to 26.
9. Count by 7's from 18 to 53, and back again to 11.
10. Count by 8's from 25 to 65, and back again to 1.
11. Jane is 11 years old, and Mary is 7 years younger ; what is Mary's age ?
12. A grocer sold tea for 10 dollars, and thus gained 3 dollars ; what did the tea cost him ?
13. If I buy cloth for 7 dollars, at what price must I sell in order to lose 4 dollars ?
14. John has 11 dollars ; he pays 2 dollars for books, and 3 dollars for a hat ; how much money has he left ?
15. Mary has 9 dollars ; she pays 7 dollars for a dress, and then earns 3 dollars more ; how much has she now ?
16. A boy having 12 apples, bought 6 more, and then sold 8 ; how many had he left ?
17. James had 5 dollars, he earned 5 dollars more, and then spent 6 dollars ; how much did he then have ?
18. A merchant gave 8 dollars for a certain article, and paid 4 dollars for carriage ; at what price must he sell to gain 3 dollars ?

25. Finding the difference between two numbers is called **Subtraction.**

26. The number found by taking one number from another is called the **Difference** or **Remainder.**

27. The number from which the other is taken is called the **Minuend.**

28. That which is taken from the Minuend is called the **Subtrahend.**

29. The sign of subtraction, —, is called *Minus*, and when placed between two numbers shows that the one on the right of the sign is to be taken from the one on the left of it. Thus, 6—2 is read 6 minus 2, and means that 2 is to be taken from 6.

30. Principle.—Only similar numbers can be subtracted ; thus, 4 boys from 7 boys ; 6 cents from 8 cents, &c.

31. **Subtraction** may be divided into two cases :

1. *When no figure of the subtrahend is greater than its corresponding figure of the minuend.*
2. *When a figure of the subtrahend is greater than its corresponding figure of the minuend.*

Case I.

32. To subtract when no figure of the subtrahend is greater than its corresponding figure of the minuend.

Ex. I. A grocer bought 678 oranges, and sold 335 of them; how many has he left?

678 oranges.
335 "
343 "

Here we are required to find the *difference* between 678 and 335. We write the less number under the greater, placing *units* under *units* and *tens* under *tens*. Beginning with the units we say 5 units from 8 units leave 3 units, and we set the 3 in the units' column below. Then 3 tens from 7 tens leave 4 tens, and we set the 4 in the tens' column. Lastly, 3 hundreds from 6 hundreds leave 3 hundreds, and we set the three in the hundreds' column. Hence we have as the whole remainder 3 hundreds 4 tens and 3 units, or 343.

Exercise x.

(1)	(2)	(3)	(4)	(5)	(6)
625	456	763	617	767	896
312	215	512	215	123	432
(7)	(8)	(9)	(10)	(11)	(12)
279	867	756	736	967	875
136	502	452	432	234	345
(13)	(14)	(15)	(16)	(17)	(18)
8763	9076	8769	5076	4872	7659
4321	4054	1546	3075	2342	3237
(19)	(20)	(21)	(22)	(23)	(24)
8769	4876	8275	8799	8591	5857
3257	2142	3251	2542	7230	1234
(25)	(26)	(27)	(28)	(29)	30
784	82345	57596	72578	27397	67385
361	22121	21321	41362	22315	24123

(31)	(32)	(33)	(34)	(35)
57897	67853	87578	96754	81296
21472	32721	21335	21423	20135

(36)	(37)	(38)	(39)	(40)
253786	472589	87695	56728	98785
213123	212324	23542	21306	21342

(41)	(42)	(43)	(44)	(45)
373967	873972	72587	95837	89976
212851	132421	51234	51321	32742

46. 314 from 678.
47. 425 from 658.
48. 561 from 780.
49. 254 from 576.
50. 437 from 869.

51. 1235 from 3768.
52. 3726 from 4969.
53. 2532 from 8748.
54. 4720 from 87856.
55. 12345 from 68799.

Exercise xi.

Practical Problems.

1. In a school of 74 pupils, 31 are boys, how many girls are there?

2. A girl had 75 cents and paid 31 cents for a slate; how many cents has she left?

3. A man bought a horse for 98 dollars, and sold it for 82 dollars; what did he lose?

4. Two parties played a game of base ball and made 87 runs. One party made 53 runs; how many did the other party make?

5. Jane and Susan together answered 87 questions in geography Jane answered 43 of them; how many did Susan answer?

6. A gentleman bought a buggy for 225 dollars, and sold it for 268 dollars; what was his profit?

7. A man bought a horse for 265 dollars, and sold it for 232 dollars; how much did he lose?

8. A man deposited 5237 dollars in the bank; he afterwards drew out 3125 dollars; how much remained?

9. A man dying left 27894 dollars to his son and his daughter. The share of the son was 13452 dollars; what was the daughter's share?

Case II.

33. To subtract when a figure in the Subtrahend is greater than its corresponding figure in the Minuend.

Ex. 2. From 522 dollars subtract 285 dollars.

522 dollars.
285 "
---
237 "

We begin at the right, but as we cannot take 5 *units* from 2 *units*, we borrow 1 *ten* from the 2 *tens*, and adding the 1 ten, = 10 *units*, to the 2 units, we have 12 *units*. Then 5 units from 12 units leave 7 units, which we write under the units' column. Now, as we borrowed 1 ten from the 2 tens, we left only 1 ten. As we cannot take 8 *tens* from 1 *ten*, we borrow 1 *hundred* from the 5 *hundreds*, and considering the 1 *hundred* borrowed as 10 *tens* we add it to the 1 *ten*, making it 11 tens; then 8 tens from 11 tens leave 3 tens, which we write in the tens' column. Now, as we borrowed 1 hundred from 5 hundreds, we left only 4 hundreds: hence we say, 2 hundreds from 4 hundreds leave 2 hundreds, which we write in the hundreds' column, making the remainder 2 hundreds 3 tens and 7 units, or 237.

There is another method of performing subtraction, which depends on the following principle:

The difference between two numbers remains the same when each of them is increased by the same number.

For example, 5—2 = 3. Now, if we add 10 to each, we have 15—12 = 3, as before.

In Ex. 2, if we add 10 units to 2 units we have 12 units. Then 5 units from 12 units leave 7 units, which we write in the units place. Now, as we added 10 units to the minuend, if we add an equal number to the subtrahend the difference will remain the same. But 10 units = 1 ten. Adding 1 ten to 8 tens we have 9 tens; and as we cannot take 9 tens from 2 tens, we add 10 tens, thereby making 12 tens; then 9 tens from 12 tens leave 3 tens, which we write in the tens' place. Since we added 10

tens to the minuend, we must add an equal number to the subtrahend, in order that the difference may remain the same. But 10 tens = 1 hundred. Adding 1 hundred to 2 hundreds we get 3 hundreds; and taking 3 hundreds from 5 hundreds we get 2 hundreds, which we write in the hundreds' place. *This is the method usually employed.*

34. *PROOF.*—Add the remainder to the subtrahend; the sum will equal the minuend if the work is correct.

Exercise xii.

(1)	(2)	(3)	(4)	(5)	(6)
573	748	835	968	839	538
248	375	573	675	584	394
(7)	(8)	(9)	(10)	(11)	(12)
650	839	547	658	735	848
475	585	284	372	373	539
(13)	(14)	(15)	(16)	(17)	(18)
524	752	845	307	456	450
356	587	579	138	387	382
(19)	(20)	(21)	(22)	(23)	(24)
854	943	607	500	704	403
396	765	309	325	507	285
(25)	(26)	(27)	(28)	(29)	(30)
726	857	735	792	807	650
387	359	558	295	328	357
(31)	(32)	(33)	(34)	(35)	(36)
8876	6385	6735	4076	4070	4135
2379	3527	2547	3128	2137	1216
(37)	(38)	(39)	(40)	(41)	(42)
8672	5283	8175	2534	6735	7219
3728	2426	2836	1235	5376	1979

(43)	(44)	(45)	(46)	(47)	(48)
8522	7135	6347	8135	7345	4372
6243	1872	2563	2453	2876	2583
(49)	(50)	(51)	(52)	(53)	(54)
35672	43763	87253	73875	63527	53413
23828	24235	34365	38376	14238	28401

Exercise xiii.

Practical Problems.

1. A horse was bought for 125 dollars, and sold for 117 dollars; how much was lost by the sale?
2. A roll of carpet contained 156 yards, but 79 yards were sold from it; how much remained?
3. A house cost 5410 dollars, and was sold for 6000 dollars; how much was the gain?
4. A man died in 1878 at the age of 75 years; when was he born?
5. A town which ten years ago had a population of 3745, has now a population of 6996; what is the gain?
6. I went to a store and bought a knife for 56 cents, and gave the storekeeper a four dollar bill [400 cents] to pay for it; how much change did he give me back?
7. Two little girls picked 74 quarts of blackberries one summer; if one picked 37 quarts, how many quarts did the other pick?
8. Mr. Robinson's horse and carriage cost four hundred dollars; what did the horse cost, if the carriage cost two hundred and twenty-five dollars?
9. Suppose a man's income is 2453 dollars a year, and his expenses are 1947 dollars, how much can he save in a year?
10. Smith bought of Brown 875 acres of land for 23'00 dollars. For 500 acres of the tract he paid 11379 dollars; how many acres were in the remainder of the tract? And for what sum was it purchased?

Addition and Subtraction.

Exercise xiv.

Find the result of

1. 768+276—360+284—782.
2. 369+784+468—266—368—248.

3. 1764 – 839 + 786 + 724 – 368 – 256.
4. 136 – 769 – 284 + 96 + 268 + 372.
5. 269 – 1846 + 368 – 274 + 2976 + 769.
6. 769 + 735 + 368 – 784 – 369 – 249.
7. 1869 – 2846 + 362 – 489 + 3007 + 249.
8. 2845 + 3624 – 78695 + 784 + 93768.
9. 7369 – 245 – 12456 + 85769 – 2572.
10. 3004 + 2006 – 5008 – 3601 + 7200.

Exercise XV.

Practical Problems.

1. A man owing 1369 dollars, paid at one time 264 dollars and at another 748 dollars; how much does he still owe?

2. A man bought a farm for 6780 dollars; he spent 1875 dollars for improvements and 977 dollars for stock. He then sold the whole for 9000 dollars; did he gain or lose, and how much?

3. The sum of four numbers is 936287; the first is 23789, the second is 11892 less than the first, the third is 35416 more than the second; what is the fourth?

4. What number increased by the difference between 1458 and 2362 will make the sum of 3641, 789 and 7008?

5. A collector received 1260 dollars from four men; from the first he got 352 dollars; from the second 67 dollars more, and from the third 94 dollars less than this; how much did he receive from the fourth?

6. At an election, in which there were two candidates, the whole number of votes was 3694; the defeated candidate received 1369 votes; what was the majority?

7. A boy shot an arrow up the road 175 feet and another down the road 234 feet; his little brother brought them to him; how far did he walk to get them?

8. John and James play marbles, John has 24 at the beginning and James 36. The first game John wins 4, the next he wins 6, the next he loses 5, the next he loses 3, the next he wins 2; how many marbles has each now?

9. Find the final remainder in subtracting 54368 as many times as possible from 476209.

10. From the difference between 576 and 7852, take the difference between 19101 and 18453.

11. The sum of two numbers is 8764; the difference of the same two numbers is 1658; what are the numbers?

Section IV.—Multiplication.

1. There are 5 oranges in each of three dishes; how many are there altogether? 5 and 5 and 5 are how many? Three 5's or three times 5 are how many?

2. If there are 3 berries in one cluster, how many berries are there in 5 clusters? 3 + 3 + 3 + 3 + 3 is how many?

3. There are 3 feet in one yard, how many feet are there in 2 yards? In 4 yards? In 6 yards?

4. There are 6 working days in 1 week, how many working days are there in 2 weeks? In 5 weeks? 6 + 6 + 6 + 6 + 6 is how many?

5. What will 3 hats cost at 2 dollars each?

Since 1 hat costs 2 dollars, 3 hats will cost 2 + 2 + 2 dollars or 3 times 2 dollars, or 6 dollars. Hence 3 hats will cost 6 dollars.

6. If John walks 3 miles an hour, how far will he go in 4 hours?

7. If a First Book costs 3 cents, what will 5 First Books cost?

8. What will 4 buns cost at 2 cents each?

9. If little James takes 2 steps in a yard, how many steps will he take in going 5 yards?

10. John bought 4 tops at 3 cents a piece, how much money did he spend?

35. When any number is to be *added* to itself a given number of times the work may be shortened by a process called **Multiplication.**

36. The number resulting from the Multiplication is called the **Product.**

37. The number to be added or repeated is called the **Multiplicand.**

38. The number denoting how many times the Multiplicand is to be repeated is called the **Multiplier.**

39. The *Sign* of Multiplication is formed by two short lines crossing each other slantingly; thus, ×.

It shows that the second of the two numbers between which it is placed is to be multiplied by the first, thus 4 times 8 is written 4 × 8.

40. Principles.—

1. The *Multiplicand* may be either an abstract or a concrete number. The *multiplier* must always be regarded as an abstract number.
2. The *Product* is always of the same kind as the Multiplicand. Thus 3 × 3 cents are 9 cents; 2 × 5 boys are 10 boys.

MULTIPLICATION TABLE.

Twice	Three times	Four times	Five times	Six times	Seven times
1 is 2	1 is 3	1 is 4	1 is 5	1 is 6	1 is 7
2 .. 4	2 .. 6	2 .. 8	2 .. 10	2 .. 12	2 .. 14
3 .. 6	3 .. 9	3 .. 12	3 .. 15	3 .. 18	3 .. 21
4 .. 8	4 .. 12	4 .. 16	4 .. 20	4 .. 24	4 .. 28
5 .. 10	5 .. 15	5 .. 20	5 .. 25	5 .. 30	5 .. 35
6 .. 12	6 .. 18	6 .. 24	6 .. 30	6 .. 36	6 .. 42
7 .. 14	7 .. 21	7 .. 28	7 .. 35	7 .. 42	7 .. 49
8 .. 16	8 .. 24	8 .. 32	8 .. 40	8 .. 48	8 .. 56
9 .. 18	9 .. 27	9 .. 36	9 .. 45	9 .. 54	9 .. 63
10 .. 20	10 .. 30	10 .. 40	10 .. 50	10 .. 60	10 .. 70
11 .. 22	11 .. 33	11 .. 44	11 .. 55	11 .. 66	11 .. 77
12 .. 24	12 .. 36	12 .. 48	12 .. 60	12 .. 72	12 .. 84

Eight times	Nine times	Ten times	Eleven times	Twelve times
1 is 8	1 is 9	1 is 10	1 .. 11	1 is 12
2 .. 16	2 .. 18	2 .. 20	2 .. 22	2 .. 24
3 .. 24	3 .. 27	3 .. 30	3 .. 33	3 .. 36
4 .. 32	4 .. 36	4 .. 40	4 .. 44	4 .. 48
5 .. 40	5 .. 45	5 .. 50	5 .. 55	5 .. 60
6 .. 48	6 .. 54	6 .. 60	6 .. 66	6 .. 72
7 .. 56	7 .. 63	7 .. 70	7 .. 77	7 .. 84
8 .. 64	8 .. 72	8 .. 80	8 .. 88	8 .. 96
9 .. 72	9 .. 81	9 .. 90	9 .. 99	9 .. 108
10 .. 80	10 .. 90	10 .. 100	10 .. 110	10 .. 120
11 .. 88	11 .. 99	11 .. 110	11 .. 121	11 .. 132
12 .. 96	12 .. 108	12 .. 120	12 .. 132	12 .. 144

Oral Exercise.

1. Multiply by 2 from 1 to 12; by 3 from 1 to 6.
2. Multiply by 4 from 3 to 9; by 5 from 12 to 4.
3. Multiply by 6 from 3 to 10; by 7 from 12 to 5.
4. Multiply by 8 from 12 to 2; by 9 from 1 to 11.

5. What will 5 hats cost at 7 dollars each?

If 1 hat costs 7 dollars, 5 hats will cost 5 times 7 dollars, or 35 dollars.

6. What will 4 pair of boots cost at 6 dollars a pair?

7. A sheep costs 7 dollars; how much will 6 sheep cost at the same rate?

8. How much will 12 tons of coal cost at 6 dollars a ton?

9. A man can earn 4 dollars a day; how much can he earn in 9 days?

10. At the rate of 8 marbles for a cent, how many can be bought for 12 cents?

11. What will 7 lead pencils cost at 7 cents each?

12. If a man cuts 3 cords of wood in a day, how many cords can he cut in twelve days?

13. If 8 men can do a piece of work in 7 days, how many days will it take one man to do it?

14. How many dollars will buy 9 tons of hay at 12 dollars a ton?

15. In an orchard there are 12 rows of trees, and 11 trees in each row; how many trees are there in the orchard?

16. If a quantity of provisions will last 8 men 7 days, how many days will the same provisions last one man?

17. If 12 bushels of apples are picked from each of 8 trees, how many bushels are picked from all?

41. Multiplication may be divided into two cases:

1. *When the Multiplier does not exceed twelve.*
2. *When the Multiplier exceeds twelve.*

Case I.

42. When the Multiplier does not exceed twelve.

Ex. 1.—How many are 4 times 87 boys?

1st Operation.	2nd Operation.
87 boys.	87 boys.
87 "	4
87 "	
87 "	348 boys.

Sum 348 boys.

In the first operation we find the result by Addition.

In the second operation, which is much shorter

write down 87 once, and we put 4, the number of times it is to be taken, under the units' figure of the Multiplicand. We then begin at the right hand side to multiply by 4; 4 times 7 units are 28 units, or 2 tens and 8 units. We write the 8 units under the units and add the 2 tens to the product of the tens. We next take 4 times 8 tens. 4 times 8 tens are 32 tens and 2 tens make 34 tens, or 3 hundreds and 4 tens. Then we write down 4 in the tens' place and 3 in the hundreds' place.

Exercise xvi.

	(1)	(2)	(3)	(4)
Multiply	7432	8432	72312	92123
By	2	2	3	4

(5)	(6)	(7)	(8)	(9)
39 boys.	47 cents.	137 cows.	186 apples.	234 girls
5	6	7	8	9

Multiply

10. 315 by 6.
11. 480 by 7.
12. 614 by 5.
13. 7842 by 3.
14. 6843 by 7
15. 8742 by 5.
16. 9764 by 8.
17. 8973 by 6.
18. 14068 by 5.
19. 18007 by 4.
20. 82709 by 8.
21. 21876 by 7.
22. 70095 by 9.
23. 58799 by 6.
24. 71873 by 9.
25. 6742 by 8.
26. 6040 by 9.
27. 61783 by 7.
28. 60784 by 6.
29. 85643 by 5.
30. 170504 by 6.
31. 683471 by 5.
32. 863478 by 7.
33. 785473 by 8.
34. 246353 by 9.
35. 786549 by 10.
36. 832957 by 11.
37. 987356 by 11.
38. 757638 by 12.
39. 895324 by 12.

Exercise xvii.

Practical Problems.

1. What will 4070 lemons cost at 4 cents each?
2. What will 37086 oranges cost at 5 cents each?
3. A man paid 387 dollars for a house; how much would he give for 7 such houses?
4. What will 3043 pair of boots cost at 5 dollars a pair?

5. There are 56 sheep in one flock; how many sheep are there in 6 such flocks? What is the value of each flock at 7 dollars a head?

6. A man bought 384 pounds of sugar; he sold 290 pounds; how much had he left? How much did he receive for what he sold, at 9 cents a pound? What is the remainder worth at 8 cents a pound? At 7 cents a pound?

7. A merchant sold 378 kegs of nails at 9 dollars a keg; 32 hundred weight of iron at 7 dollars a hundred weight; what did each of the articles come to? What did both come to? He paid away 1389 dollars; how much money has he left;

8. I have a book with 220 pages; there are 6 paragraphs on each page; there are 9 lines in each paragraph; there are 8 words in each line; there are, on an average, 5 letters in each word; how many paragraphs are there in the book? How many lines? How many words? How many letters?

9. A grocer sold 37 pounds of rice at 8 cents a pound; 46 pounds of sugar at 9 cents a pound; what did the rice come to? What did the sugar come to? What did both come to? How much did one cost more than the other?

10. A man bought 137 pints of chestnuts at 8 cents a pint; 246 pints of peanuts at 9 cents a pint; what did each cost? What did both cost? How much did one cost more than the other?

43. To multiply by the factors of a number

44. The **Factors** of a number are those numbers which multiplied together will produce it. Thus, 3 and 5 are the factors of 15.

Ex. 2.—Multiply 742 by 36.

$36 = 6 \times 6$, or 9×4, or 12×3.

742	742	742	742
36	6	9	12
4452	4452	6678	8904
2226	6	4	3
26712	26712	26712	26712

It is thus seen that the *Multiplicand multiplied by the Multiplier, gives the same product as when multiplied by any set of factors into which the Multiplier can be separated.*

Exercise xviii.

Multiply

1. 478 by 25.
2. 976 by 42.
3. 1879 by 63.
4. 1362 by 49.
5. 8936 by 54.
6. 4729 by 72.
7. 2345 by 81.
8. 3764 by 64.
9. 2978 by 45.
10. 3475 by 18.
11. 7649 by 24.
12. 9365 by 144.

13. In one mile there are 1760 yards, how many yards are there in 56 miles?

14. If sound travels 1142 feet in one second, how far will it move in one minute or 60 seconds

15. What will 72 bushels of wheat cost at 118 cents for one bushel?

16. If 27 men can do a piece of work in 17 days, how long will it take one man to do the same work?

17. What is the cost of 24 horses at the rate of 125 dollars each?

18. If a yoke of oxen costs 135 dollars, what will 63 yoke cost?

19. If a man spends 945 dollars in a year, how much will he spend at the same rate in 21 years?

20. There are 1440 minutes in a day; how many minutes are there in 28 days?

Case II.

45. When the Multiplier exceeds Twelve.

Ex. 3. Multiply 479 by 57.

479
57

1st partial product 3353 = 7 times the Multiplicand
2nd " " 2395 = 50 " "

Entire " 27303 = 57 " "

Since 57 is composed of 7 units and 5 tens or 50, 5 times the number must be equal to 7 times the number, plus 50 times the number. 7 times 479 is 3353, the *first partial* product We get 50 times 479 by first finding 5 times 479 and then multiplying this result by 10. 5 times 479 is 2395 and 10 times 2395 is 23950, the *second partial* product. We write this under the first product so that

units may come under units, tens under tens, &c., and then we add the two partial products together.

In actual practice we always omit the *0* and write the second partial product as above.

46. *PROOF.*—Multiply the Multiplier by the Multiplicand. If the product is the same as before, the work is likely to be correct.

Exercise xix

Multiply

1. 744 by 635.
2. 895 by 336.
3. 972 by 243.
4. 825 by 682.
5. 973 by 745.
6. 8462 by 781.
7. 9643 by 683.
8. 8532 by 763.
9. 8984 by 133.
10. 4659 by 886.
11. 28352 by 345
12. 41678 by 287.
13. 34073 by 435.
14. 40735 by 628
15. 29304 by 789
16. 90705 by 897
17. 43445 by 678.
18. 37436 by 835
19. 88888 by 789.
20. 23567 by 597.
21. 6484 by 6372.
22. 7856 by 3375.
23. 6748 by 6334.
24. 4878 by 3437.
25. 8547 by 7733.
26. 85474 by 2547.
27. 46887 by 3489
28. 56184 by 5474
29. 56664 by 4871
30. 25473 by 4487.
31. 73519 by 4735.
32. 81897 by 3456.
33. 21346 by 31452.
34. 47309 by 45233.
35. 25737 by 63252.
36. 43629 by 28516.
37. 10786 by 31672.
38. 47396 by 73462.
39. 76448 by 54173.
40. 28354 by 31867.

47. To multiply when the Multiplicand, the Multiplier, or both, contain ciphers.

Ex. 4. Multiply 2479 by 4006.

2479	4006 times 2479 equals 4000 times
4006	2479 plus 6 times 2479. 6 times 2479
14874	is 14874; 4000 times 2479 is 9916000.
9916	These partial products are written one
9930874	under the other as before, the 0's being omitted.

Exercise xx.

Multiply

1. 415 by 307.
2. 7004 by 902.
3. 1364 by 5004.
4. 2769 by 708.
5. 9006 by 7036.
6. 1684 by 4008.
7. 2002 by 4103.
8. 3678 by 7068.
9. 9999 by 8008.
10. 3674 by 200901.

Ex. 5. Multiply 614000 by 700.

614000
700
429800000

This result is the same as that obtained by multiplying 614 by 7, and then annexing to the right *five* naughts, which is the sum of the number of naughts to the right of both the multiplicand, 614, and the multiplier, 7.

Exercise xxi.

Find the value

1. Of 743×600.
2. Of 847×700.
3. Of 9642×6300.
4. Of 1875×6340.
5. Of 27×9000.
6. Of 6000×43.
7. Of 18000×623.
8. Of 6400×640.
9. Of 650×650.
10. Of 83600×7500
11. Of 9230×7000.
12. Of 8000×61000

Exercise xxii.

Practical Problems.

1. In 1 ream of paper there are 480 sheets; how many sheets are there in 947 reams?

2. If a cotton mill manufactures 637 yards of cloth in one day, how many yards will it make in 307 days

3. At 125 dollars each what will 49 horses cost?

4. A merchant bought 29 pieces of cloth; in each piece there were 57 yards; how many yards were there in the whole?

5. If 19003 pounds of hay are required for the horses of a cavalry regiment for one day, how many pounds will be needed for 206 days?

6. What would be the cost of constructing 309 miles of plank road, at 3975 dollars a mile?

7. How many apples will an orchard containing 208 trees produce, if the average yield is 1269 apples for each tree?

8. In 3 editions of 750 books each, how many pages are there, if each book contains 407 pages?

9. How many yards of sheeting are there in 57 bales, each bale containing 25 pieces and each piece 43 yards?

10. In a cotton mill there are 29 looms; each loom can weave 42 yards daily. At this rate, how many yards can be woven in 159 days?

11. A lot cost 420 dollars; how much will 105 lots cost at the same rate?

12. A drover has 406 cows worth 30 dollars each; how much are they all worth?

13. How much will it cost to build 307 miles of railroad at 4060 dollars a mile?

14. A contractor built 604 miles of railroad at 6500 dollars a mile; how much did he get for it?

15. If it requires 720 barrels of provisions to supply an army for one day, how many barrels will be required for 365 days?

16. If one acre of land costs 9620 dollars, how much will 736 acres cost?

17. If it costs 98650 dollars to build one mile of railroad, how much will it cost to build 2809 miles?

18. There are 15 fields of corn; in each field there are 97 rows, and 256 hills in each row; how many hills are there in the 15 fields?

19. How many yards of cloth are there in 43 bales, each bale containing 72 pieces, and each piece 29 yards?

20. If a railway train goes 18 miles an hour, how far will it go in 17 days of 24 hours each?

Exercise xxiii.

Practical Problems involving the Previous Rules.

1. B bought a house for 2960 dollars, and gave for it 98 cows at 24 dollars each, and the rest in money; how much money did he pay?

2. One army contains 4575 men, and another 36 times as many, lacking 1936 men; how many men are there in the second army?

3. Mr. Peters has 2461 gallons of coal oil, Mr. Martin has 1146 gallons, and Mr. Benson has 147 times as much as both; how much has Mr. Benson?

4. A farmer sold 129 cows at 37 dollars each, and re-

ceived in payment 2000 dollars; how much yet remains due?

5. B sold 76 hens at 73 cents each, 96 turkeys at 324 cents each, and received in payment 24000 cents; how much remains due?

6. A's barn cost 2485 dollars, his house cost 3 times as much, and his farm cost as much as both; what was the cost of the house? what was the cost of the farm?

7. A drover bought 36 horses at 145 dollars a head, and 96 cows at 28 dollars a head; which cost the most, and how much?

8. A's book contains 248 pages, with 2850 letters on a page, and B's contains 325 pages, with 3465 letters on a page; how many letters in A's book? how many in B's?

9 A man has 75 bags of apples, each bag containing 2 bushels; how much will he receive for them, at 125 cents a bushel?

10. A farmer sold 25 firkins of butter, each firkin containing 126 pounds, and received for each pound 37 cents; how much did he receive for it all?

11. Find the product of the sum and difference of 784 and 397.

12. If 472 men cut 800 cords of wood in two days, how long would it take one man to do it?

13. A farmer sold 129 cows at 29 dollars each, and received in payment 2300 dollars; how much yet remains due?

14. A s barn cost 175 dollars; his house cost 16 times as much, and his farm cost as much as both; what was the cost of the house? what was the cost of the farm?

15 A man bought 56 acres of land at 45 dollars an acre, and 78 acres at 62 dollars an acre, and sold the whole at 53 dollars an acre. Did he gain or lose, and how much?

16. A merchant bought 1600 barrels of flour at 7 dollars a barrel; he sold 900 barrels at 12 dollars a barrel; and the remainder at 5 dollars a barrel. Did he gain or lose, and how much?

17. If a house is worth 3250 dollars, and the farm on which it stands 3 times as much and 450 dollars more, and the stock on the farm twice as much as the house lacking 2368 dollars; what is the value of the whole?

18. A has 4278 dollars more than B, and 1225 dollars less than C, who has 7864 dollars; and D has as much as A and B together. How much has D?

19. A man invests in trade 450 dollars at one time, at another 840 dollars, at another 1125 dollars, and at another 1640 dollars; how much must be added to these sums that the amount invested by him shall be increased three fold?

20. A man sold his house for 4500 dollars, and 250 acres of land at 75 dollars an acre; he got in payment 5000 dollars in cash, 239 cattle at 25 dollars each, and 317 sheep at 5 dollars each; how much is still due him?

Section IV.—Division.

1. John has 9 apples which he wishes to divide equally among his 3 brothers; how many apples can he give to each?

Here we are required to divide 9 apples into 3 equal parts. If John gives each brother one apple, it will require 3 apples, and 6 apples would be left. If, now, he gives each of them another apple, it will require 3 more apples, and 3 apples would be left. If he gives them one apiece a *third* time there would be none left. Hence, it is plain that he can give each of his brothers 3 apples.	9 apples. 3 — 6 1st remainder. 3 — 3 2nd remainder. 3 — 0 3rd remainder.	

In this example we see that 9 contains 3 *three* times, for if we subtract 3 from 9 *three* times there is no remainder. A number, therefore, may be divided into equal parts by subtraction.

Hence, we see that Division is simply a short method of performing several successive subtractions of the same number.

We might have obtained the result in a shorter way, as follows: Since 3 times 3 is 9, we see that 3 is contained in 9 *three* times.

Hence, *to find how many times one number is contained in a second, we have merely to find what number multiplied by the first will produce the second.*

2. How many times 2 horses are 6 horses?
3. How many times 3 cents are 12 cents?
4. How many times is 5 contained in 15?
   Since 3 times 5 is 15, 5 is contained 3 times in 15
5. How many times is 6 contained in 30?

6. How many times 6 boys are 30 boys?

7. Three dogs have 12 feet; how many feet has 1 dog?

8. How many times 4 feet are 12 feet?

9. A bush has 8 roses; how many times 2 roses has it? How many times 4 roses?

10. How many times 9 boys are 27 boys?

11. A house has 12 doors; how many times 3 doors has it?

12. How many times 7 horses are 21 horses?

13. How many times is 7 contained in 28?

14. How many times is 4 contained in 20?

15. How many times is 5 contained in 30?

48. When it is required to find how many times one number contains another the process is called **Division.**

49. The number to be divided is called the **Dividend.**

50. The number by which we divide is called the **Divisor.**

51. The number of times the Divisor is contained in the Dividend is called the **Quotient.**

52. When the Divisor does not go an *exact* number of times into the Dividend, the *excess* is called the **Remainder.**

53. The remainder, being part of the Dividend, will always be of the same kind or denomination as the Dividend.

54. The **Sign of Division** is a short horizontal line, with a dot above it and another below it; thus, ÷. It shows that the number *before* it is to be divided by the number *after* it. Thus $8 \div 2 = 4$ is read, 8 divided by 2 is equal to 4.

55 Division frequently indicated by a line, with the divided it and the divisor below it; thus signifies to be divided by 3.

56 Division be divided into two cases:

1. *When the divisor does not exceed twelve.*
2. *When the divisor exceeds twelve.*

DIVISION TABLE.

1 in			2 in			3 in			4 in		
1	1	time	2	1	time	3	1	time	4	1	time
2	2	times	4	2	times	6	2	times	8	2	times
3	3	"	6	3	"	9	3	"	12	3	"
4	4	"	8	4	"	12	4	"	16	4	"
5	5	"	10	5	"	15	5	"	20	5	"
6	6	"	12	6	"	18	6	"	24	6	"
7	7	"	14	7	"	21	7	"	28	7	"
8	8	"	16	8	"	24	8	"	32	8	"
9	9	"	18	9	"	27	9	"	36	9	"
10	10	"	20	10	"	30	10	"	40	10	"
11	11	"	22	11	"	33	11	"	44	11	"
12	12	"	24	12	"	36	12	"	48	12	"

5 in			6 in			7 in			8 in		
5	1	time	6	1	time	7	1	time	8	1	time
10	2	times	12	2	times	14	2	times	16	2	times
15	3	"	18	3	"	21	3	"	24	3	"
20	4	"	24	4	"	28	4	"	32	4	"
25	5	"	30	5	"	35	5	"	40	5	"
30	6	"	36	6	"	42	6	"	48	6	"
35	7	"	42	7	"	49	7	"	56	7	"
40	8	"	48	8	"	56	8	"	64	8	"
45	9	"	54	9	"	63	9	"	72	9	"
50	10	"	60	10	"	70	10	"	80	10	"
55	11	"	66	11	"	77	11	"	88	11	"
60	12	"	72	12	"	84	12	"	96	12	"

9 in			10 in			11 in			12 in		
9	1	time	10	1	time	11	1	time	12	1	time
18	2	times	20	2	times	22	2	times	24	2	times
27	3	"	30	3	"	33	3	"	36	3	"
36	4	"	40	4	"	44	4	"	48	4	"
45	5	"	50	5	"	55	5	"	60	5	"
54	6	"	60	6	"	66	6	"	72	6	"
63	7	"	70	7	"	77	7	"	84	7	"
72	8	"	80	8	"	88	8	"	96	8	"
81	9	"	90	9	"	99	9	"	108	9	"
90	10	"	100	10	"	110	10	"	120	10	"
99	11	"	110	11	"	121	11	"	132	11	"
108	12	"	120	12	"	132	12	"	144	12	"

Oral Exercises.

1. 36 is how many times 4? How many times 12?
2. How many times 7 is 28? Is 42? Is 84? Is 35?
3. How many times 9 in 27? In 45? In 63? In 99?
4. A farmer received 8 dollars for 2 sheep; what was the price of each?

Since he received 8 dollars for 2 sheep, for 1 sheep he must get as many dollars as the number of times 2 is contained in 8. 2 is contained 4 times in 8, because 4 times 2 is 8; hence 4 dollars was the price of each sheep.

5. If a man walks 24 miles in 6 hours, how far will he walk in 1 hour?

6. If 1 man can do a piece of work in 32 days, how long will it take 8 men to do it?

7. If 7 yards of silk can be got for 21 dollars, how much will 1 yard cost?

8. If 27 yards of cloth can be bought for 3 dollars, how many yards can be bought for 1 dollar.

9. If 3 hats cost 9 dollars, how much will 1 hat cost? How much will 7 cost? How much will 12 cost?

10. How many times 5 oranges are 50 oranges? Is the result a concrete number, or an abstract number?

11. If you can buy a lead pencil for 3 cents, how many can you buy for 24 cents?

12. How many barrels of apples, at 2 dollars a barrel, can be bought for 24 dollars?

13. If a man walks 3 miles an hour, how many hours will it take him to walk 18 miles?

14. A farmer divides 84 bushels of apples equally among 12 men; how many bushels does each receive?

15. 72 cents are paid for 12 eggs; how much will 1 cost at the same rate?

16. How long will it take 12 men to perform a piece of work that 1 man can do in 60 days?

17. A man planted an orchard of 120 trees and put 10 in each row; how many rows are there in the orchard?

18. How many men at 9 dollars a month can be hired 1 month for 81 dollars?

19. If 6 barrels of flour cost 54 dollars, how much will 1 barrel cost?

Case I.

57. When the divisor does not exceed Twelve.

Ex. I. How many times is 7 contained in 952?

Divisor.	*Dividend.*	*Quotient.*
7)	952	(136
	7	
	25	
	21	
	42	
	42	

We write the Divisor at the left, and the Quotient at the right of the Dividend, and begin at the left to divide. 7 is contained in 9 hundreds 1 hundred times and a remainder. We write the 1 hundred in the Quotient, and multiply the Divisor 7 by the 1 hundred. This gives us 7 hundreds, which we write under the hundreds of the Dividend. We then subtract the 7 hundreds from the 9 hundreds and the remainder is 2 hundreds, or 20 tens. We add the 5 tens of the Dividend to these 20 tens and set down the 25 tens. 7 is contained in 25 tens 3 tens times, and a remainder. We write the 3 tens in the Quotient and multiply the Divisor by the 3. This gives 21 tens, which we write under the *partial* Dividend, 25 tens. We subtract, and the remainder is 4 tens or 40 units. We add the 2 units of the Dividend to these 40 units and set down the 42 units. 7 is contained in 42 units 6 units times. We write the 6 units in the Quotient and multiply the Divisor by the 6. This gives us 42 units, which we subtract as before, and there is no remainder.

The working of the preceding example may be shortened as follows:—

Divisor 7)952 Dividend.
136 Quotient.

We write the Divisor to the left of the Dividend and proceed as follows:—

7 is contained in 9, 1 time and 2 over. We place the 2 before the 5 and thus make 25. 7 is contained in 25, 3 times and 4 over. We place this 4 before 2 and thus make 42. 7 is contained in 42, 6 times.

When the *Divisor does not exceed* 12 the multiplication and subtraction are performed mentally, the quotient *only* being written down, the work being thus greatly shortened. This is called **Short Division.**

When all the different steps of the solution are written, the process is called **Long Division.**

Exercise xxiv.

(1) 2)86(	(2) 2)58(	(3) 2)54(	(4) 2)92(	(5) 2)96(
(6) 3)576(	(7) 3)465(	(8) 3)723(	(9) 3)873(	(10) 3)975(
(11) 4)852(	(12) 4)764(	(13) 4)932(	(14) 4)576(	(15) 4)748(
(16) 5)735(	(17) 5)850(	(18) 5)975(	(19) 5)745(	(20) 5)835(
(21) 6)732(	(22) 6)846(	(23) 6)924(	(24) 6)972(	(25) 6)834(
(26) 7)784(	(27) 7)798(	(28) 7)833(	(29) 7)966(	(30) 7)959(
(31) 8)896(	(32) 8)936(	(33) 8)944(	(34) 8)976(	(35) 8)992(
(36) 9)468(	(37) 9)576(	(38) 9)864(	(39) 9)738(	(40) 9)666(

Exercise xxv.

(1) 2)456	(2) 2)736	(3) 2)548	(4) 2)574	(5) 2)538
(6) 3)735	(7) 3)816	(8) 3)522	(9) 3)414	(10) 3)738
(11) 3)669	(12) 3)513	(13) 3)546	(14) 3)765	(15) 3)825
(16) 4)512	(17) 4)624	(18) 4)732	(19) 4)576	(20) 4)824
(21) 4)736	(22) 4)816	(23) 4)972	(24) 4)608	(25) 4)436
(26) 5)615	(27) 5(735	(28) 5)645	(29) 5)785	(30) 5)840

The pupil is expected to work Exercise xxiv., first by Long Division, and next by Short Division.

(31) 5)815	(32) 5)935	(33) 5)780	(34) 5)765	(35) 5)880
(36) 6)834	(37) 6)648	(38) 6)654	(39) 6)774	(40) 6)864
(41) 6)1476	(42) 6)3336	(43) 6)2514	(44) 6)3654	(45) 6)7338
(46) 7)2569	(47) 7)4732	(48) 7)8456	(49) 7)9359	(50) 7)9870
(51) 8)7256	(52) 8)3656	(53) 8)7576	(54) 8)29352	(55) 8)111032
(56) 9)8892	(57) 9)3978	(58) 9)2565	(59) 9)63288	(60) 9)67356

Exercise xxvi.

Practical Problems.

1. At 6 cents each, how many oranges can be bought for 354 cents?

2. At 2 dollars a day, how many days' work can I hire for 346 dollars?

3. How many pounds of rice at 4 cents a pound can I buy for 3672 cents?

4. In 3 feet there is 1 yard; how many yards are there in 693 feet?

5. If 8 men can dig 768 rods of ditch in 3 weeks, how many rods can 1 man dig in the same time?

6. If 7 yards of cloth cost 637 cents, what will 1 yard cost?

7. If 9 men can dig 135 bushels of potatoes in 1 day, how many bushels can 1 man dig in 1 day?

8. When 7 is multiplied by a certain number the product is 861, what is the number?

9. If 6 bins of equal size are exactly filled by 36312 bushels of grain, how much does each bin hold?

10. If 7 men can cut 56 cords of wood in 4 days how much can 1 man cut in the same time?

Ex. 2. Divide 70268 by 7

Divisor 7) 70268 Dividend.

10038 Quotient. 2 Remainder.

In this example we say 7 is contained in 7 ten thousands, 1 ten thousand times and no remainder. We put down this 1 in the ten thousands' place. 7 is not contained in 0 thousands. We put a 0 in the thousands' place. 7 is not contained in 2 hundreds. We again write a 0 in the hundreds' place. 7 is contained in 26 tens, 3 tens times and 5 over. We write the 3 in the tens' place. 7 is contained 8 times in 58 units and 2 over. We write 8 in the units' place and indicate the division of the 2, thus, $\frac{2}{7}$; this is annexed to 10038, thus, $10038\frac{2}{7}$.

58. *PROOF.*—Multiply the Quotient by the Divisor, and to the Product add the Remainder, if any, and if the result is the same as the Dividend the work is likely to be correct.

Exercise xxvii.

Divide

1. 6532 by 3.
2. 11236 by 9.
3. 57636 by 6.
4. 11485 by 7.
5. 98537 by 8.
6. 345246 by 5.
7. 1680245 by 4.
8. 3432026 by 6.
9. 6216563 by 8.
10. 7295849 by 10.
11. 16779120 by 12
12. 37000305 by 5.
13. 5767692 by 7.
14. 56464237 by 9.
15. 46626289 by 11.
16. 2523360 by 6.
17. 160590736 by 8.
18. 370370480 by 10.
19. 101650247 by 12.
20. 51088982 by 7.
21. 67320837 by 9.
22. 30040526 by 11.
23. 106131923 by 12.
24. 740048200 by 8.
25. 45603875 by 10
26. 336384072 by 9

Exercise xxviii.

Practical Problems.

1. When flour is worth 8 dollars a barrel, how many barrels could be bought for 3456 dollars?

2. If 7 casks of sugar weigh 8792 pounds, what is the average weight of each cask?

3. A father dying left an estate of 37356 dollars to be divided equally among his wife, his two sons and his three daughters; what was the share of each?

4. Five men bought a horse for 160 dollars; they hire him out at 4 dollars a day for 24 days, and sell him for 120 dollars; how much will each one gain?

5. A grocer bought 15 barrels of flour for 100 dollars; he sold it so as to gain 20 dollars; how much did he receive per barrel?

6. How long will it take two boys, starting at the same place, and travelling in opposite directions, to be 29076 rods apart, if one goes 5 and the other 7 rods in a minute?

7. If 66 apples are divided equally among 5 boys, how many does each boy receive?

In performing this division we see that each boy receives 13 *whole* apples, and that there is *one* apple left. This apple, being part of the Dividend, is also to be divided among the 5 boys, but when anything is divided into *five* equal parts *one* of the parts is called *one-fifth* and is written $\frac{1}{5}$. Each boy will, therefore, receive $13\frac{1}{5}$ apples.

5) 66 Dividend.
$13\frac{1}{5}$ Quotient.

8. If 4 sacks of coffee weigh 523 pounds; what is the weight of each?

9. If 626 dollars are divided equally among 5 men, what will be the share of each?

10. In one week there are 7 days; how many weeks are there in 365 days?

11. John, James and William have altogether 756 marbles, which they wish to divide equally; what will be the share of each?

12. A man has 4 equal lots of land, containing in all 2759 acres; how many acres are there in each lot?

13. If 9 car-loads of freight weigh 141712 pounds, what is the weight in each car-load?

14. If 8 waggons carry 4384 bricks, how many bricks can be carried on one waggon?

Case II.

50. When the Divisor is greater than Twelve.

Ex. 3. Divide 4839 by 17

Divisor.	*Dividend.*	*Quotient.*	
17)	4839	(284	* 1— 17
	34		2— 34
	143		3— 51
	136		4— 68
	79		5— 85
	68		6—102
	11 Remainder.		7—119
			8—136
			9—153

Since 17 is not contained in 4 thousands any thousand times, we unite the 4 thousands to the 8 hundreds, making 48 hundreds; 17 is contained in 48 hundreds 2 hundred times. We set down 2 as the first figure in the quotient, then multiply 17 by 2, and subtract the product 34 from 48. The remainder is 14. To this remainder we annex the 3 tens of the dividend, making 143 tens; 17 is contained in 143 tens 8 tens times. We set down 8 as the next figure of the quotient, then multiply 17 by 8, and subtract the product, 136 from 143. The remainder is 7; to this remainder annex the next figure of the dividend and continue as before.

Ex. 4. Divide 74198 by 37.

37)	74198	(2005
	74	
	198	
	185	
	13	

In this example we find there is no remainder on subtracting 74 from 74, and on bringing down 1, the third figure of the dividend, 37 is not contained in it; we therefore write 0 as the second figure of the quotient. When we bring down 9, the next figure of the dividend, 37 is not contained in 19; we therefore write another 0 as the third figure of the quotient. When we bring down 8, the last figure of the dividend, 37 is contained in 198, 5 times, and we go on as before.

Note.—*For every figure of the dividend brought down* one figure *must be written in the quotient.*

* Let the pupil, before commencing the operation of dividing, construct a table by multiplying the divisor by each number successively up to 9 in the manner indicated in the example. The proper quotient will then be apparent on inspection.

The proof is the same as in *Short Division*.

2095 Quotient.
37 Divisor.

14035
6015

74185 Product.
13 Remainder.

74198 Dividend.

Exercise xxix.

Divide

1. 764 by 31.
2. 367 by 41.
3. 987 by 53.
4. 4567 by 61.
5. 2936 by 74.
6. 38271 by 65.
7. 29781 by 56.
8. 71847 by 76.
9. 67654 by 122.
10. 39298 by 801.
11. 80157 by 346.
12. 466281 by 9[illegible].
13. 159750 by 4[illegible].
14. 539902 by 239.
15. 999999 by 198.
16. 2802690 by 990.
17. 8991207 by 1449.
18. 9672160 by 1560.
19. 6192138 by 1653.
20. 3515772 by 1736.
21. 9876480 by 1976.
22. 24197460 by 2492.
23. 8231505 by 1905.
24. 13896225 by 2975.
25. 16084440 by 5058.
26. 23103465 by 6391.
27. 18356508 by 16074.
28. 572105376 by 78617.
29. 344943192 by 134376
30. 1806147420 by 85805.

Exercise xxx.

Practical Problems.

1. There are 24 hours in a day; how many days are there in 1032 hours?
2. If a man walks 25 miles in a day, how long will it take him to walk 950 miles?
3. Sound moves 37060 feet in 34 seconds; how far will it move in 1 second?
4. A drover bought 23 head of cattle for 736 dollars; what was the price per head?
5. In 1 year there are 52 weeks; how many years are there in 6708 weeks?
6. If 75 shares of bank stock sell for 9225 dollars, what is the price per share?

7. A man bought a farm of 524 acres for 24104 dollars, what was the average price per acre?

8. How many bales could be made out of 281765 pounds of cotton, allowing 517 pounds to the bale?

9. If a steamship sails 5836 miles in 17 days, what would be the average daily distance?

10. A flour barrel holds 196 pounds of flour; how many barrels will it take to hold 406700 pounds?

ABBREVIATED PROCESSES IN LONG DIVISION

Case I.

60. To divide by a composite number.

61. A Composite Number is one which may be produced by multiplying together two or more numbers, neither of which is 1. Since $16 = 8 \times 2$, 16 is a *composite* number.

Ex. 5. Divide 8769 by 42.

7)8769

6)1252 and 5 units over= 5.

208 " 4 sevens over=28.

Remainder=33.

Since the factors of 42 are 7 and 6 we divide by these factors in succession. First, dividing by seven we obtain 1252 for quotient and 5 for remainder. This is 5 units. We then divide the quotient by six and obtain 208 for quotient and 4 for remainder. This is 4 groups of 7 units each or 28 units. The remainder is, therefore, 28 units + 5 units=33 units.

Hence, **The true remainder is found by multiplying the last remainder by the first divisor and adding to the product the first remainder.**

Exercise xxxi.

Divide

1. 46827 by 27.
2. 87468 by 64.
3. 97648 by 63.
4. 13853 by 45.
5. 8642896 by 35.
6. 7808216 by 49.
7. 8742 by 25.
8. 76842 by 96.
9. 87648 by 81.
10. 419421 by 99.
11. 339240 by 132.
12. 806345 by 144.

Case II.

62. To Divide when there are ciphers at the right of the Divisor.

Ex. 6. Divide 8593 by 700.

The divisor, 700, may be resolved into the factors 7 and 100. We first divide by the factor 100 by cutting off two figures at the right, and get 85 for the quotient and 93 for the remainder. We then divide the quotient, 85, by the other factor, 7, and obtain 12 for the quotient and 1 for a remainder. The last remainder, 1, being multiplied by the divisor, 100, and 93, the first remainder, added, we obtain 193 for the true remainder.

7,00)85,93

12 and 193 rem.

Hence, *To divide, when there are ciphers at the right of the divisor, we cut off the ciphers from the divisor, and the same number of figures from the right of the dividend; we then divide the remaining figures of the dividend by the remaining figures of the divisor and prefix the remainder to the figures cut off, and the result will be the true remainder.*

Exercise xxxii.

Divide

1. 725 by 30.
2. 7642 by 60.
3. 8642 by 700.
4. 97861 by 300.
5. 72369 by 90.
6. 94678 by 80.
7. 3786 by 1700.
8. 21500 by 3600.
9. 378751 by 12300.
10. 984721 by 6400.
11. 1684273 by 2500.
12. 9486279 by 15000.

Exercise xxxiii.

Practical Problems.

1. In a yard there are 36 inches; how many yards are there in 3888 inches?
2. There are 60 minutes in an hour; how many hours are there in 3900 minutes?
3. There are 16 ounces in a pound; how many pounds are there in 1968 ounces?
4. How many pounds of beef at 18 cents a pound can be bought for 540 cents?
5. There are 64 pints in a bushel; how many bushels are there in 2688 pints?

6. A farmer sold 24 horses for 5640 dollars; how much did he receive apiece for them?

7. There are 25 pounds in a quarter; how many quarters are there in 34450 pounds?

8. How many bushels of oats, at 56 cents a bushel, can be bought for 13272 cents?

9. If 48 acres of land produce 2064 bushels of corn, how much will be produced from one acre?

10. If a man travels 2052 miles in 54 days, what is the average travel per day?

63. If any *three* of the four numbers, that form the Divisor, Dividend, Quotient and Remainder be given, we can find the *fourth*.

1. Let Divisor, Dividend, and Quotient be given. Multiply the Divisor by the Quotient, subtract the result from the Dividend, and we have the Remainder.

2. Let Divisor, Quotient, and Remainder be given. Multiply the Divisor by the Quotient, add the Remainder to the result, and we have the Dividend.

3. Let Divisor, Dividend, and Remainder be given Subtract the Remainder from the Dividend, divide the result by the Divisor, and we have the Quotient.

4. Let Quotient, Dividend, and Remainder be given. Subtract the Remainder from the Dividend, divide the result by the Quotient, and we have the Divisor.

Exercise xxxiv.

1. What number divided by 75 will give a quotient of 117 and remainder of 39?

2. What number must be taken from 9765 so that it may be exactly divisible by 132?

3. Of what number is 483 both divisor and quotient?

4. What number larger than 216 will divide 75168 without a remainder?

5. What number must be added to 38472 so that it may be exactly divisible by 379?

6. The answer to a question in Multiplication is 1404336 and the multiplicand is 5163; what is the multiplier?

7. If the quotient be 5000 when the divisor is 2001 and the remainder 100, what is the dividend?

8. What number divided by 528 will give 36 for quotient and leave 44 as a remainder?

9. If the dividend be 784622 and the quotient be 4044, what is the divisor and the remainder?

10. If the quotient be 194, the divisor 4044, and the remainder 87, what is the dividend?

Exercise xxxv.

Practical Problems Involving the Previous Rules.

Ex. 1. A carpenter can earn 45 dollars a month, his expenses are at the rate of 24 dollars a month. He wishes to purchase a lot of ground which contains 19 acres, and is held at 42 dollars per acre; in what time may he save enough to make the purchase?

He saves $45 - 24 = 21$ dollars a month.

The lot will cost $19 \times 42 = 798$ dollars;

then the number of months in which he can save enough to purchase the lot is $798 \div 21 = 38$ months.

1. A farmer bought land from A at 60 dollars an acre, and the same quantity from B at 85 dollars an acre. The whole amounted to 53215 dollars: how many acres did he buy from each?

2. A merchant sold a piece of cloth containing 45 yards, another piece containing 57 yards, and another containing 63 yards, at 14 dollars a yard; what did the whole amount to?

3. A man left 2535 dollars each to his four children, but one of them dying the three remaining children divided the money equally among them; how much did each receive?

4. A man earns 25 dollars a week, and spends 12 dollars a week: he saves 195 dollars: how many weeks does he work?

5. A farmer has 24 cows and 93 sheep, worth 1521 dollars; if the sheep are worth 5 dollars each, how much is each cow worth?

6. If 29 men earn 7946 cents in a day, and 25 boys earn 5450 cents in a day, how much more does one man earn in a day than one boy?

7. How many barrels of flour at 6 dollars a barrel are equal in value to 1100 tons of coal at 9 dollars a ton?

8. If a mechanic earns 52 dollars a month, and his expenses are 34 dollars a month, how long will it take him to pay for a farm of 36 acres, worth 12 dollars an acre?

9. A clerk's salary is 1200 dollars a year; he pays 5 dollars a week for board, 2 dollars a month for car fare, and his other expenses amount to 1 dollar a day; how much can he save in a year?

10. Mr. Jones bought a farm of 100 acres at 75 dollars an acre, 2200 dollars to be paid down, and the remainder in five equal yearly instalments; what must he pay each year?

11. A man has 13 piles of wood, each containing 25 cords, and each cord 128 cubic feet; how many cubic feet of wood has he?

12. A man exchanges 159 cords of wood at 5 dollars a cord, for a horse valued at 144 dollars, and the balance in sheep at 3 dollars each; how many sheep did he receive?

13. A merchant balancing his accounts found that he had on hand merchandise worth 475 dollars, and cash amounting to 2570 dollars; he had lost by bad debts 250 dollars, and owed 525 dollars; if his original capital was 2000 dollars what had he gained?

14. A cistern containing 13500 gallons is filled by two pipes, one discharging 250 gallons an hour, and the other 300 gallons, but, by a leak in one of the pipes, 100 gallons are lost in an hour; if the cistern is empty, how long will it take to fill it?

Ex. 2. If 3 pounds of coffee cost 30 cents, what will 8 pounds cost?

The cost of 3 pounds of coffee = 30 cents;

" 1 pound " $= \frac{30}{3} = 10$ cents;

" 8 pounds " $= 8 \times 10$ cents $= 80$ cents.

15. What will 15 slates cost, if 5 slates cost 80 cents?

16. If 4 trees cost 72 cents, what will 3 trees cost?

17. If 6 barrels of flour cost 48 dollars, what will 7 barrels cost?

18. What will be the cost of 13 cords of wood, if 4 cords cost 24 dollars?

19. If 15 yards of cloth cost 75 dollars, what will 20 yards cost?

20. If 7 pounds of beef cost 56 cents, what will 5 pounds cost?

21. If 12 men can earn 36 dollars in a day, how much can 4 men earn in the same time?

22. If 28 acres of land cost 4480 dollars, how much will 43 acres cost at the same rate?

23. In 52 years there are 18980 days, how many days are there in 65 years?

24. Twenty-five barrels of flour weigh 4900 pounds; what is the weight of 36 barrels?

25. If you can buy 765 yards of cloth for 51 dollars, how many yards can you get for 376 dollars?

26. If 38 acres of land cost 11172 dollars, how many acres can be bought for 107310 dollars?

27. If 13 houses cost 16250 dollars, what will 25 houses cost?

28. If 17 horses cost 1802 dollars, how much will 9 horses cost?

29. Fifteen men can husk 1095 bushels of corn in a day, how many bushels can 27 husk?

Ex. 3. If 7 men do a piece of work in 36 days, in how many days can 28 men do it?

Time for 7 men to do the work = 36 days;

" 1 man " " $= 7 \times 36$ days;

" 28 men " " $= \frac{7 \times 36}{28} = 9$ days.

30. If 15 workmen can do a piece of work in 25 days, in what time can 25 men do the same?

31. A field can be mowed by 40 men in 9 days, in how many days would it be finished by 30 men?

32. If 16 men can build a house in 20 days, how long would it take 10 men to build it?

33. If 19 men can finish a work in 437 days, how long will it take 23 men to do the same work?

34. If 18 horses can cart away the earth from a cellar in 75 days, in how many days would 27 horses do this work?

35. Ten men engage to build a house in 63 days, but 3 of them being taken sick, how long will it take the rest to build the house?

36. If 6 carpenters can build a house in 72 days, how long would it take 9 carpenters to build the same?

37. How long will it take 40 men to build a wall, if 12 men can do it in 20 days?

38. How long will it take 9 men to do the same amount of work that 6 men can do in 15 days?

39. How long will 19 men take to do a piece of work which requires 17 men 133 days to do?

Ex. 4. If 30 men build a wall in 18 days, how many men will be required to do it in 12 days?

Men required to build the wall in 18 days=30 men ;
" " " " 1 day $=18 \times 30$ men ,
" " " " 12 days$=\frac{18\times30}{12}=45$ men.

40. If 4 men can dig a garden in 7 days, how many men would be required to dig it in 1 day ?

41. If 28 men can mow a field of grass in 12 days, how many men will be required to mow it in 4 days ?

42. If 7 men can reap a field of wheat in 18 days, how many men would be required to do the same work in 6 days ?

43. A piece of work was to have been performed by 144 men in 36 days, but a number of them having been discharged the work was performed in 48 days ; how many men worked ?

44. If 20 men can perform a piece of work in 15 days, how many men will it take to do it in 12 days ?

45. How many men in 26 days can perform the same amount of work that 39 men can do in 76 days ?

46. A drain is dug by 49 men in 96 days ; how many men would have been required to dig it in 84 days ?

47 If 8 workmen can build a wall in 27 days, how many workmen would be required to build it in 3 days ?

48. If 100 workmen can perform a piece of work in 12 ays, how many men are sufficient to perform the work in days ?

49. A gentleman met a number of beggars, and relieved 9 of them by giving 25 cents to each one; how many would he have relieved for the same sum had he given them only 15 cents apiece ?

EXAMINATION PAPERS.

I.

1. Define the following terms: Unit, Number, Notation, and Numeration.

2. Add together four millions twenty thousand and seventy-nine, twelve millions two thousands and seven, and one million and five thousand, and subtract 16538107 from the sum.

3. Find the remainder after subtracting the numbers 4444, 9999, 666, 77, 1, in succession from 1000000.

4. Add together the sum, difference, product, and quotient of the two numbers 825 and 9318375.

5. I bought a farm of 136 acres for 8568 dollars, and sold 93 acres of it at 75 dollars an acre, and the remainder for what it cost; how much did I gain by the bargain?

II.

1. Explain the meaning of the following terms, and give an example of each: Subtrahend, Multiplicand, Product, Divisor, Quotient, Remainder.

2. Find the sum of the following numbers, and express the result in words: 1234567, 8765433, 6894703, 3105297, 5712843, and 4187157.

3. What is the difference between the aggregate of 1050, 325, 1769, 150801, and a million. Show that the same difference is obtained by taking one of the numbers from a million, another from the remainder, and so on for the rest of the numbers.

4. Express MMDCXCIX. and CCCXXIX. in the ordinary numerical characters; find their product, and express the result in Roman characters.

5. How many bushels of wheat, at 125 cents per bushel, should be exchanged for 250 pounds of sugar, at 8 cents per pound?

III.

1. From 7503 take 871, and explain the process of "borrowing and carrying" in the common rule of subtraction.

2. How may the process of subtraction be verified? Give an example.

3. By how much does the sum of the numbers 27182818284 and 31415926535 exceed their difference?

4. What arithmetical operation bears the same relation to subtraction that multiplication bears to addition?

5. Bought a farm for 35380 dollars, and having made improvements valued at 3420 dollars, I sold one-half of it for 21750 dollars, at 75 dollars an acre; how many acres did I purchase, and at what price per acre?

IV.

1. What is the object of division? Show that it may be considered a shortened subtraction.

2. What are the *factors* of a number?

3. If division by a composite number be performed by successively dividing by its factors, show how the complete remainder may be found. Ex. 1437281 divided by 105.

4. How much can a man earn in 114 days, if he can earn 48 dollars in 24 days?

5. A man bought a number of sheep at the rate of 3 for 18 dollars: how many did he buy for 3648 dollars?

V

1. What is multiplication? Show by an example that it is a short method of performing addition.

2. Show by an example that two or more factors will give the same product in whatever order they are multiplied

3. How many times must 1874 be added to itself to make a total of 163038?

4. The product of 75 by 43 is 3225; how much must be added to it to obtain the product of 77 by 43?

5. A drover bought 79 oxen at 42 dollars each; he sold 25 at 40 dollars each; for how much per head must he sell the rest so as to gain 1544 dollars on the whole transaction?

VI.

1. Given the divisor, quotient and remainder, how is the dividend found?

2. I bought a farm of 150 acres for 12000 dollars; I sold 29 acres at 95 dollars an acre, 75 at 112 dollars an acre, and the rest at 96 dollars an acre; what did I gain by this transaction?

3. What number is that, which being multiplied by 15, the product divided by 16, the quotient multiplied by 7, 35 subtracted from the product, the remainder divided by ten, and 52 subtracted from the quotient, the remainder is 18?

4. I bought a farm for 6480 dollars, and after spending 890 dollars on improvements on it, I sold one half of it for 4050 dollars at 45 dollars an acre; how many acres did I buy, and at what price per acre?

5. If 16 men can perform a piece of work in 36 days, in how many days can they do it with the aid of 8 more men?

VII.

1. Explain why in addition of numbers the operation is begun at the units' place. Is this necessary? Illustrate by an example.

2. A person willed his property to his three children. To the youngest he gave 2149 dollars; to the second 3 times as much; and to the eldest 5 times as much as to the second; find the value of the property

3. Two ships 3120 miles apart approach each other, the one sailing at the rate of 146 miles a day, and the other at 127 miles; how far will they be apart at the end of 9 days?

4. John found a bagful of coins. On counting them he found there were 5 cent pieces, 10 cent pieces, and 20 cent pieces in it, and the same number of each; how many of each were there, if the whole amounted to 8645 cents?

5. A gentleman dying disposed of his property worth 53175 dollars, as follows: he left 1500 dollars to a church; 4 times this sum to a college; and he divided the remainder equally among his 5 sons and 2 daughters; what was the share of each child?

VIII.

1. If a man has a salary of 2400 dollars a year, and spends 4 dollars a day, how much will he save in 5 years, allowing 365 days in a year?

2. What number must be taken 708 times from 688953 so as to leave 69 for remainder?

3. A drover bought 12 head of cattle at 22 dollars each; 9 head at 25 dollars each; and 4 head at 32 dollars each; at how much per head must he sell them so as to gain 158 dollars?

4. Three boys go picking berries and agree to divide the proceeds equally; the first picks 15 quarts and sells them at 13 cents a quart; the second picks 16 quarts and sells them at 12 cents a quart; and the third picks 12 quarts and sells them at 18 cents a quart; find what each one gets.

5. Two travellers, *A* and *B*, meeting on a journey, found that the whole distance both had travelled was 2145 miles, and that *B* had gone 217 miles further than *A*; how far had each travelled?

CHAPTER II.

Canadian Money.

64. **Canadian Money** is the legal currency of the Dominion of Canada. It is composed of *dollars*, *cents* and *mills*. The **dollar** is the unit, and is denoted by the symbol $.

65. 10 mills = 1 cent.
100 cents = $1.

66. Dollars are separated from cents, in writing, by a *point*. Thus $6.75 is read six dollars and seventy-five cents. Any number of cents less than ten, when written with dollars, occupies the second place to the right of the *point*, and the first place is occupied by a cipher; thus, $4.05 is read four dollars and five cents. The *mill* is one-tenth of a cent and is written one place to the right of the cents thus, $3.755 is read 3 dollars, 75 cents and 5 mills.

67. The present silver coins of the Dominion are the fifty-cent piece, the twenty-five-cent piece, the ten-cent piece, and the five-cent piece. The only copper coin is the one-cent piece.

NOTE.—The *mill* is not coined; it is used only in computation. When the final result of a business computation contains mills, if 5, or more, they are reckoned 1 cent, and if less than 5 they are rejected.

68. Since numbers expressing mills, cents and dollars increase from right to left in the same manner as the numbers with which we have been dealing, **they may be added, subtracted, multiplied, and divided in the same manner.**

Exercise xxxvi.

Read the following:

$1.15.	$14.25.	$21.50.	$.243.
$3.24.	$6.00.	$107.16.	$.803.
$11.17.	$18.05.	$107.60.	$8.03.
$19.30.	$25.07.	$100.70.	$6.003.

Write in figures:

1. Five dollars and twenty-five cents; eighty-seven dollars and forty cents.

2. Seventy dollars and sixty-seven cents; two dollars and four cents.

3. Ninety dollars and nine cents; one hundred and one dollars and ten cents.

4. One hundred and twenty-nine dollars and one cent; nine hundred dollars and ninety cents.

5. One thousand dollars; one thousand and seven dollars and three cents.

6. Five thousand three hundred dollars and forty-three cents.

7. Twenty-three thousand and five dollars; forty thousand dollars, fort cents and five mills.

8. Five thousand dollars and five cents; five hundred thousand and nine dollars and thirty-seven cents.

9. Four hundred and eighty thousand dollars; five hundred thousand five hundred dollars, fifty cents and seven mills.

10. One million dollars; one million, one thousand and one dollars, one cent and one mill.

Reduction.

Oral Exercise.

1 How many cents are there in $3?
2. How many cents are there in $2?
3. How many cents are there in $3.16?
4. How many cents are equal to a five-dollar bill?
5 How many cents are equal to a dollar bill and 25 cents?
6. How many cents are there in a half-dollar and a quarter-dollar?

7. How many cents are there in one dollar and a half?
8. How many ten-cent pieces are there in $4?
9. How many cents are equal to 2 five-dollar bills?
10. How many five-cent pieces are there in $2?

69. **Reduction** is the process of changing the denomination or name of a number without changing its value.

Ex. 1. How many cents are there in $3.29?

Since $1 = 100 cents;
$3 = 3 × 100 cents or 300 cents;
300 cents and 29 cents make 329 cents,
therefore, $3.29 = 329 cents.

Hence, *In reducing a number of dollars and cents to cents, we simply remove the point.*

Ex. 2. How many dollars are there in 6904 cents?

Since 100 cents = $1;
1 cent = $\frac{1}{100}$;
6904 cents = $\frac{6904}{100}$ = $69.04.

For when 6904 cents are divided by 100 the quotient is 69, and the remainder 4 cents.

Hence, *In reducing cents to dollars the point must be placed two places from the right.*

Exercise xxxvii.

Reduce to cents

1	$5;	$7.36;	$17.04.
2.	$29.18;	$141.36;	$200.09.
3.	$361.07;	$500.75;	$1000.10
4.	$1875.63;	$3647.29;	$76841.06.
5.	$20063.07;	$141368.79;	$10010010.01.

Reduce to dollars and cents

6	368 cents;	700 cents;	1236 cents.
7	3641 cents;	7008 cents;	910988 cents.
8	54168 cents;	500709 cents,	684007 cents
9.	300041 cents;	280014 cents;	340001 cents.
10	2000009 cents;	7010013 cents.	10000091 cents

Addition.

Oral Exercises.

1. A book cost $1.25, and a slate 50 cents; how much did they both cost?

2. A pound of tea cost $1, a pound of coffee 25 cents, and a ham $1.75; what was the total cost?

3. If I pay $1.20 for a turkey, $1.15 for a goose, and 60 cents for butter, how much do I pay for all?

4. Bought a pig for $6, a bag of flour for $4, and a cord of wood for $7.50; how much did I pay for all?

5. Paid 90 cents for paper, 10 cents for pins, and $1.25 for a book; how much did I pay for all?

6. A book costs 90 cents, a pen-holder 10 cents, and a slate 35 cents; how much do they all cost?

Ex. 3. Add together $7.37, $29.78, $0.29, $187.04 and $500.

$ 7.37
29.78
0.29
187.04
500.00
$724.48

As we must add things of the same kind, we write dollars under dollars and cents under cents, letting the points range in a straight line. Then regarding the dollars and cents as so many cents, we add as in simple numbers and place the point in the sum two places from the right to reduce the cents to dollars.

Exercise xxxviii.

(1)	(2)	(3)	(4)
$71.36	$184.36	$1843.21	$105.20
169.08	769.28	978.89	110.00
208.72	41.07	36.07	409.05
714.39	869.36	362.48	1000.65

5. A farmer receives $15.37 for a cow, $75 for a horse, $3 13 for some potatoes, and $5.55 for some poultry; how much does he receive in all?

6. Sold some velvet for $3.33, broadcloth for $13.75, silk for $12.50, muslin for $5.40, carpeting for $30.05, a shawl for $12.25; what is the amount of the bill?

7. If a house costs $3487.75; repairs, $53.37; painting, $119.23; furniture, $1563.39; moving, $9; what was the whole cost?

8. A lady gives 25 cents for needles, $17.50 for a dress, $2.63 for trimmings, $1.50 for a cap, and 12 cents for thread; how much does she lay out?

Subtraction.

Oral Exercises.

1. John bought a book for $1.50 and sold it for $1.75; how much did he gain?

2. A merchant bought goods for $4.75 and sold them for $6; how much did he gain?

3. John had $10; he paid $2.50 for some books, and $1.50 for a satchel; how much money has he left?

4. Mary had $1.25; she paid 75 cents for some ribbons, and 25 cents for car tickets; how much has she left?

5. Bought some rice for 60 cents, some sugar for 45 cents and some tea for $1; how much change should I get from a five-dollar bill?

6. Bought a horse for $120, a saddle for $15, and sold both for $150; what was my gain?

7. I bought a pound of rice for 8 cents, crackers for 15 cents, raisins for 18 cents, candy for 10 cents; how much change should I get back if I gave the clerk $1.00?

Ex. 4. John owes $137.35 and pays $29.17; how much does he still owe?

$137.35	Writing dollars under dollars and
29.17	cents under cents, we regard the dollars
	and cents as so many cents and sub
$108.18	tract as in simple numbers. We then

place the point two places from the right of the remainder to reduce the cents to dollars.

Exercise xxxix.

(1)	(2)	(3)	(4)
$104.36	$76.14	$200.00	$782.36
9.78	17.39	156.81	189.75

5. A man has $4000; he buys a house worth $1829.36; how much money has he remaining?

6. John has $17.21, James has $41.00; how much has James more than John?

7. My salary is $1600 a year; I pay for rent $150, for groceries $325.40, for butter $50.30, for dry goods $227.63, and for other expenses $75.60; how much do I save?

8. A man worth $10000 gave away $956.38, and lost $1127.82; what was he then worth?

9. If a lady gives 12 cents for ink, 63 cents for pens,

$13.30 for books, and $1.87 for paper; how much change must she get from a twenty-dollar bill?

10. Bought $75 worth of hay, and $25.25 worth of corn; paid $49.88; how much is still due?

11. I paid $4637.25 for a farm, $3675.25 for building a house, and $2896.87 for building a barn; I sold my property for $13000; how much did I gain?

12 I paid $246.75 for a horse, $325.45 for a mule, $42.25 for an ox, $37.50 for a cow; I sold them all for $603.50; what was the loss?

Multiplication.

Oral Exercises.

1. What will 10 pounds of fish cost at 12 cents a pound?

2. What will 3 pair of boots cost at $5.25 a pair?

3. If I earn $10.50 in 1 week, how much can I earn in 2 weeks?

4. Bought 2 hats at $1.25 each, and 3 collars at 25 cents each; how much did I pay for them?

5. Thomas earns 75 cents a day; his expenses are 52 cents a day: how much does he save in 7 days?

6. A man bought 4 bushels of wheat at $1.12, and sold the flour for $5; how much did he gain?

7. Bought 5 barrels of flour at $8.50 a barrel, and 6 bushels of wheat at $1.25 a bushel; what was the cost of both?

8. What is the cost of 2 pair of chickens at 75 cents a pair, and 5 pair of ducks at 60 cents a pair?

9. Bought 5 pounds of coffee at 35 cents a pound, and 12 pounds of ham at 22 cents a pound; how much change did I get from a five-dollar bill?

Ex. 5. Multiply $78.39 by 8.

$78.39	We regard the dollars and cents as
8	so many cents, and multiply as in
———	Simple Multiplication, and then we
$627.12	place the point two places from the

right of the product, to reduce the cents to dollars.

Exercise xl.

	(1)	(2)	(3)	(4)
Multiply	$78.37	$247.16	$48.75	$781.36
By	5	6	19	125

5. A farmer sold 175 acres of land at $37.50 an acre; how much did he get for the land?

6. A miller sold 525 barrels of flour at $6.71 a barrel; how much did he receive for all of it?

7. What will 42 calves cost at $3.75 each?

8. At 37 cents each, what will 75 geese cost?

9. What will 890 cords of wood cost at $3.78 a cord?

10. What will be the cost of 14 yards of black silk at $1.20 a yard?

11. If a boy's wages are $4.75 a week, how much will he earn in a year, or 52 weeks?

12. If a clerk earns $8 a week, and spends $4.75 a week, how much will he lay by in a year?

13. What will it cost six persons to board for a year at the rate of $5.75 each for a week?

14. What is the value of 17 chests of tea, each weighing 59 pounds, at $0.72 a pound?

15. A merchant sold 15 barrels of pork, each weighing 200 pounds at 12 cents a pound; what did he receive?

16. A lady goes to market with 10 dollars; she buys 6 dozen eggs at 27 cents, 7 pounds of meat at 16 cents, and 3 bushels of potatoes at $1.25; how much money has she remaining?

17. A drover bought 95 cows at $37.25 each, and sold them at $40 each; how much did he make?

18. Mr. Good bought 15 hogsheads of molasses, containing 63 gallons each, at 65 cents a gallon, and sold it at $1.10 a gallon; what was his gain?

Division.

Oral Exercises.

1. If 7 hens cost $3.57, what will one cost?

2. At 5 cents each, how many oranges can I buy for $1.

3. I paid $18.24 for 6 weeks' board; how much did I pay a week?

4. At 6 cents each, how many lemons can I buy for $3.72?

5. If 4 hats cost $5, what will 7 such hats cost?

6. A yard of calico is worth 12 cents; if I buy 15 yards and give a two-dollar bill in payment, how much oranges at 5 cents each can I buy with the change?

7. If a barrel of flour costs $6.25, how many barrels can be bought for $50?

8. At the rate of 15 cents a dozen, how many dozen buttons can be bought for $3?

9. If I buy 17 pounds of sugar at 10 cents a pound, how many oranges at 5 cents each can I get for the change due me from a five-dollar bill?

10. A yard of calico is worth 9 cents; how many yards can I get for 10 dozen of eggs, worth 18 cents a dozen?

11. If I trade 6 pounds of butter at 20 cents a pound, and 10 pounds of lard at 12 cents a pound, for sugar at 12 cents a pound, how many pounds of sugar do I get?

Ex. 6. Divide $639.75 by 5.

5)$639.75	We regard the dollars and cents as so many cents, and divide as in simple division. Then we place the point in the quotient to separate the dollars from the cents.
$127.95	

Ex. 7. When potatoes are worth $1.25 a bag, how many bagfuls can be bought for $46.25?

125)4625(37	We are required to find how often $1.25 is contained in $46.25. We regard $1.25 as 125 cents and $47.25 as 4625 cents and then we divide in the usual way.
375	
875	
875	

Exercise xli.

(1)	(2)	(3)	(4)
6)$76.32	7)$149.59	8)$145.36	9)$237.06

5. If a person spends $410.28 in a year, how much is that a week, allowing 52 weeks to a year?

6. Divide $2117.71 equally among 35 families, and find the share of each.

7. A man pays for some land $400 cash and $192.80 in produce. If there were 57 acres, how much does the land cost him per acre?

8. How many sheep can be bought for $302.95 at $4.15 each.

9. If 93 oranges cost $5.58, what will 37 cost?

10. I bought a house for $3453, and paid for it in instalments of $575.50 each; how many payments did I have to make?

11. William earned $3.25 a day, and paid 75 cents for board; in how many days would he save $912.50?

12. A merchant received $53.25 for a case of silk, including $1.25 cost of box. How many pieces of silk were in the case, if he received $53.25 for each piece?

BILLS.

70. A **Bill** of Goods is a written statement of the goods sold, giving the quantity and price of each article and total cost, also the date of the sale, with the names of the buyer and seller.

71. The party who owes is called a *Debtor*, and the party to whom a debt is owed is called a *Creditor*.

SPECIMEN OF A BILL.

Toronto, February 23, 1878.

James Brown, Esq.,

Bought of C. Meredith.

1878.			$	c.
Jan.	19	15 lb. Coffee at 32c.....	4	80
"	23	16 " Lard at 15c.....	2	40
Feb.	2	25 " Sugar at 13c.....	3	25
"	20	16 " Ham at 16c.....	2	56
			$13	01

SPECIMEN OF RECEIPTED BILL.

Toronto, March 1, 1878.

John Smith, Dr.

To George Brown.

1873.					$	c.
Jan.	1	To 75 lbs. of sugar at $0.12,	$9	00		
Feb.	2	" 47 yds. of cloth " 3.25,	152	75		
					161	75
		Cr.				
Jan.	7	By 75 bu. of corn, at $0.73,	$58	50		
Feb.	2	" 43 bu. of apples " 1.25,	53	75		
					112	25
		Balance due,			$49	50

1878, March 15th.

Received Payment,

George Brown.

Exercise xlii.

Make out bills for the following accounts, supplying dates:

1. Mr. J. Jones bought of R. Walker 10 yards silk, at $2.50; 12 yards flannel, at 40 cents; 16 yards calico, at 15 cents.

2. Mr. Brown bought of McClung & Bros. 10 pounds tea, at 75 cents; 8 lbs. raisins, at 18 cents; 5 pounds rice, at 10 cents; 12 lbs. butter, at 21 cents.

3. James Taylor bought of Thomas Yellowlees 5 quires foolscap, at 25 cents; 1 Hamblin Smith's Arithmetic, at 75 cents; 3 rolls wall paper, at 45 cents; 4 dolls, at 25 cents.

4. David Montgomery bought of F F. McArthur 20 yards cotton, at 11 cents; 15 yards print, at 16 cents; 12 yards braid, at 6 cents; 3 pair gloves, at 27 cents; 26 yards dress goods, at 63 cents; 1 hat, at $5.25.

5. Robert Davey bought of Murdoch Bros. 18 bags salt, at 75 cents; 4 barrels plaster, at 98 cents; 10 pounds coffee, at 35 cents; 1 chest tea. 18 pounds, at 65 cents; 48 grain bags, at $3.60 a doz.

6. Levi Van Camp sold Wm. Burns & Co. 257 bushels wheat, at $1.12; 475 bushels oats, at 36 cents; 45 bushels corn, at 76 cents; 175 bushels pease, at 82 cents; 367 bushels barley, at 69 cents.

7. A. Thompson bought of A. Harrison 32 pounds sugar, at 12 cents; 11 pounds coffee, at 35 cents; 26 pounds soap, at 8 cents; 14 pounds rice, at 9 cents; 7 pounds fish, at 15 cents; 18 pounds crackers, at 12 cents.

8. W. West bought of T. Brown 27 pair calfskin boots, at $4.50; 96 pair gaiters, at $3.25; 126 pair overshoes, at 91 cents; 18 pair slippers, at 95 cents; 75 pair heavy boots, at $2.75.

9. Mrs. Jones bought of R. Walker & Co. 25 yards calico, at 12 cents; 12 spools cotton, at 5 cents; 16 yards alpaca, at 75 cents; 17 yards muslin, at 18 cents; 6 skeins tape, at 2 cents.

10. Murdoch Bros. sold to A. Preston the following: 27 yards calico, at 13 cents; 45 yards muslin, at 18 cents; 16 yards linen, at 45 cents; 17 yards cambric, at 15 cents; and 9 handkerchiefs, at 45 cents; and took in exchange 12 bushels potatoes, at 65 cents; 3 barrels apples, at $3.25; 13 pounds butter, at 35 cents, and the remainder in cash. How much cash was paid? Make out a receipted bill.

EXAMINATION PAPERS

I.

1. A farmer gave $43.50 for sheep, at the rate of $7.25 for 3 sheep; how many did he buy?

No. of sheep bought for $7.25 = 3 sheep;

" " $1 = $\frac{3}{7.25}$ "

" " $43.50 = $\frac{43.50 \times 3}{7.25}$ sheep,

= 18 sheep.

2. If 18 chickens cost $4.20, how much will 3 chickens cost?

3. A merchant bought 9 pieces of cloth, each containing 50 yards, for which he paid $317.50; what was the cost of a single yard?

4. A banker has $20000 in cash; he pays for 50 shares of stock, at $97.50 a share; and 100 shares, at $110 a share; how many shares, at $41.25 each, can he buy with the remainder of his money?

5. I owed $276 and paid $17.25 on it; how many times must I pay such a sum to cancel the debt?

II.

1. I retail envelopes at 12 cents a pack, gaining 3 cents on each pack of 24; what did they cost me per 1000?

Cost of 24 envelopes = 9 cents;

" 1 " = $\frac{9}{24}$ "

" 1000 " = $\frac{9 \times 1000}{24}$ "

= $3.75.

2. A grocer sold 9760 pounds of flour, at $4.25 per 100 lbs.; what was the amount of the sale?

3. Messrs. Smith & Co. burn in their store, in a year, 62500 cubic feet of gas, what is their gas bill for a year at $1.50 per 1000 feet?

4. A man bought a quantity of coal for $250, and by retailing it at $5.75 a ton, he gained $37.50; how many tons did he buy?

5. The charge of sending a telegram to a certain place is 40 cents for ten words, and 5 cents for each additional word; what would a despatch of 24 words cost me?

III.

1. A horse worth $150, and 7 cows at $25 each, were exchanged for 57 sheep and $25.75 in money; what was the price of a sheep?

Value of horse and cows $=\$150+7\times\$25=\$325$.
Value of sheep $=\$325-\$25.75=\$299.25$.
Hence " 57 sheep $=\$299.25$;
therefore " 1 sheep $=\$\frac{299.25}{57}$
$=\$5.25$.

2. A merchant bought 5 pieces of cloth of equal lengths, at $3.25 a yard; he gained $18.75 on the whole cost by selling 4 of the pieces for $750; how many yards were there in each piece?

3. At an election there were three candidates A, B, and C; the total number of votes polled was 7734. The successful candidate, A, got 203 votes more than C, who got 107 votes less than one-third of the total vote polled; what was A's majority over B?

4. A father divided his property worth $4767 among his three sons A, B, and C, in such a way that A got as much as B and C together, and B and C shared alike; what was C's share?

5. If the continued product of 275, 376, 484 and 196 be divided by $77\times28\times47\times55$, what will be the quotient?

IV.

1. A merchant expended $547.40 for cloth. He sold a certain number of yards for $522, at $1.45 per yard, and gained on what he sold $108. How many yards did he buy and how much did he gain per yard on the cloth he sold?

2. A farmer exchanged 390 bushels of wheat worth $1.20 a bushel, for an equal number of bushels of barley at 75 cents a bushel, and oats at 42 cents a bushel; how many bushels of each did he receive?

3. John Turner has manufactured in 4 years 7740 pair of shoes, making each successive year 250 pair more than the year before; how many pair did he manufacture the first year?

4. If 80 men have sufficient provisions for 75 days, and 20 men go away, how long will they last the rest?

5. The product of 275 and 86 is 23650; how much must be taken from the product to give the product of 275 and 82; and to give the product of 270 and 86?

CHAPTER III.

MEASURES AND MULTIPLES

Section 1.—Prime Numbers, Prime Factors, &c.

72. In the series of numbers 1, 2, 3, 4, &c., a distinction may be observed of *odd* and *even* numbers.

An **Odd** number is one which cannot be divided into two equal whole numbers, as 1, 3, 5, &c.

An **Even** number is one which can be divided into two equal whole numbers, as 2, 4, 6, &c.

73. There is another, and a more important division of numbers into two classes, one class consisting of numbers, each of which is divisible only by 1 and a number equal to itself, as 2, 3, 5, &c.; and the other class consisting of numbers which admit of other divisors, as 4, 6, 8, &c. The numbers in the former class are called *prime numbers;* and those in the latter class *composite numbers.* (Art. 61.)

74. A **Prime Number** is one which can be exactly divided only by unity and a number equal to itself.

75. The **Prime Factors** of a number are the prime numbers, which when multiplied together will produce it; thus, 2, 2 and 3 are the prime factors of 12.

Oral Exercises.

1. What are the prime factors of 30?

   The prime factors of 30 are 3, 2 and 5, since these are the only prime numbers which multiplied together will produce 30.

2. Name the prime numbers from 16 to 53? from 53 to 101?
3. What are the prime factors of 12? 16? 15? 18?
4. What are the prime factors of 21? 25? 27? 32? 33? 34?
5. What prime factor is found in both 6 and 9?
6. What prime factor is found in both 20 and 26?

7. What prime factor is common to 12 and 30? 21 and 28?

8. What prime factor is common to 35 and 50? 14 and 70? 33 and 99? 42 and 48? 26 and 39?

76. To resolve a number into its Prime Factors.

Ex. 1. Find the prime factors of 105.

3)105
5)35
7

Dividing 105 by 3, a prime factor, we have 35; dividing 35 by 5, a prime factor, we have 7, a prime number, therefore the prime factors of 105 are 3, 5, 7.

Exercise xliii.

Find the prime factors of

1. 48.	5. 175.	9. 429.	13. 313.
2. 72.	6. 270.	10. 276.	14. 336.
3. 81.	7. 160.	11. 800.	15. 855.
4. 108.	8. 325.	12. 180.	16. 1155.

What prime factors are common to

17. 50 and 70?	19. 63 and 147?
18. 81 and 96?	20. 120 and 600?

Section II.—Cancellation.

77. Cancellation is the process of shortening operations in division by rejecting or *cancelling* equal factors common to both dividend and divisor.

Ex. 1. Divide 28 by 8.

$$\frac{28}{8}=\frac{4\times 7}{4\times 2}=\frac{7}{2}=3\tfrac{1}{2}$$

Write the divisor 8, under the dividend 28. Resolve 28 into 4×7, and 8 into 4×2. Cancelling the common factor 4 in dividend and divisor, we have 7 divided by 2 or $3\frac{1}{2}$.

The same result will be obtained by dividing both dividend and divisor by 4.

Hence, *Cancelling a common factor from both dividend and divisor does not change the quotient.*

Exercise xliv.

1. Divide $16\times4\times5$ by $8\times2\times10$.
2. Divide $7\times16\times6$ by $14\times3\times8$.

3. Divide $9 \times 7 \times 16 \times 16$ by $21 \times 32 \times 2$.

4. Divide $27 \times 12 \times 14$ by $9 \times 4 \times 7$.

5. Divide $72 \times 45 \times 140$ by $18 \times 24 \times 35$.

6. Divide $24 \times 32 \times 36 \times 144$ by $64 \times 108 \times 8$.

7. How many yards of muslin, worth 12 cents a yard, may be bought for 16 pounds of butter, worth 15 cents a pound?

8. How many bushels of potatoes at 75 cents a bushel must a farmer give for 36 yards of carpet worth $1.50 a yard?

9. A tailor bought 12 pieces of cloth, each containing 22 yards, worth $2.50 a yard; he made 27 suits of clothes; how much must he get per suit so as not to lose?

10. If a farmer exchange 25 bushels of wheat at $1.20 a bushel for cloth at 40 cents a yard, how many yards does he get?

11. Three pieces of cloth containing 30 yards each, worth $5 a yard, were exchanged for 5 pieces of cloth containing 45 yards each; what was the second kind worth per yard?

12. Divide the continued product of 16, 18, 24, 25, 36 and 45 by the continued product of 27, 72 and 100.

Section III.—The Highest Common Factor.

Oral Exercise.

Name a common factor

1. Of 6 and 9.
2. Of 12 and 10.
3. Of 27 and 24.
4. Of 16 and 20.
5. Of 12 and 18.
6. Of 15 and 40.

What is the highest common factor

7. Of 15 and 10?
8. Of 20 and 15?
9. Of 25 and 50?
10. Of 24 and 72?
11. Of 24 and 12?
12. Of 72 and 144?

78. A **Common Factor** of two or more numbers is a number that will exactly divide each of the given numbers.

79. The **Highest Common Factor,** called also the *Greatest Common Measure,* of two or more numbers is the largest number that will exactly divide each of the given numbers.

Ex. 1. Find the highest common factor of 18, 36 and 72.

6) 18, 36, 72
3) 3, 6, 12
1, 2, 4.

We place the numbers as in the margin. By dividing each number by 6, we take out the common factor 6; by dividing each of the quotients by 3 we take out the common factor 3; since the quotients, 1, 2, 4 have no factor common to all of them 6 and 3 are all the common factors of the given numbers, hence 6×3, or 18 is their H. C. F.

Hence, *To find the H. C. F. of two or more numbers, we divide by any common factor of all the numbers; we then divide the quotients in the same manner, and thus continue until the quotients have no common factor; the product of all the divisors will be the highest common factor.*

Exercise xlv.

Find the H. C. F.

1. Of 15, 20, 30.
2. Of 16, 20. 24.
3. Of 24, 96, 80.
4. Of 28, 56, 42.
5. Of 30, 50, 60.
6. Of 84, 126, 210.
7. Of 120, 240, 72.
8. Of 44, 110, 77.
9. Of 75, 300, 450.
10. Of 144, 576, 720.

11. A man has two logs which he wishes to cut into boards of equal length; one is 24 feet, and the other 16 feet long; what is the greatest length into which the boards can be cut?

12. What is the greatest equal length into which two trees can be cut, one being 105 feet in length and the other 84 feet?

13. Three pieces of carpet, of 48, 64 and 80 yards respectively, if cut into the longest possible equal lengths, will exactly cover a parlor floor, each piece being the length of the parlor; how long is the parlor?

14. A grocer has 136 quarts of strawberries, and 152 quarts of plums, which he wishes to put into boxes, each box to hold the same number of quarts, and the largest number possible; how many quarts may be put into each box?

15. If a pear costs an exact number of cents, what is the greatest number of pears you could buy with 180 cents, or 225 cents, or 315 cents, so as to get the same number each time?

16. A certain school consists of 132 pupils in the lower school, and 99 in the upper school; how might each of

these be divided so that the whole school should be distributed into equal sections?

80. To find the H. C. F. when the numbers are large.

Ex. 2. Find the H. C. F. of 91 and 143.

91) 143 (1
91
—
52) 91 (1
52
—
39) 52 (1
39
—
13) 39 (3
39
—

That which we are seeking to find is the largest number that will divide both numbers. Now *any number that will divide two other numbers will also divide their difference or their sum*, and as we can see the factors of a small number more easily than those of a large one, we divide the greater of the two numbers by the less; then we divide the less number by the remainder, and each former remainder by the new remainder, till we find a number that will divide the last remainder exactly. This will be the H. C. F. of the two numbers.

To find the H. C. F. of more than two numbers, first find the H. C. F. of two of them; then find the H. C. F. of the common factor thus found and a third number; and so on through all the numbers. The last common factor found will be the H. C. F. of all the numbers.

Exercise xlvi.

Find the H. C. F. of

1. 115 and 161.
2. 333 and 592.
3. 697 and 899.
4. 392 and 672.
5. 405 and 900.
6. 1220 and 2013.
7. 6006 and 3818.
8. 2871 and 4213.
9. 43902 and 49533.
10. 23940 and 28350.
11. 1435, 1084 and 2135.
12. 14385, 20391 and 49287.

13. A grocer has two hogsheads of sugar, one containing 1104 pounds, and the other 1288 pounds. He wishes to put this sugar into barrels, each barrel to contain the same number of pounds, and this the greatest number possible; of how many pounds must each barrel consist?

14. *A* and *B* purchased horses at the same rate per head; the value of *A*'s horses was $623; and of *B*'s $1068; what was the number purchased by each?

Section IV.—Least Common Multiple.

Oral Exercises.

1. What number is three times 5? four times 7?

A number which is one or more times another number is called a *multiple* of that number.

2. What number is a multiple of 3? of 5? of 9?
3. Name two multiples of 8; three multiples of 7.
4. What number is a multiple of both 4 and 6? 3 and 5?
5. What multiple is common to both 3 and 4? 4 and 7?
6. Name all the multiples of 4 from 3 to 30.
7. What is the least number of which 3 and 5 are factors?
8. What is the least number exactly divisible by 3, 4, and 8?
9. What is the least number exactly divisible by 10 and 12? by 8 and 12? by 6 and 10? by 12 and 18?
10. James has just enough money to buy oranges at 5 cents each, pears at 4 cents each, or tops at 6 cents each; how much money has he?

81. A **Multiple** of a number is a number that is exactly divisible by that number.

82. A **Common Multiple** of two or more numbers is a number that is exactly divisible by each of the given numbers. Thus, 24 is a common multiple of 4 and 6, because it is exactly divisible by each of them.

83. The **Least Common Multiple** (L. C. M.), of two or more numbers is the least number that is exactly divisible by each of them.

Ex. 1. Find the least common multiple of 24, 20, and 33.

$24 = 2 \times 2 \times 2 \times 3$
$20 = 2 \times 2 \times 5$
$33 = 3 \times 11$
L. C. M. $= 2 \times 2 \times 2 \times 3 \times 5 \times 11 = 1320$.

The L. C. M. of the given numbers must contain the factors 2, 2, 2 and 3 to be divisible by 24; it must contain the factors 2, 2 and 5, to be divisible by 20; it must contain the factors 3 and 11 to be divisible by 33. Since the number 1320 contains all these factors and no others, it is the least common multiple of 24, 20, and 33.

Hence, *To find the L. C. M. of two or more numbers we find the prime factors of the numbers, and take the product of these factors, using each the greatest number of times it occurs in any of the given numbers.*

84. When the several numbers are not large, the process may be shortened by successive divisions of the given numbers, by *prime factors* which are common to two or more of the given numbers. By this means, all the divisors will consist of the prime factors common to two or more of the numbers, and the numbers left after the divisions will be the factors which are not common to any two of the numbers. Then the product of these common prime factors, and the factors which are not common, will be the least common multiple of the given numbers.

Ex. 2. Find the L. C. M. of 15, 24, 36, and 42.

2) 15, 24, 36, 42	Here 2, 2, 3 are the prime factors
2) 15, 12, 18, 21	common to two or more of the
3) 15, 6, 9, 21	numbers, and 5, 2, 3, 7 are the
5, 2, 3, 7	factors not common.

L. C. M. $= 2 \times 2 \times 3 \times 5 \times 2 \times 3 \times 7 = 2520$.

Exercise xlvii.

Find the L. C. M.

1. Of 15, 10 and 5.
2. Of 20, 10 and 30.
3. Of 9, 12 and 18.
4. Of 10, 25 and 30.
5. Of 24, 30 and 36.
6. Of 5, 9, 12 and 15.
7. Of 12, 15, 18 and 24.
8. Of 22, 55, 77 and 110.
9. Of 15, 30, 42 and 72.
10. Of 21, 54, 56 and 84.
11. Of 5, 7, 16, 23, 48 and 21.
12. Of 16, 12, 14, 32, 50 and 75.
13. Of 15, 18, 24, 40, 50, 60 and 90.
14. Of the even numbers from 14 to 28 inclusive
15. Of the odd numbers from 13 to 25 inclusive.
16. What is the least number which divided by 8, by 12, and by 14 gives in each case the remainder 5?
17. What is the least sum of money for which I can purchase either sheep at $6, cows at $28, or horses at $150 a head?
18. What is the least number of bushels of wheat that would make an exact number of full loads for three drays hauling respectively 24, 30 or 36 bushels a load?
19. What is the least number of cents with which you could buy an exact number of lemons at 6 cents each; or oranges at 8 cents; or bananas at 10 cents; or pine apples at 16 cents?

20. How many bushels would fill a number of barrels each containing 3 bushels, or a number of sacks, each containing 4 bushels, or a number of casks, each containing 14 bushels, the quantity to be the same in each case, and the smallest possible?

21. *A*, *B*, *C*, and *D* start together, and travel the same way round an island which is 600 miles in circuit. *A* goes 20 miles per day, *B*, 30, *C*, 25, and *D*, 40. How long must their journeyings continue, in order that they may all come together again?

EXAMINATION PAPERS.

I.

1. How do you determine whether a given number is prime or composite? Which of the following numbers are prime and which composite:—611, 645, 707, 757, 901, 1089?

2. Divide the continued product of 6, 15, 16, 24, 12, 21, and 17 by the continued product of 2, 10, 9, 8, 36, 7, and 51.

3. What is the least number of dollars that will purchase an exact number of cows at $24 each, sheep at $6 each, or horses at $127 each?

4. What is the least number which divided by 18, 21, and 30 gives 13 for remainder in each case?

5. A man owns 3 tracts of land, containing 525, 725, and 875 acres, respectively. He wishes to divide each tract into lots that will contain the same number of acres, and this the largest number possible; of how many acres must his lots consist?

II.

1. Define Highest Common Factor and Prime Factor, and explain when a number is Odd and when Even.

2. Find the largest number which will divide 942 and 1484, leaving as remainders respectively 16 and 9.

3. What is the quotient of $144 \times 75 \times 15 \times 32 \times 23$ divided by $432 \times 25 \times 8 \times 30$?

4. What is the least number of marbles that can be divided equally among 16, 21, 24 or 30 boys?

5. A can dig 24 post holes in a day; B can dig 25, and C 30 in the same time. What is the smallest number which will furnish exact days' labour either for each working alone or for all working together?

III.

1. The product of four consecutive numbers is 73440; find the numbers.

2. What is the least number of acres in a farm that can be exactly divided into lots of 12 acres, 15 acres, 18 acres or 25 acres each?

3. A farmer sold 4 loads of apples, each containing 15 barrels, and each barrel 3 bushels at 60 cents a bushel. He received as payment 6 barrels of pork, each weighing 200 pounds; what was the pork worth a pound?

4. The product of two numbers is 152303, and 7 times one of them is 2996; what is the other one?

5. How many rails will enclose a field 7163 feet long by 3315 feet wide, provided the fence is straight, 6 rails high, the rails of equal length, and the longest that can be used?

IV.

1. A farmer exchanged 9 tubs of butter, each containing 56 pounds, worth 25 cents per pound, for 4 chests of tea, each containing 42 pounds; what was the tea worth per pound?

2. What is the smallest sum of money with which I can buy sheep at $5 each, cows at $24 each, oxen at $54 each, or horses at $165 each?

3. Divide the continued product of 51, 72, 144, 972, and 720 by the continued product of 9, 17, 18, 24, 36 and 45.

4. Find the least number which divided by 1595, 2530, and 3168, will leave the same remainder, 719.

5. The following are the prime factors of a number: 2, 2, 3, 5, 5, 7, 11, 11, 13, 19, 89, and 227; find the number.

V.

1. State and prove the rule for finding the H. C. F. of two numbers, and find the H. C. F. of 125000 and 501504.

2. Find the L. C. M. of 16, 24, and 30, and explain the method.

3. A school was found to contain such a number of boys, that when arranged in sixes, sevens, nines or twelves, there were always five over; how many children, *at least*, did the school contain?

4. The fore and hind wheels of a carriage are 12 and 15 feet in circumference; find the least number of revolutions of each that will give the same length.

5. Explain the terms *measure* and *common measure*; and prove, by means of an example, that every common measure of the dividend and divisor is a measure of the remainder.

CHAPTER IV.

FRACTIONS.

Section I.—Definitions.

Oral Exercises.

1. If an apple is divided into two equal parts, what is one of these equal parts called?

2. How many halves are there in anything? Write down one-half. (See example 7, page 42.)

3. When I divide an orange into three equal parts, what is one of these equal parts called? What are two of them called?

4. How many thirds are there in anything? How many fourths are there in anything?

5. How would you get fourths? fifths? sixths?

6. How many thirds make a whole? How many fourths? sevenths? tenths?

7. Into how many equal parts must a thing be divided to get halves? fifths? sevenths? eighths?

8. Two halves of an apple are equal to how many whole apples?

9. What are four fourths of a pear equal to?

10. Which are the *smaller*, halves or thirds? Halves or fourths? thirds or fourths?

The value of the part varies according to the number of equal parts into which the whole is divided. The more parts it is divided into, the smaller they must be.

Half		Half
Third	Third	Third
Fourth	Fourth	Fourth	Fourth

One half of a thing is greater than one third; one third is greater than one fourth.

85. A Fraction is an expression representing one or more of the equal parts of a unit.

86. Fractions are divided into **two classes, Common, or Vulgar Fractions,** and **Decimal Fractions.**

87. A Common Fraction is one which is expressed by two numbers one placed above the other with a line between them; thus four-fifths is written $\frac{4}{5}$; nine-elevenths, $\frac{9}{11}$; ten thirty-fifths, $\frac{10}{35}$.

88. One of these equal parts is called the **Fractional Unit** and instead of the name of this unit being written *after* the number of such units as in whole numbers, it is placed *under* it. Thus, three apples is written 3 apples and 3 fourths, $\frac{3}{4}$.

89. The number written below the line is called the **Denominator** or "*name-giver*" because it indicates the name of the fractional unit, *i.e.*, it shows into how many equal parts the whole is divided.

90. The number written above the line is called the **Numerator,** *i.e.*, the "*numberer*" or "*counter,*" because it indicates how many of the parts named by the denominator are to be taken.

91. The Terms of a fraction are the numerator and the denominator. Thus, $\frac{5}{8}$ is a fraction—5 and 8 are its *terms.*

92. A Proper Fraction is one whose numerator is *less* than its denominator. Thus, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$ are proper fractions.

93. If we cut an apple into *two* equal parts, one-half will be represented by $\frac{1}{2}$.

If we cut an apple into *four* equal parts, one-half will be represented by $\frac{2}{4}$.

If we cut an apple into *eight* equal parts, one-half will be represented by $\frac{4}{8}$.

$$\therefore \frac{1}{2} = \frac{2}{4} = \frac{4}{8}.$$

Similarly,

If we cut an apple into *three* equal parts, one-third will be represented by $\frac{1}{3}$.

If we cut an apple into *nine* equal parts, one-third will be represented by $\frac{3}{9}$.

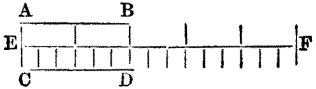
$\therefore$ stands for the word *therefore.*

If we cut an apple into *eighteen* equal parts, one-third will be represented by $\frac{6}{18}$.

$$\therefore \frac{1}{3} = \frac{3}{9} = \frac{6}{18}.$$

Hence we conclude that, The value of a fraction is not altered by multiplying or dividing both its numerator and its denominator by the same number.

The following is another proof of this important proposition:



CD = $\frac{6}{15}$ of EF,
and AB = $\frac{2}{5}$ of EF;
but AB = CD;

$$\therefore \frac{2}{5} = \frac{6}{15} = \frac{3\times2}{3\times5}.$$

94. The usual definition of a fraction is given in Art. 85, but by the help of the following proposition, which is best explained by an example, we shall be able to obtain another definition of a fraction, which is sometimes useful.

Ex. 1. Prove that $\frac{2}{5}$ of 1 = $\frac{1}{5}$ of 2.

Since 1 = *five*-fifths of a unit,
2 = *ten*-fifths of a unit;
$\therefore \frac{1}{5}$ of 2 = $\frac{1}{5}$ of *ten*-fifths of a unit
= *two*-fifths of a unit
= $\frac{2}{5}$ of 1;
$\therefore \frac{1}{5}$ of 2 = $\frac{2}{5}$ of 1.

Hence, *We may define a fraction as a simple manner of indicating that its numerator is to be divided by its denominator.*

95. Since 3 apples multiplied by 2 = 6 apples,
so 3 eighths ($\frac{3}{8}$) " 2 = 6 eighths ($\frac{6}{8}$);
$\therefore 2 \times \frac{3}{8} = \frac{6}{8}$.

Hence, *To multiply a fraction by a whole number, we simply multiply the numerator by the whole number, and retain the denominator.*

Since 8 marbles divided by 2 = 4 marbles,
so 8 ninths ($\frac{8}{9}$) " 2 = 4 ninths ($\frac{4}{9}$),
$\therefore \frac{8}{9} \div 2 = \frac{4}{9}$.

Hence, *A fraction is divided by a whole number by dividing the numerator by the number and retaining the denominator.*

93. From the preceding article it appears that fractions may, in general, be treated as whole numbers. Indeed, they differ from whole numbers simply in the unit employed. Thus in 20 feet, 1 foot is the unit or base of the collection and in $\frac{1}{5}$, the fractional unit is $\frac{1}{5}$, four of them being taken or collected to form the fraction.

The fractional unit is always equal to 1 divided by the denominator.

Section II.—Reduction of Fractions.

Case I.

97. To reduce whole or mixed numbers to improper Fractions.

98. A Mixed Number consists of a whole number and a fraction; as $3\frac{1}{5}$, $4\frac{3}{4}$, &c.

99. An Improper Fraction is one whose numerator is not less than its denominator.

Oral Exercises.

1. How many halves in 5 apples?
   In 1 apple there are 2 halves, and in 5 apples there are 5 times 2 halves, or 10 halves.
2. How many halves in 6? In 10? In 15? In 40?
3. How many fourths in 4? In 6? In 9? In 12?
4. How many fifths in $4\frac{3}{5}$?
   In 1 there are 5 *fifths*, and in 4 there are 4 times 5 fifths or 20 fifths, which added to 3 fifths, make 23 fifths; therefore $4\frac{3}{5} = \frac{23}{5}$.
5. How many fourths in $7\frac{1}{4}$? In $5\frac{2}{4}$? In $9\frac{1}{4}$? In $12\frac{3}{4}$?
6. How many sixths in $4\frac{5}{6}$? In $8\frac{3}{6}$? In $9\frac{1}{6}$? In $11\frac{5}{6}$?

Ex. 1. Reduce $27\frac{3}{4}$ to fourths.

$$27\tfrac{3}{4}=27+\tfrac{3}{4};$$

$$\text{Now,}\quad 1=\tfrac{4}{4};$$

$$\therefore\quad 27=\frac{27\times4}{4}=\frac{108}{4};$$

$$\therefore\quad 27+\tfrac{3}{4}=\frac{108}{4}+\frac{3}{4}=\frac{111}{4}=\frac{27\times4+3}{4}.$$

Hence, *To reduce mixed numbers to improper fractions, we multiply the whole number by the denominator of the fraction, add the numerator to the product, and write the denominator under the sum.*

Exercise xlviii.

Reduce to improper fractions

1. $3\frac{4}{7}$.	7. $35\frac{11}{12}$.	13. $237\frac{13}{152}$.
2. $4\frac{1}{5}$.	8. $82\frac{15}{17}$.	14. $304\frac{75}{238}$.
3. $9\frac{2}{7}$.	9. $51\frac{17}{23}$.	15. $1306\frac{9}{178}$.
4. $4\frac{5}{9}$.	10. $86\frac{5}{10}$.	16. $2500\frac{184}{973}$.
5. $11\frac{1}{12}$.	11. $99\frac{1}{11}$.	17. $1001\frac{11}{101}$.
6. $12\frac{3}{8}$.	12. $78\frac{19}{99}$.	18. $2397\frac{716}{999}$.

Case II.

100. To reduce improper fractions to whole or mixed numbers.

Ex. 2. Reduce $\frac{87}{11}$ to a mixed number.

Since dividing both terms of a fraction by the same number does not change its value (Art. 93),

$$\frac{87}{11}=\frac{87\div11}{11\div11}=\frac{7\frac{10}{11}}{1}=7\tfrac{10}{11}.$$

Hence, *To reduce an improper fraction to a whole or mixed number we simply divide the numerator by the denominator.*

Exercise xlix.

Reduce the following fractions to whole or mixed numbers:

1. $\frac{19}{5}$.	7. $\frac{183}{16}$.	13. $\frac{4407}{153}$.
2. $\frac{31}{6}$.	8. $\frac{982}{15}$.	14. $\frac{5919}{59}$.
3. $\frac{45}{7}$.	9. $\frac{725}{[illegible]}$.	15. $\frac{17007}{33}$.
4. $\frac{75}{10}$.	10. $\frac{[illegible]}{5}$.	16. $\frac{32460}{48}$.
5. $\frac{99}{3}$.	11. $\frac{475}{7}$.	17. $\frac{25650}{604}$.
6. $\frac{102}{6}$.	12. $\frac{932}{19}$.	18. $\frac{72648}{139}$.

Case III.

101. To reduce a fraction to its lowest rms.

102. A fraction is in its **Lowest Terms** when the numerator and denominator have no common factor.

Ex. Reduce $\frac{48}{108}$ to its lowest terms.

$$\frac{48}{108} = \frac{1}{2}\frac{2}{7} = \frac{4}{9}.$$

Dividing both terms of $\frac{48}{108}$ by the common factor 4, reduces it to $\frac{12}{27}$; dividing both terms of this fraction by 3, reduces it to $\frac{4}{9}$. Since $\frac{4}{9}$ has its numerator and denominator prime to each other it is in its *lowest terms.*

We might have found the H. C. F. of the numerator and denominator and divided both terms by it at once.

Hence, *To reduce a fraction to its lowest terms we divide both terms by a common factor, and the result again by a common factor, and so on till the terms have no common factor.*

Or *we may divide both terms of the fraction by their Highest Common Factor.*

Exercise I.

Reduce the following fractions to their lowest terms.

1. $\frac{15}{20}$.
2. $\frac{18}{27}$.
3. $\frac{14}{35}$.
4. $\frac{24}{64}$.
5. $\frac{75}{90}$.
6. $\frac{80}{112}$.
7. $\frac{210}{301}$.
8. $\frac{648}{720}$.
9. $\frac{288}{864}$.
10. $\frac{792}{864}$.
11. $\frac{840}{4312}$.
12. $\frac{1176}{1512}$.
13. $\frac{6161}{7171}$.
14. $\frac{5403}{7203}$.
15. $\frac{1309}{2618}$.
16. $\frac{1853}{2502}$.
17. $\frac{304}{1072}$.
18. $\frac{1040}{1155}$.
19. $\frac{872}{1054}$.
20. $\frac{1584}{5040}$.
21. $\frac{4235}{7315}$.
22. $\frac{3276}{4914}$.
23. $\frac{2109}{2550}$.
24. $\frac{2418}{3597}$.

Case IV.

103. To reduce a Compound Fraction to a Simple one.

104. A Compound Fraction is a fraction of a fraction; as $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{3}{7}$ of $\frac{4}{5}$, &c.

105. A Simple Fraction is one in which both numerator and denominator are whole numbers; as $\frac{3}{4}$, $\frac{5}{6}$, &c.

Oral Exercises.

1. What is $\frac{1}{2}$ of 6 apples? of 10 boys? of 16 cents?

2. What is $\frac{1}{2}$ of 6 ninths? of 10 elevenths? of 16 twentieths?

3. What is $\frac{1}{2}$ of $\frac{6}{9}$?
   Since $\frac{1}{2}$ of 6 apples=3 apples,
   $\frac{1}{2}$ of 6 ninths $(\frac{6}{9})$=3 ninths $(\frac{3}{9})$.

4. What is $\frac{1}{3}$ of $\frac{2}{17}$? of $\frac{5}{21}$? of $\frac{3}{25}$? of $\frac{8}{27}$?

5. What is $\frac{1}{5}$ of $\frac{5}{7}$? $\frac{2}{5}$ of $\frac{5}{7}$? $\frac{3}{5}$ of $\frac{15}{16}$? $\frac{4}{5}$ of $\frac{20}{21}$?

6. What is $\frac{1}{3}$ of $\frac{1}{4}$?
   $\frac{1}{4}=\frac{3}{12}$,
   $\frac{1}{3}$ of $\frac{1}{4}=\frac{1}{3}$ of $\frac{3}{12}=\frac{1}{12}$.

7. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{5}$?

8. A boy had $\frac{1}{5}$ of a dollar, and lost $\frac{1}{4}$ of it? what part of a dollar did he lose?

9. A man owned $\frac{1}{7}$ of a farm, and sold $\frac{1}{6}$ of his share; how much did he sell?

10. I had $\frac{1}{2}$ a ton of coal, and gave my neighbour $\frac{1}{5}$ of it; he gave his brother $\frac{1}{3}$ of his share; how much did his brother get?

Ex. 4. Reduce $\frac{3}{5}$ of $\frac{7}{8}$ to a simple fraction.

$\frac{7}{8}=\frac{35}{40}$,

$\frac{1}{5}$ of $\frac{7}{8}=\frac{1}{5}$ of $\frac{35}{40}=\frac{7}{40}$;

$\therefore$ $\frac{3}{5}$ of $\frac{7}{8}=3\times\frac{7}{40}=\frac{21}{40}=\frac{3\times 7}{5\times 8}$

$=\frac{\text{the product of the numerators}}{\text{the product of the denominators}}$

Hence, *To reduce a compound fraction to a simple one, multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

NOTE.—Before performing the multiplication, mixed numbers should be reduced to improper fractions, and any factor common to a numerator and denominator cancelled.

Exercise li.

Simplify the following fractions:

1. $\frac{4}{7}$ of $\frac{5}{11}$.
2. $\frac{5}{8}$ of $\frac{9}{22}$.
3. $\frac{3}{4}$ of $4\frac{1}{2}$.
4. $2\frac{2}{3}$ of $\frac{3}{8}$.
5. $\frac{5}{9}$ of $\frac{4}{5}$ of $\frac{3}{5}$.
6. $\frac{4}{9}$ of $\frac{3}{11}$ of $\frac{5}{8}$.
7. $\frac{3}{11}$ of $\frac{5}{7}$ of $2\frac{4}{5}$.
8. $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$.
9. $\frac{1}{7}$ of $\frac{4}{4}$ of $\frac{9}{10}$ of $\frac{7}{15}$.
10. $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{7}{6}$ of $\frac{8}{11}$.
11. $\frac{6}{8}$ of $\frac{2}{3}$ of $1\frac{9}{12}$ of $\frac{2}{8}$.
12. $\frac{1}{2}$ of $\frac{2}{4}$ of $\frac{3}{28}$ of 7.
13. $\frac{3}{5}$ of $\frac{4}{11}$ of $\frac{17}{23}$ of $5\frac{3}{4}$.
14. $\frac{33}{35}$ of $\frac{8}{11}$ of $\frac{1}{2}\frac{3}{2}$ of $9\frac{5}{8}$.
15. $\frac{2}{3}$ of $8\frac{1}{4}$ of $\frac{1}{5}$ of $2\frac{1}{2}$.
16. $\frac{5}{16}$ of $\frac{3}{9}$ of $\frac{1}{5}$ of $4\frac{1}{2}$.
17. $\frac{1}{9}$ of $\frac{5}{3}$ of $\frac{3}{8}$ of 9.
18. $\frac{1}{5}$ of $\frac{8}{15}$ of $3\frac{1}{16}$ of 6.

Exercise lii.

1. Some boys owned $\frac{3}{8}$ of a boat; they sold $\frac{2}{4}$ of their share; what part of the boat did they sell?

2. Having $\frac{3}{4}$ of a bushel of potatoes I gave away $\frac{5}{6}$ of what I had; what part or a bushel did I give away?

3. A boy had $\frac{9}{10}$ of a dollar, and spent $\frac{2}{3}$ of it; how much did he spend?

4. A gentleman owning $\frac{3}{5}$ of a factory gave $\frac{2}{6}$ of what he owned to his son; what part of the whole factory was the son's share?

5. A has $\frac{4}{7}$ of a ton of hay, which is $\frac{3}{4}$ as much as B has; how much has B?

$\frac{3}{4}$ of what B has $= \frac{4}{7}$ of a ton;
$\therefore \frac{1}{4}$ " " $= \frac{1}{3}$ of $\frac{4}{7}$ of a ton $= \frac{4}{21}$ of a ton;
$\therefore \frac{4}{4}$ " " $= 4 \times \frac{4}{21}$ of a ton $= \frac{16}{21}$ of a ton;
$\therefore$ B has $\frac{16}{21}$ of a ton.

6. A owns $\frac{4}{7}$ of a railroad, and $\frac{3}{4}$ of this is $3\frac{3}{7}$ times what B owns; how much does B own?

7. How many acres of land has B, if $\frac{3}{15}$ of 18 is $\frac{6}{25}$ of his number?

8. A's money equals $\frac{8}{15}$ of $8750, and A's is $\frac{32}{66}$ of B's money; how much money has B?

Case v.

106. To reduce Fractions to equivalent ones having the least common denominator.

1. Let the denominators be prime to each other and the fractions in their simplest form.

Ex. 5. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to equivalent fractions with *least* common denominator.

Since multiplying the terms of a fraction by the same number does not change its value, we have,

$$\frac{2}{3}=\frac{4\times5\times2}{4\times5\times3}=\frac{40}{60},$$

$$\frac{3}{4}=\frac{3\times5\times3}{3\times5\times4}=\frac{45}{60},$$

and $$\frac{4}{5}=\frac{5\times4\times4}{3\times4\times5}=\frac{48}{60}.$$

Hence, *To reduce fractions to equivalent ones having a common denominator, we multiply both terms of each fraction by all the denominators except its own.*

Exercise liii.

Reduce to equivalent fractions having a common denominator.

1. $\frac{3}{5}$, $\frac{7}{9}$.
2. $\frac{7}{9}$, $\frac{9}{10}$.
3. $\frac{8}{9}$, $\frac{7}{11}$.
4. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{7}$.
5. $\frac{5}{7}$, $\frac{7}{8}$, $\frac{8}{9}$.
6. $\frac{3}{5}$, $\frac{3}{3}$, $\frac{3}{4}$.
7. $\frac{3}{5}$, $\frac{1}{3}$, $\frac{2}{7}$, $\frac{1}{4}$.
8. $\frac{5}{7}$, $\frac{3}{5}$, $\frac{4}{3}$, $\frac{11}{13}$.
9. $\frac{3}{5}$, $\frac{5}{8}$, $\frac{4}{9}$, $\frac{8}{11}$.

2. Let the denominators be not prime to each other.

Ex. 6. Reduce $\frac{3}{2}$, $\frac{4}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to equivalent fractions with least common denominator.

L. C. M. of 3, 4, 6, 8=24.

$\frac{2}{3}=\frac{8\times2}{8\times3}=\frac{16}{24}$.

$\frac{3}{4}=\frac{6\times3}{6\times4}=\frac{18}{24}$.

$\frac{5}{6}=\frac{4\times5}{4\times6}=\frac{20}{24}$.

$\frac{7}{8}=\frac{3\times7}{3\times8}=\frac{21}{24}$.

We find the L. C. M. of the denominators to be 24, hence 24 is the least common denominator. Dividing 24 by 3, the denominator of $\frac{2}{3}$, we find we must multiply 3 by 8 to produce 24, similarly with the other fractions.

Hence, *To reduce fractions to equivalent ones having a common denominator, when the denominators are not prime to each other, we find the least common multiple of the denominators, divide this by each denominator, and multiply both terms of the fraction by the quotient.*

Exercise liv.

Reduce the following fractions to equivalent ones with the least common denominator.

1. $\frac{2}{3}, \frac{5}{6}, \frac{3}{5}$.
2. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$.
3. $\frac{7}{8}, \frac{9}{10}, \frac{7}{20}$.
4. $\frac{5}{9}, \frac{7}{12}, \frac{11}{18}$.
5. $\frac{5}{6}, \frac{4}{9}, \frac{7}{12}$.
6. $\frac{3}{4}, \frac{7}{10}, \frac{5}{6}$.
7. $\frac{1}{4}, \frac{7}{12}, \frac{11}{20}$.
8. $\frac{5}{6}, \frac{4}{5}, \frac{2}{3}$.
9. $\frac{7}{8}, \frac{3}{4}, \frac{4}{5}$.
10. $2, \frac{3}{10}, \frac{4}{5}$.
11. $2\frac{1}{2}, 3\frac{1}{3}, \frac{5}{6}$.
12. $\frac{7}{9}, 4\frac{1}{3}, 5$.
13. $3, 4\frac{1}{2}, \frac{1}{16}$.
14. $\frac{7}{8}, \frac{5}{14}, \frac{5}{12}, \frac{13}{42}$.
15. $\frac{4}{7}, \frac{5}{11}, \frac{9}{77}, 6$.
16. $\frac{4}{15}, 4\frac{1}{3}, \frac{4}{7}, 2$.
17. $\frac{7}{12}, \frac{17}{24}, \frac{11}{36}, \frac{55}{64}$.
18. $2\frac{1}{4}, 7\frac{1}{5}, 3\frac{2}{9}, 4\frac{7}{12}$.

107. Comparison of Fractions with respect to Magnitude.

To compare fractions we must express them in terms of the same fractional unit, that is, we must bring them to a common denominator. When they are so expressed they are compared as other numbers.

Ex. 7. Arrange the fractions $\frac{5}{6}, \frac{4}{9}, \frac{7}{12}$, in order of magnitude.

Reducing to equivalent fractions having a common denominator we have $\frac{30}{36}, \frac{16}{36}, \frac{21}{36}$; hence the order of magnitude is $\frac{4}{9}, \frac{7}{12}, \frac{5}{6}$.

If two fractions happen to have the same numerator, that which has the smaller denominator is the greater; for its units are greater, and there are the same number of them in each.

Exercise lv.

Find which is the greater,

1. $\frac{4}{7}$ or $\frac{5}{9}$.
2. $\frac{8}{9}$ or $\frac{25}{27}$.
3. $\frac{7}{15}$ or $\frac{10}{21}$.
4. $\frac{7}{15}$ or $\frac{11}{18}$.
5. $\frac{17}{20}$ or $\frac{21}{25}$.
6. $\frac{17}{19}$ or $\frac{17}{21}$.
7. $\frac{5}{84}$ or $\frac{7}{51}$.
8. $\frac{7}{40}$ or $\frac{11}{70}$.
9. $\frac{9}{11}$ or $\frac{11}{13}$.

Which is the greatest and which the least of the following:

10. $\frac{7}{15}, \frac{10}{20}, \frac{12}{35}$?
11. $\frac{6}{11}, \frac{9}{22}, \frac{10}{33}$?
12. $\frac{3}{16}, \frac{2}{9}, \frac{7}{32}$?
13. $\frac{9}{10}, \frac{17}{20}, \frac{21}{25}$?
14. $\frac{9}{11}, \frac{13}{15}, \frac{17}{21}$?
15. $\frac{5}{6}, \frac{6}{7}, \frac{8}{9}$?

Arrange in ascending order of magnitude

16. $\frac{1}{8}\frac{7}{1}$, $1\frac{3}{5}$, $2\frac{5}{7}$, $6\frac{4}{1}$, $\frac{3}{5}$ | 7. $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}\frac{3}{0}$.

Section III.—Addition.

Case I.

108. To add proper fractions.

Oral Exercises.

1. What is the sum of 2 apples, 3 apples and 5 apples?
2. What is the sum of 2 elevenths, 3 elevenths, and 5 elevenths?
3. How many ninths are $\frac{2}{9}$, $\frac{4}{9}$, $\frac{1}{9}$, and $\frac{7}{9}$?
4. James paid $\$\frac{1}{2}$ for a slate, $\$\frac{3}{5}$ for a reader, and $\$\frac{1}{5}$ for an arithmetic; how much did he pay for all?
5. Mary paid $\$\frac{3}{5}$ for some ribbon and $\$\frac{4}{5}$ for a pair of gloves; how much did she pay for both?
6. Jane bought $\frac{3}{4}$ of a yard of ribbon at one time and $\frac{7}{8}$ of a yard at another time; how much did she buy at both times?

$\frac{3}{4}=\frac{6}{8}$, and $\frac{6}{8}+\frac{7}{8}=\frac{1}{8}\frac{3}{8}=1\frac{5}{8}$. She, therefore, bought $1\frac{5}{8}$ yards.

7. A farmer sold $\frac{1}{5}$ of his grain to one man, and $\frac{1}{6}$ of it to another; how much did he sell altogether?

$\frac{1}{5}=\frac{6}{30}$, and $\frac{1}{6}=\frac{5}{30}$; $\therefore \frac{1}{5}+\frac{1}{6}=\frac{6}{30}+\frac{5}{30}=\frac{11}{30}$.

8. If I pay $\frac{1}{4}$ of a dollar for butter, $\frac{3}{5}$ of a dollar for eggs, and $\frac{1}{2}$ a dollar for cheese, how much do I pay for all?
9. What is the sum of $\frac{1}{4}$ and $\frac{1}{5}$? of $\frac{1}{6}$ and $\frac{1}{4}$? of $\frac{1}{4}$ and $\frac{3}{5}$?
10. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$? of $\frac{1}{2}$ and $\frac{1}{5}$? of $\frac{2}{3}$ and $\frac{3}{4}$?

Ex. 1. Find the sum of $\frac{2}{5}$, $\frac{5}{6}$, and $\frac{3}{10}$.

$\frac{2}{5}+\frac{5}{6}+\frac{3}{10}=\frac{12}{30}+\frac{25}{30}+\frac{9}{30}=\frac{46}{30}=\frac{23}{15}=1\frac{8}{15}$.

In this example we are required to add fifths, sixths, and tenths together. As the addends have not the same name, we cannot add them till they are changed into others having the same fractional unit. We, therefore, change the fractions into others having a

common denominator, we then add the numerators together for a new numerator, and call the sum 30ths. We reduce the improper fraction to its lowest terms, and then to the mixed number, $1\frac{8}{15}$.

Hence, *To add fractions, we reduce them to equivalent ones having a common denominator; we then add the numerators together for a new numerator and place the sum over the common denominator.*

Exercise lvi.

Add together the following fractions:

1. $\frac{2}{3}$ and $\frac{3}{4}$.
2. $\frac{4}{5}$ and $\frac{7}{8}$.
3. $\frac{5}{12}$ and $\frac{7}{18}$.
4. $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{3}{4}$.
5. $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{7}{8}$.
6. $\frac{4}{25}$, $\frac{9}{40}$ and $\frac{3}{20}$.
7. $\frac{5}{6}$, $\frac{2}{7}$, $\frac{3}{4}$ and $\frac{5}{12}$.
8. $\frac{3}{4}$, $\frac{7}{16}$, $\frac{5}{6}$ and $\frac{5}{12}$.
9. $\frac{3}{5}$, $\frac{4}{7}$, $\frac{13}{21}$ and $\frac{7}{15}$.
10. $\frac{1}{4}$, $\frac{1}{5}$, $\frac{5}{6}$ and $\frac{2}{9}$.
11. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{11}{12}$ and $\frac{29}{42}$.
12. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{7}{12}$ and $\frac{1}{17}$.

Case II.

109. To add mixed numbers.

Ex. 2. Add together $2\frac{1}{4}$, $3\frac{4}{5}$, $7\frac{5}{12}$ and $\frac{7}{10}$.

$$2\frac{1}{4}+3\frac{4}{5}+7\frac{5}{12}+\frac{7}{10}$$
$$=2+3+7+\frac{1}{4}+\frac{4}{5}+\frac{5}{12}+\frac{7}{10}$$
$$=12+\frac{15}{60}+\frac{48}{60}+\frac{25}{60}+\frac{42}{60}$$
$$=12+\frac{130}{60}=12+2\frac{1}{6}=14\frac{1}{6}.$$

NOTE.—When there are mixed numbers in the example we add the sum of the whole numbers to the sum of the fractions.

Exercise lvii.

Find the sum of the following fractions:

1. $2\frac{1}{2}$, $3\frac{2}{3}$ and $4\frac{1}{7}$.
2. [illegible], $\frac{5}{12}$ and $7\frac{11}{16}$.
3. $2\frac{2}{5}$, $3\frac{1}{3}$ and $4\frac{3}{11}$.
4. $3\frac{5}{6}$, $1\frac{2}{7}$ and $\frac{1}{15}$.
5. $1\frac{5}{16}$, $\frac{11}{84}$ and $2\frac{2}{7}$.
6. $30\frac{4}{5}$, $4\frac{9}{10}$ and $10\frac{5}{6}$.
7. $2\frac{1}{3}$, $4\frac{1}{5}$, $7\frac{1}{12}$ and $8\frac{5}{18}$.
8. [illegible], $10\frac{4}{15}$, $4\frac{1}{6}$ and $7\frac{7}{30}$.
9. [illegible], $3\frac{1}{3}$, $2\frac{3}{4}$ and $5\frac{7}{12}$.
10. $4\frac{1}{2}$, [illegible], $3\frac{1}{4}$ and $9\frac{1}{5}$.
11. $4\frac{2}{3}$, [illegible], $4\frac{2}{5}$ and $\frac{4}{15}$.
12. $5\frac{1}{3}$, $6\frac{3}{4}$, [illegible] and $\frac{11}{12}$.

Section IV.—Subtraction.

Case I.

110. To subtract one fraction from another.

Oral Exercise.

1. John has 7 marbles, James has 4: how many marbles has John more than James?

2. John has $\frac{7}{12}$ of an apple, James has $\frac{4}{12}$ of an apple; how much has John more than James?

3. How much less is $\frac{3}{8}$ than $\frac{7}{8}$? $\frac{5}{8}$ than $\frac{5}{8}$? $\frac{3}{8}$ than $\frac{4}{8}$?

4. John has $\frac{1}{3}$ of an apple, James has $\frac{1}{4}$ of an apple; how much has John more than James?

$\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$; $\therefore \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$.

5. A boy spent $\frac{1}{5}$ of his money for a coat and $\frac{1}{4}$ of it for a hat; how much had he left?

6. What is the difference between $\frac{1}{3}$ and $\frac{1}{5}$? $\frac{2}{5}$ and $\frac{2}{4}$?

7. What is the difference between $\frac{2}{3}$ and $\frac{5}{12}$? between $\frac{1}{2}$ and $\frac{1}{3}$?

Ex. 1. From $\frac{7}{12}$ take $\frac{7}{20}$.

$\frac{7}{12} - \frac{7}{20} = \frac{35}{60} - \frac{21}{60} = \frac{14}{60} = \frac{7}{30}$.

In this example we are required to take twentieths from twelfths. As we can only subtract numbers that have the same name, we must change the fractions into equivalent ones, having a common denominator. $\frac{7}{12}$ becomes $\frac{35}{60}$, and $\frac{7}{20}$ becomes $\frac{21}{60}$. We now find the difference between $\frac{35}{60}$ and $\frac{21}{60}$ to be $\frac{14}{60}$ which reduced to its lowest terms is $\frac{7}{30}$.

Hence, *To subtract one fraction from another we reduce the fractions to equivalent ones having a common denominator; we then subtract the numerator of the subtrahend from the numerator of the minuend; and place the difference over the common denominator.*

Exercise lviii.

Find the difference between.

1. $\frac{5}{7}$ and $\frac{3}{7}$.	5. $\frac{23}{24}$ and $\frac{13}{18}$.	9. $\frac{3}{10}$ and $\frac{4}{15}$.
2. $\frac{7}{9}$ and $\frac{2}{9}$.	6. $\frac{2}{3}$ and $\frac{4}{5}$.	10. $\frac{11}{17}$ and $\frac{13}{51}$.
3. $\frac{5}{8}$ and $\frac{11}{12}$.	7. $\frac{7}{8}$ and $\frac{9}{10}$.	11. $\frac{14}{36}$ and $\frac{18}{45}$.
4. $\frac{17}{20}$ and $\frac{4}{5}$.	8. $\frac{5}{7}$ and $\frac{13}{105}$.	12. $\frac{6}{55}$ and $\frac{3}{50}$.

Case II.

111. To subtract when one or both the fractions are mixed numbers.

Ex. 2. From $16\frac{1}{3}$ take $9\frac{3}{4}$.

$16\frac{1}{3} = 16\frac{4}{12}$; $16\frac{4}{12} + \frac{12}{12} = 16\frac{16}{12}$.

$9\frac{3}{4} = 9\frac{9}{12}$; $9\frac{9}{12} + 1 = 10\frac{9}{12}$.

$6\frac{7}{12}$.

We first reduce the fractional parts of the given numbers to their least common denominator, 12. Then, since $\frac{9}{12}$ cannot be taken from $\frac{4}{12}$, we add 1 or $\frac{12}{12}$ to both minuend and subtrahend (Art. 33). This gives $16\frac{16}{12}$ for minuend and $10\frac{9}{12}$ for subtrahend. Then subtracting the fractions and integers separately we have $16\frac{16}{12} - 10\frac{9}{12} = 6\frac{7}{12}$, the required result.

NOTE.—The fractional part of the result can be obtained much more readily by subtracting the numerator of the fraction in the subtrahend from the common denominator and adding the remainder to the numerator of the fraction in the minuend. Thus $12-9=3$; $3+4=7$. This is the numerator of the fractional part of the result. The integral part is obtained as above.

Exercise lix.

Find the value of

1. $3\frac{3}{5} - 1\frac{1}{5}$.
2. $2\frac{1}{6} - 1\frac{11}{12}$.
3. $6\frac{1}{2} - 2\frac{11}{12}$.
4. $8\frac{3}{7} - 5\frac{1}{2}$.
5. $3\frac{5}{12} - 1\frac{5}{6}$.
6. $2\frac{3}{8} - 1\frac{7}{12}$.
7. $5\frac{3}{14} - 2\frac{5}{21}$.
8. $4\frac{7}{9} - 1\frac{11}{14}$.
9. $9 - 4\frac{3}{7}$.
10. $18\frac{2}{5} - 5\frac{8}{9}$.
11. $28\frac{16}{63} - 3\frac{9}{14}$.
12. $25\frac{1}{16} - 14\frac{13}{24}$.

Exercise lx.

Practical Problems.

1. The sum of two numbers is $26\frac{1}{4}$ and the less is $7\frac{2}{3}$; what is the greater?
2. From a barrel of vinegar containing $31\frac{1}{2}$ gallons, $14\frac{1}{8}$ gallons were drawn; how much was there left?
3. To what fraction must the sum of $\frac{1}{2}$ and $\frac{1}{18}$ be added that the sum may be $\frac{1}{15}$?
4. From a piece of silk containing $35\frac{2}{5}$ yards, $14\frac{3}{4}$ yards were sold; how much remained in the piece?

5. From \$10, \$2$\frac{1}{8}$ were given to James, \$3$\frac{1}{4}$ to Jane, \$1$\frac{1}{2}$ to Emily, and the remainder to Mary; what did she receive?

6. A has two farms, one of $70\frac{3}{7}$ acres and the other of $118\frac{19}{35}$ acres; if he sells $87\frac{11}{21}$ acres, how much land has he left?

7. How much paper has a printer left if he had on hand $30\frac{7}{12}$ reams, and has used $7\frac{2}{5}$ reams for one job, and $3\frac{3}{4}$ reams for another?

8. A grocer, having mixed $15\frac{3}{4}$ pounds of tea with $32\frac{1}{12}$ pounds of a different kind, sold all the mixture but $13\frac{5}{8}$ pounds; how much did he sell?

9. B started on a journey of 100 miles; the first day he travelled $30\frac{5}{12}$ miles, the second day $36\frac{5}{9}$ miles; how far has he yet to go?

10. Henry had \$47$\frac{1}{9}$, and James as much lacking \$9$\frac{4}{15}$; how many dollars had James?

11. The selling price of a horse was \$125$\frac{3}{7}$; the gain was \$26$\frac{3}{5}$; what was the cost price?

12. Find the sum of the greatest and least of the fractions $\frac{3}{8}$, $\frac{5}{11}$, $\frac{4}{9}$, $\frac{7}{10}$, the sum of the other two, and the difference of these sums.

Exercise lxi.

Practical Problems.

1. \$249$\frac{1}{4}$ is \$134$\frac{1}{8}$ less than the value of my horse and carriage; what are they worth?

2. A boy paid \$$\frac{1}{8}$ for a ball, \$$\frac{1}{6}$ for a slate, \$$\frac{7}{8}$ for a knife and \$$\frac{7}{10}$ for a book; how much did he spend?

3. What is the entire weight of 4 crocks of butter weighing as follows: the first $10\frac{1}{4}$ pounds; the second, $11\frac{3}{8}$ pounds; the third, $13\frac{5}{16}$ pounds, and the fourth $14\frac{1}{2}$ pounds?

4. Three men bought a horse; the first paid \$415, the second paid \$$383\frac{1}{2}$, and the third as much as the other two; what was the cost of the horse?

5. A grocer has 3 barrels of molasses; the first contains $27\frac{7}{8}$ gallons, the second, $42\frac{7}{9}$, and the third, $36\frac{2}{3}$ gallons; how many gallons are there in the three barrels?

6. What number is that from which if $5\frac{1}{12}$ is taken the remainder will be $2\frac{5}{16}$?

7. A merchant sold $34\frac{5}{9}$ yards of cloth for \$$94\frac{5}{16}$, \$$39\frac{3}{17}$

yards for $124\frac{5}{9}$, and $70\frac{5}{8}$ yards for $184\frac{2}{3}$; how many yards of cloth did he sell and how much did he receive for the whole?

8. Four geese weigh respectively $9\frac{5}{8}$, $10\frac{3}{5}$, $12\frac{7}{16}$ and $11\frac{5}{8}$ pounds; what is their entire weight?

9. A lady hired a gardener at 15 cents an hour for 3 days; how much did she pay him if he worked $6\frac{5}{12}$ hours the first day, $7\frac{5}{8}$ the second, and $5\frac{3}{4}$ the third?

10. If $5\frac{2}{7}$ gallons of brandy are mixed with $1\frac{9}{14}$ gallons of water and $3\frac{5}{21}$ gallons of whiskey, how many gallons are there in the mixture?

11. A paid $\$46\frac{1}{2}$ for an ox, and $\$57\frac{5}{12}$ more than this for a horse; for how much must he sell them to gain $\$26\frac{7}{9}$?

12. A owns $71\frac{5}{8}$ acres of land, B owns $112\frac{5}{18}$ acres, C owns $217\frac{1}{2}\frac{7}{4}$ acres, and D owns $372\frac{17}{42}$ acres, how many acres do they together own?

Section V.—Multiplication and Division of Fractions.

Case I.

112. To multiply a fraction by a whole number.

Oral Exercises.

1. If 5 cents are multiplied by 3, what is the product?
2. If 3 marbles are multiplied by 5, what is the product?
3. If 3 sevenths are multiplied by 5, what is the product? 3 sevenths ($\frac{3}{7}$) multiplied by 5=15 sevenths ($\frac{15}{7}=2\frac{1}{7}$).
4. How much will 5 pair of ducks cost at $\$\frac{3}{4}$ a pair?
5. How much will 10 yards of cloth cost at $\$\frac{1}{4}$ a yard?
6. If it requires $\frac{3}{4}$ of a yard of cloth to make a vest, how many yards will it require to make 8 vests?
7. If a man earns $\$\frac{1}{4}$ in 1 day, how much will he earn at the same rate in 10 days?
8. If a hat costs $\$\frac{3}{5}$, how much will 12 hats cost?
9. If 1 yard of muslin costs $\$\frac{5}{8}$, what would be the cost of 4 yards?
10. If a man can plough $\frac{3}{5}$ of an acre of land in 1 day, how many acres could he plough in 7 days?
11. If a barrel of flour costs $\$8\frac{3}{4}$, what will 5 barrels cost?

Multiply the fractional and integral parts separately, and add the products.

12. How much is 6 times $\frac{7}{8}$? $\frac{9}{10}$? $\frac{11}{12}$? $\frac{3}{6}$?
13. How much is 8 times $\frac{10}{12}$? $\frac{9}{12}$? $\frac{5}{8}$? $\frac{5}{6}$?
14. How much is 10 times $1\frac{3}{20}$? $2\frac{3}{4}$? $6\frac{1}{3}$? $3\frac{5}{12}$?

Ex. 1. Multiply $\frac{3}{10}$ by 5.

Since 3 apples multiplied by $5 = 15$ apples,
so 3 tenths ($\frac{3}{10}$) multiplied by $5 = 15$ tenths ($\frac{15}{10}$);

$$\therefore\ 5 \times \tfrac{3}{10} = \tfrac{15}{10} = \tfrac{5 \times 3}{10};$$

$$\text{but } \tfrac{5 \times 3}{10} = \tfrac{3}{2} \text{ (Art. 77)} = \tfrac{3}{10 \div 5}.$$

Hence, *To multiply a fraction by any number we either multiply the numerator or divide the denominator by it.*

Exercise lxii.

Multiply

1. $\frac{15}{16}$ by 9.	4. $\frac{14}{15}$ by 10.	7. $\frac{15}{17}$ by 21.
2. $\frac{13}{16}$ by 8.	5. $\frac{16}{21}$ by 49.	8. $\frac{8}{19}$ by 24.
3. $\frac{9}{28}$ by 7.	6. $\frac{11}{14}$ by 91.	9. $\frac{15}{112}$ by 36.

10. At $\$1\frac{1}{4}$ a day, how much does a man earn in 4 weeks of 6 days each?
11. What is the cost of 36 dozen eggs, at $36\frac{1}{2}$ cents a dozen?
12. At $\$16\frac{3}{4}$ a month, what will a boy earn in 12 months?
13. What is the cost of 12 pounds of beef at $14\frac{3}{4}$ cents a pound?
14. What is the cost of 14 bushels of oats at $62\frac{1}{2}$ cents a bushel?

Case II.

113. To divide a fraction by a whole number.

Oral Exercises.

1. If 8 apples are divided by 4, what is the quotient?
2. If 8 ninths are divided by 4, what is the quotient?
3. Divide $\frac{8}{9}$ by 2; $\frac{6}{11}$ by 3; $\frac{16}{17}$ by 8.
4. If 3 ducks cost $\frac{6}{10}$ of a dollar, how much will 1 duck cost?

Cost of 3 ducks $= \$\frac{6}{10}$;
" 1 duck $= \frac{1}{3}$ of $\$\frac{6}{10} = \$\frac{2}{10} = \$\frac{1}{5} = 20$ cents.

5. If 3 caps cost $\frac{6}{10}$ of a dollar, how much will 1 cap cost?

6. William had $\frac{9}{12}$ of an orange, and divided it equally among 3 of his schoolmates; what part of an orange did he give to each?

7. A man shares $\frac{8}{9}$ of a ton of coal among 5 persons; how much does each get?

Share of 5 persons $= \frac{8}{9}$ of a ton;
" 1 person $= \frac{1}{5}$ of $\frac{8}{9}$ of a ton
$= \frac{8}{45}$ of a ton.

8. If 8 men can do $\frac{3}{5}$ of a piece of work, how much can 1 man do in the same time?

9. If 3 men together own $\frac{3}{4}$ of a vessel, what part of the vessel does 1 man own, if their shares are equal?

10. A lady gave $\frac{3}{5}$ of a pound of candy to her 4 sons and 2 daughters; what was the share of each?

Ex. 2. Divide $\frac{8}{9}$ by 4.

Since 8 apples divided by $4 = 2$ apples,
so 8 ninths $(\frac{8}{9})$ divided by $4 = 2$ ninths $(\frac{2}{9})$;
$\therefore \frac{8}{9} \div 4 = \frac{2}{9} = \frac{8 \div 4}{9}$.

We may obtain the same result by multiplying the denominator of $\frac{8}{9}$ by 4, and reducing the resulting fraction to its lowest terms; thus,

$$\frac{8}{9} \div 4 = \frac{8}{4 \times 9} = \frac{2}{9}.$$

Hence, *To divide a fraction by any number we either divide the numerator by the number or multiply the denominator by it.*

Exercise lxiii.

NOTE.—Reduce mixed numbers to improper fractions.

Divide

1. $\frac{15}{16}$ by 5.	4. $\frac{7}{8}$ by 17.	7. $4\frac{2}{7}$ by 10.
2. $\frac{35}{40}$ by 7.	5. $7\frac{1}{3}$ by 6.	8. $129\frac{1}{5}$ by 16.
3. $\frac{63}{64}$ by 9.	6. $4\frac{2}{3}$ by 7.	9. $287\frac{5}{8}$ by 12.

10. If a man can reap $22\frac{3}{4}$ acres of wheat in 7 days, how much could he reap in 1 day?

11. If a man can cut $15\frac{3}{4}$ cords of wood in 7 days how many cords could he cut in 1 day?

12. If a man can walk $38\frac{1}{3}$ miles in 10 hours, how far could he walk in 1 hour?

13. If 7 tons of coal cost $\$60\frac{2}{3}$, what is the price per ton?

Case III.

114. To multiply a whole number or a fraction by a fraction.

Oral Exercises.

1. If a yard of muslin cost 12 cents, how much will $\frac{1}{3}$ of a yard cost?

2. If a man earns $60 a month, how much will he earn in $\frac{1}{6}$ of a month?

3. If a ton of hay costs $25, how much will $\frac{4}{5}$ of a ton cost?

Cost of 1 ton = $25;
" $\frac{1}{5}$ " = $\frac{1}{5}$ of $25 = $5;
" $\frac{4}{5}$ " = $4 \times \$5 = \$20 = \$\frac{4 \times 25}{5}$.

4. If a house costs $800 and a barn $\frac{3}{4}$ as much, how much does the barn cost?

5. If $\frac{4}{5}$ of $50 is 8 times the cost of a shawl, what does the shawl cost?

6. How much will $\frac{3}{4}$ of a bushel of potatoes cost at $\$\frac{4}{5}$ a bushel?

Cost of 1 bushel = $\$\frac{4}{5}$;
" $\frac{1}{4}$ " = $\frac{1}{4}$ of $\frac{4}{5} = \$\frac{4}{4 \times 5}$;
" $\frac{3}{4}$ " = $3 \times \$\frac{4}{4 \times 5} = \$\frac{3 \times 4}{4 \times 5}$.

7. John had $\$\frac{5}{6}$, and lost $\frac{2}{3}$ of it; what part of a dollar did he lose?

8. Robert had $\frac{3}{4}$ of a melon, and gave his brother $\frac{2}{3}$ of it; what part of the melon did he give away?

9. A man owning $\frac{9}{10}$ of a mill, sold $\frac{1}{3}$ of $\frac{2}{5}$ of what he owned; what part of the mill did he sell?

10. Thomas had $\frac{5}{8}$ of an orange, and gave to John $\frac{2}{3}$ of $\frac{3}{4}$ of what he had; what part of the whole orange did he give away?

Ex. 3. Multiply $\frac{4}{19}$ by $\frac{3}{7}$.

Here, $3 \times \frac{4}{19} = \frac{12}{19}$. (Art. 112.)

This result is evidently 7 times too great, because $\frac{4}{19}$ is not to be multiplied by 3 but by $\frac{1}{7}$ of 3 (Art. 94); we must therefore divide $\frac{12}{19}$ by 7. Hence we have

$\frac{3}{7} \times \frac{4}{19} = \frac{12}{19} \div 7 = \frac{12}{133}$ (Art. 113)

$= \frac{3 \times 4}{7 \times 19}$

$= \frac{\text{the product of the numerators}}{\text{the product of the denominators}}$

Hence, *The product of two fractions is found by multiplying the two numerators together for the numerator, and the two denominators together for the denominator of the product.*

In a similar way it may be shown that the product of more fractions than two is found by multiplying all the numerators together for the numerator, and all the denominators together for the denominator of the product.

Exercise lxiv.

NOTE.—Cancel the factors common to the numerators and the denominators.

Find the value of

1. $\frac{5}{6} \times 18$.
2. $\frac{8}{9} \times 45$.
3. $\frac{5}{6} \times 43$.
4. $\frac{7}{16} \times 124$.
5. $1\frac{2}{5} \times \frac{9}{10}$.
6. $\frac{9}{16} \times \frac{5}{21} \times \frac{7}{25}$.
7. $\frac{9}{7} \times \frac{7}{12} \times \frac{14}{15}$.
8. $\frac{10}{11} \times \frac{5}{10} \times \frac{7}{12}$.
9. $\frac{45}{49} \times \frac{50}{65} \times \frac{91}{75}$.
10. $\frac{16}{21} \times \frac{21}{28} \times \frac{35}{54}$.
11. $\frac{9}{10} \times \frac{10}{27} \times \frac{33}{48}$.
12. $\frac{91}{72} \times \frac{20}{21} \times \frac{7}{9}$.

13. What should be paid for $\frac{1}{2}$ of $\frac{7}{8}$ of a pound of tea, at the rate of $\frac{13}{16}$ of a dollar per pound?

14. What should be paid for $\frac{2}{3}$ of a barrel of apples, if the whole barrel is worth $\frac{15}{16}$ of a dollar?

15. *A* has $\frac{3}{4}$ of \$375, *B* has $\frac{4}{5}$ as much, and *C* $\frac{3}{5}$ as much as both; how many dollars has each, and how many have they all?

Ex. 4. Multiply $6\frac{2}{3}$ by $7\frac{4}{5}$.

$$7\tfrac{4}{5} = \tfrac{39}{5}, \text{ and } 6\tfrac{2}{3} = \tfrac{20}{3};$$

$$\therefore\ 7\tfrac{4}{5} \times 6\tfrac{2}{3} = \tfrac{39}{5} \times \tfrac{20}{3} = \tfrac{13}{1} \times \tfrac{4}{1} = 52.$$

Exercise lxv.

Find the value of

1. $3\frac{1}{4} \times 5\frac{2}{7}$.
2. $6\frac{3}{5} \times 7\frac{2}{3}$.
3. $17\frac{2}{5} \times 16\frac{2}{3}$.
4. $39\frac{3}{5} \times 33\frac{1}{3}$.
5. $6\frac{2}{7} \times 4\frac{3}{11} \times 77 \times 4\frac{1}{4}$.
6. $3 \times 7\frac{1}{2} \times \frac{11}{15} \times 3\frac{8}{11}$.

7. If a cord of wood costs $\$4\frac{1}{4}$, what will $3\frac{1}{2}$ cords cost?

Cost of 1 cord $= \$4\frac{1}{4}$;

" $3\frac{1}{2}$ cords $= 3\frac{1}{2} \times \$4\frac{1}{4}$

$= \frac{7}{2} \times \$\frac{17}{4} = \$\frac{119}{8} = \$14\frac{7}{8}$.

8. If a pound of sugar is worth $9\frac{1}{2}$ cents, what will $4\frac{1}{2}$ pounds cost?

9. If a man reaps $3\frac{3}{4}$ acres of wheat in a day, how many acres could he reap in $2\frac{2}{3}$ days?

10. What would be the cost of $18\frac{3}{4}$ acres of land at $\$18\frac{3}{4}$ per acre?

11. If a ton of coal costs $\$6\frac{7}{8}$, what will be the cost of $9\frac{4}{5}$ tons at the same rate?

12. Mr. Jones rented a house at $\$42\frac{3}{4}$ a month, taking a lease for 5 years, but disposed of the lease at the end of $3\frac{1}{4}$ years; how much rent did he pay?

13. A bill of books at retail amounts to $\$375\frac{5}{8}$, but I got a reduction of $\frac{1}{5}$ for wholesale and $\frac{5}{50}$ for cash; what was the exact amount of the bill?

Case IV.

115. To divide a whole number or a fraction by a fraction.

Oral Exercises.

1. How many parcels of sugar, each containing 3 pounds, can you make out of 24 pounds?

2. How often is 3 lbs. contained in 24 lbs.

3. If 24 apples are divided equally among 6 boys, how many apples will each boy receive?

4. How often is 6 apples contained in 24 apples?

5. When both *dividend* and *divisor* are concrete numbers, what kind of number is the quotient? Give examples.

6. How often is 4 ninths ($\frac{4}{9}$) contained in 8 ninths ($\frac{8}{9}$)?

7. How often is 2 fifths ($\frac{2}{5}$) contained in 12 fifteenths $\frac{12}{15}$?

Reduce the fractions to equivalent ones having a common denominator.

8. How often is $\frac{2}{10}$ contained in $\frac{4}{5}$?

9. At $\frac{2}{10}$ of a dollar each, how many caps can I buy for $\$\frac{4}{5}$?

10. If a pound of coffee costs $\$\frac{1}{4}$, how much can be bought for $\$1\frac{1}{4}$?

11. At $\$\frac{3}{5}$ a yard, how many yards of cloth can be bought for $\$6$?

Yards bought for $\$\frac{3}{5} = 1$ yd. $= \frac{5}{5}$ yd.;
" " $\$1 = \frac{5}{5}$ yds. $\div \frac{3}{5} = \frac{5}{3}$ yards;
" " $\$6 = 6 \times \frac{5}{3}$ yards $= 10$ yards.

12. When apples are worth $\$\frac{3}{4}$ a bushel, how many bushels can be bought for $\$\frac{3}{5}$?

13. At $\$\frac{7}{8}$ a yard, how many yards of silk can be bought for $\$4\frac{1}{2}$?

Ex. 5. Divide $\frac{2}{3}$ by $\frac{3}{5}$.

$$\frac{2}{3} = \frac{10}{15}, \text{ and } \frac{3}{5} = \frac{9}{15}.$$

Since 10 apples divided by 9 apples $= \frac{10}{9}$,
so 10 fifteenths $(\frac{10}{15})$ divided by 9 fifteenths $(\frac{9}{15}) = \frac{10}{9}$;

$$\therefore \frac{2}{3} \div \frac{3}{5} = \frac{10}{15} \div \frac{9}{15} = \frac{10}{9}$$

$$= \frac{5 \times 2}{3 \times 3} = \frac{5}{3} \times \frac{2}{3}$$

= dividend multiplied by divisor inverted.

Or we may reason as follows:

Here, $\frac{2}{3} \div 3 = \frac{2}{9}$. (Art. 113.)

This result is evidently 5 times too small, because $\frac{2}{3}$ is not to be divided by 3, but by $\frac{1}{5}$ of 3. (Art. 94.) The true quotient must therefore be 5 times $\frac{2}{9}$. Hence

$$\frac{2}{3} \div \frac{3}{5} = 5 \times \frac{2}{9} = \frac{5 \times 2}{3 \times 3} = \frac{5}{3} \times \frac{2}{3}, \text{ as before.}$$

Ex. 6. Divide 10 by $\frac{3}{4}$.

$$10 = \frac{10}{1} = \frac{40}{4}.$$

$$10 \div \frac{3}{4} = \frac{40}{4} \div \frac{3}{4} = \frac{40}{3} = \frac{4 \times 10}{3 \times 1} = \frac{4}{3} \times \frac{10}{1}.$$

= dividend multiplied by divisor inverted.

Hence, *To divide one fraction by another, invert the divisor and multiply the dividend by the fraction thus formed.*

Exercise lxvi.

Divide

1. 10 by $\frac{5}{7}$.	5. $\frac{3}{4}$ by $\frac{3}{12}$.	9. $9\frac{3}{4}$ by $\frac{9}{10}$.
2. 18 by $\frac{9}{7}$.	6. $\frac{15}{17}$ by $\frac{9}{10}$.	10. $7\frac{5}{24}$ by $12\frac{8}{15}$.
3. 30 by $\frac{7}{8}$.	7. $\frac{13}{14}$ by $\frac{7}{15}$.	11. $21\frac{3}{7}$ by $12\frac{5}{21}$.
4. 40 by $3\frac{1}{2}$.	8. $\frac{185}{286}$ by $\frac{15}{22}$.	12. $45\frac{3}{5}$ by $2\frac{8}{9}$.

Exercise lxvii.

Practical Problems.

1. If $\frac{3}{4}$ of a yard of cloth cost 24 cents, what would a whole yard cost?

Cost of $\frac{3}{4}$ of a yard = 24 cents;
" $\frac{1}{4}$ " " = 8 "
" $\frac{4}{4}$ or 1 yard = 4×8 cents = 32 cents.

2. At $\$1\frac{7}{9}$ per bushel, how many bushels of wheat can be bought for $\$42\frac{2}{3}$?

3. If a ton of coal is worth $\$6\frac{2}{5}$, how many tons can be bought for $\$89\frac{6}{10}$?

4. If a bushel of apples costs $\$2\frac{1}{4}$, how many bushels could be bought for $\$60\frac{7}{8}$?

5. If a man earns $\$7\frac{7}{8}$ in a week, how long will it require him to earn $\$20\frac{3}{4}$?

6. A man divided $30\frac{4}{5}$ pounds of flour among the poor, giving to each $2\frac{1}{5}$ pounds; how many persons were there?

7. If $21\frac{2}{4}$ pounds of tea cost $\$18\frac{3}{80}$, what will 1 pound cost?

8. If an errand boy earns $\$7\frac{7}{8}$ in a week, how long will it require him to earn $\$20\frac{1}{4}$?

9. A man raised $93\frac{3}{4}$ bushels of wheat on $8\frac{1}{8}$ acres of land; how many bushels per acre was that?

10. In how many days will a horse eat $329\frac{1}{3}$ pecks of oats, if he eats $1\frac{3}{4}$ pecks daily?

11. If $\frac{4}{5}$ of an acre of land sells for $30, what will an acre sell for at the same rate?

12. The product of two numbers is 27, and one of them is $2\frac{5}{6}$; what is the other?

Section VI.—Complex Fractions.

116. To reduce a Complex Fraction to a Simple One.

117. A **Complex Fraction** is one in which either the numerator or denominator, or both, are fractions; as $\frac{\frac{3}{4}}{7}$, $\frac{4}{2\frac{1}{2}}$, $\frac{3\frac{1}{4}}{\frac{1}{8}}$.

Ex. 1. Reduce $\frac{\frac{1}{6}}{\frac{3}{5}}$ to a simple fraction.

Since the numerator of a fraction is the dividend, and the denominator the divisor (Art. 94), we have simply to divide the numerator $\frac{1}{6}$, by the denominator, $\frac{3}{5}$, as in *division of fractions*;

hence, $\frac{\frac{1}{6}}{\frac{3}{5}} = \frac{1}{6} \div \frac{3}{5} = \frac{1}{6} \times \frac{5}{3} = \frac{5}{18}$.

Ex. 2. Reduce $\frac{8}{4\frac{1}{5}}$ to a simple fraction.

$$\frac{8}{1} \div 4\tfrac{1}{5} = \frac{8}{1} \times \frac{5}{21} = \frac{40}{21} = 1\tfrac{19}{21}.$$

In many cases it is simpler to multiply the numerator and denominator of the complex fraction by the L. C. M. of the denominators; thus,

$$\frac{8}{4\frac{1}{5}} = \frac{5 \times 8}{5 \times 4\frac{1}{5}} = \frac{40}{21} = 1\tfrac{19}{21}.$$

Ex. 3. Simplify $\dfrac{1+\frac{1}{3}}{1-\frac{1}{3}} \div \dfrac{1+\frac{3}{5}}{1-\frac{3}{5}}$.

Multiplying the numerator and denominator of the dividend by 3, and the numerator and denominator of the divisor by 5, we have

$$\frac{3+1}{3-1} \div \frac{5+3}{5-3} = \frac{4}{2} \div \frac{8}{2} = \frac{4}{8} = \frac{1}{2}.$$

Exercise lxviii.

1. $\dfrac{52}{3\frac{1}{4}}$.

2. $\dfrac{3\frac{3}{4}}{5}$.

3. $\dfrac{1\frac{4}{15}}{1\frac{7}{25}}$.

4. $\dfrac{11}{7\frac{7}{18}}$.

5. $\dfrac{4\frac{2}{9}}{2\frac{3}{8}}$.

6. $\dfrac{9\frac{7}{9}}{2\frac{1}{27}}$.

7. $\dfrac{5\frac{3}{11}}{2\frac{7}{11}}$.

8. $\dfrac{8\frac{3}{4}}{5\frac{5}{8}}$.

9. $\dfrac{15\frac{3}{5}}{7\frac{4}{5}}$.

10. $\dfrac{9}{3\frac{3}{11}}$.

11. $\dfrac{23}{2\frac{2}{3}+\frac{2}{5}}$.

12. $\dfrac{2\frac{1}{2}+1\frac{2}{3}}{3\frac{2}{3}-2\frac{1}{2}}$.

13. $\dfrac{14\frac{1}{4}-6\frac{1}{2}}{3\frac{1}{3}+7\frac{1}{5}}$.

14. $\dfrac{4\frac{1}{12}+6\frac{1}{8}}{9\frac{1}{8}-3\frac{1}{4}}$.

15. $\dfrac{2\frac{1}{3}+1\frac{3}{15}}{9\frac{2}{7}-\frac{39}{112}}$.

16. $\dfrac{3\frac{1}{3} \text{ of } 1\frac{1}{2}}{1\frac{1}{3} \text{ of } 1\frac{1}{4}}$.

17. $\dfrac{3\frac{3}{7} \text{ of } 21\frac{1}{12}}{\frac{1}{33} \text{ of } 8\frac{9}{14}}$.

18. $\dfrac{4\frac{4}{15} \text{ of } 2\frac{5}{6}}{5\frac{1}{5}-4\frac{1}{2}}$.

19. $\dfrac{2\frac{1}{4}+3\frac{1}{5}}{5\frac{1}{2} \times 1\frac{1}{2}}$.

20. $\dfrac{2\frac{1}{3}-1\frac{1}{5}}{1\frac{3}{5} \text{ of } 1\frac{7}{8}}$.

118. Brackets, which are of several kinds—*e.g.*, (), { }, [],—are used to denote that all numbers included within any pair of them are to be considered as forming but one number, and are therefore to be equally affected by any number not included within the same pair of brackets, thus

$$(6+3)\times 8 = 9\times 8 = 72.$$

Also, $[8+2\times\{9+3\times(4+3)+17\}+21]\times 9$

$=[8+2\times\{9+21+17\}+21]\times 9$

$=[8+94+21]\times 9$

$=123\times 9$

$=1107.$

119. A Vinculum is a sign sometimes used instead of brackets. It consists of a *line* drawn over the numbers to be considered as forming one number—thus, $2\times\overline{8+3}=2\times 11=22$.

In removing brackets from an expression, it is best to commence with the *innermost* and to remove the brackets one by one, the outermost last of all.

120. In finding the value of an involved fraction, it must be remembered that the signs $\times$, $\div$, and "of" *connect* the terms between which they are found into one quantity, while the signs + and − *separate* the terms between which they occur. *Brackets, however, should always be used where there is a possibility of ambiguity.*

The following cases will illustrate the generally received usage in Arithmetic respecting these signs:—

(1) *The operations indicated by "of," $\times$, and $\div$ should be performed before adding or subtracting.*

Ex. 4. $\frac{2}{4}+\frac{2}{3}$ of $\frac{9}{11}-\frac{1}{4}\div\frac{1}{3}+\frac{2}{3}\times\frac{6}{11}$

$=\frac{2}{4}+(\frac{2}{3}$ of $\frac{9}{11})-(\frac{1}{4}\div\frac{1}{3})+(\frac{2}{3}\times\frac{6}{11})$

$=\frac{2}{4}+\frac{6}{11}-\frac{3}{4}+\frac{4}{11}$

$=\frac{10}{11}.$

(2) *The operations indicated by $\times$ and $\div$ should be performed in the order in which they occur.*

Ex. 5. $\frac{2}{3}\times\frac{6}{11}\div\frac{3}{4}$

$=\frac{2}{3}\times\frac{6}{11}\times\frac{4}{3}$

$=\frac{16}{33}.$

Ex. 6. $\frac{2}{3}\div\frac{6}{11}\times\frac{3}{4}$

$=\frac{2}{3}\times\frac{11}{6}\times\frac{3}{4}$

$=\frac{11}{12}.$

Ex. 7. $\frac{3}{4}\times\frac{5}{6}\div\frac{3}{8}\times\frac{1}{4}$
$=\frac{3}{4}\times\frac{5}{6}\times\frac{8}{3}\times\frac{1}{4}$
$=\frac{1}{1}\frac{5}{4}$.

(3) *The operation indicated by "of" should be performed before that indicated by* $\div$; this is the only case in which custom makes a distinction between $\times$ and "of."

Ex. 8. $\frac{3}{8}$ of $2\frac{5}{7}\div 1\frac{9}{7}$ of $\frac{3}{4}$
$=(\frac{3}{8}\times\frac{1}{7}^{9})\div(\frac{1}{7}^{3}\times\frac{3}{4})$
$=\frac{3}{8}\times\frac{1}{7}^{9}\times\frac{7}{1}^{7}\times\frac{4}{3}$
$=\frac{1}{2}\frac{9}{1}$.

121. If a number is placed before a bracket, *with no sign after it*, it is implied that the contents of the bracket is to be multiplied by the number. In like manner, if two brackets stand side by side, with no sign between them, it is implied that the contents of one bracket are to be multiplied by the contents of the other.

Ex. 9. Simplify $\dfrac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\text{ of }3\frac{1}{3}+\frac{13}{36}}$.

$$\frac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\text{ of }3\frac{1}{3}+\frac{13}{36}}=\frac{2\frac{1}{4}-\frac{2}{3}\text{ of }\frac{11}{6}}{\frac{1}{5}\text{ of }\frac{10}{3}+\frac{13}{36}}=\frac{2\frac{1}{4}-\frac{11}{9}}{\frac{2}{3}+\frac{13}{36}}=\frac{36(2\frac{1}{4}-\frac{11}{9})}{36(\frac{2}{3}+\frac{13}{36})}$$
$$=\frac{81-44}{24+13}=\frac{37}{37}=1.$$

NOTE 1.—In multiplying $2\frac{1}{4}$ by 36, multiply the fractional and integral parts separately, and add the results.

2. In multiplying $\frac{11}{9}$ by 36, divide the denominator, 9, into 36, and multiply the numerator, 11, by the quotient, 4.

Ex. 10. Simplify $8+\cfrac{1}{2+\cfrac{5}{7+\cfrac{1}{2}}}$

Beginning at the lowest fraction, considering 5 as its numerator and $7+\frac{1}{2}$ as its denominator, and multiplying each of these terms by 2, we have

$$8+\cfrac{1}{2+\cfrac{5}{7+\cfrac{1}{2}}}=8+\cfrac{1}{2+\cfrac{10}{14+1}}=8+\frac{15}{30+10}=8\frac{3}{8}.$$

Exercise lxix.

Simplify the following expressions:

1. $\frac{3\frac{2}{3}}{4\frac{1}{7}} \times (3\frac{5}{8} \times 5\frac{4}{7}) - 17\frac{3}{4}$.
2. $(\frac{2}{3}$ of $\frac{5}{6}$ of $3\frac{3}{4} + 8\frac{2}{3}) \div (10\frac{1}{2} - 7\frac{5}{12})$.
3. $\frac{9\frac{5}{8}\text{ of }\frac{3}{11}}{6\frac{3}{10}\text{ of }\frac{5}{8}} + \frac{6\frac{1}{3}\text{ of }\frac{9}{10}}{4\frac{1}{8}\text{ of }2\frac{19}{11}}$
4. $(19\frac{4}{5} - 3\frac{3}{4}) \times (3\frac{4}{5} - 2\frac{3}{7})$.
5. $19\frac{4}{5} - 3\frac{3}{4} \times 3\frac{4}{5} - 2\frac{3}{7}$.
6. $19\frac{4}{5} - 3\frac{3}{4} \times (3\frac{4}{5} - 2\frac{3}{7})$.
7. $\left(\frac{4\frac{1}{2}+5\frac{3}{4}}{5\frac{7}{8}-2\frac{1}{4}}\right) \times \left(2 - \frac{3}{5\frac{1}{8}}\right)$.
8. $\frac{\frac{4}{7}\text{ of }\frac{3}{8}}{5\frac{1}{14} - 4\frac{81}{26}} - \frac{\frac{5}{11}\text{ of }1\frac{13}{15}}{7\frac{15}{22} - 6\frac{1}{2}}$.
9. $(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{4} + \frac{1}{5}) - (\frac{1}{2} - \frac{1}{3}) \times (\frac{1}{4} - \frac{1}{5})$.
10. $(\frac{1}{2} + \frac{1}{3}) \times (\frac{1}{4} - \frac{1}{5}) \div (\frac{1}{6} - \frac{1}{7})$.
11. $(\frac{2}{10} + \frac{1}{3}) \div (3 - \frac{1}{3}) \times (\frac{1}{3} + 2\frac{1}{5})$.
12. $(2\frac{1}{3} + 1\frac{1}{6} + 3\frac{1}{2}) \div \frac{1}{4}$ of $\frac{6}{5}$ of $1\frac{5}{9}$.
13. $\frac{1\frac{1}{4} - \frac{5}{12}}{1\frac{1}{4} + \frac{5}{12}} + \frac{6}{7}\text{ of }\frac{9\times5}{14\times3} - \frac{11\frac{1}{4}}{15}$.
14. $\frac{3\frac{1}{2} - 2\frac{1}{6}}{\frac{1}{4}\text{ of }(\frac{1}{6}+\frac{1}{7})} + 15\frac{5}{9}$.
15. $2\frac{1}{2} + \frac{3\frac{1}{2} - \frac{1}{4}}{3\frac{1}{2} + \frac{1}{4}} - 2\frac{3}{5}$ of $\frac{13}{15} - \frac{6}{7}$.
16. $2\frac{1}{2} + \frac{3\frac{1}{2} - \frac{1}{4}}{3\frac{1}{2} + \frac{1}{4}} - 2\frac{3}{5}$ of $(\frac{12}{13} - \frac{6}{7})$.
17. $\frac{39}{72} \times \frac{218}{425} \div \frac{327}{2859} \times \frac{75}{1117}$.
18. $4\frac{1}{2}$ of $\frac{5}{27} \div 5\frac{2}{5}$ of $1\frac{7}{9}$.
19. $4\frac{1}{3}$ of $2\frac{1}{4} - 2\frac{1}{15} + 3\frac{5}{9} \times 3\frac{3}{8} + 12\frac{1}{7} \div 3\frac{3}{5}$.
20. $\frac{2\frac{1}{3}}{7} + \frac{4}{10\frac{1}{2}} - \frac{8}{15}\text{ of }\frac{3}{28}$.

EXAMINATION PAPERS.

I.

1. What is a fraction? Define a Simple, a Compound, a Proper Fraction.

2. What rule of fractions is anticipated in reducing a mixed number to an improper fraction?

3. If the numerator and denominator of a fraction be multiplied by the same number, the fraction thus obtained is equivalent to the former fraction. Prove the truth of this statement by taking the fractions $\frac{2}{3}$ and $\frac{10}{15}$ and showing that they are equivalent.

4. What name is given to a fractional expression of the form of $\frac{3}{5}$ of $\frac{7}{8}$?

State and illustrate the rule for multiplying one fraction by another, and show that the product of two proper fractions must always be numerically less than either of them?

5. What is meant by a vulgar fraction? When is a vulgar fraction greater than unity? What is it then called and why?

II.

1. A vulgar fraction may be considered as expressing the division of the numerator by the denominator. Explain this.

2. Explain the principle upon which vulgar fractions are reduced to their equivalents having a common denominator. When may the common denominator be less than the product of all the denominators; and how is it then determined? Ex. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{9}$, $\frac{11}{12}$.

3. By what fraction must $\frac{2}{3}$ be divided to give a quotient $\frac{11}{12}$? Can more than one such fraction be found?

4. State and prove the rule for the division of one vulgar fraction by another. Divide $\frac{3}{2}$ by $\frac{4}{5}$; show that a proper fraction will always be increased by dividing it by another proper fraction. By what fraction must $\frac{11}{17}$ be divided to give a quotient 3?

5. A man's wages are $\$3\frac{1}{2}$ a day, and his daily expenses are $\$1\frac{7}{8}$; how many days must he labour to enable him to buy a suit of clothes worth $\$46\frac{1}{8}$?

III.

1. Define Numerator and Denominator, and show why they are properly applied to the terms of a fraction.
2. John had $\frac{3}{4}$ of a melon and gave away $\frac{5}{6}$ of what he had; what part of the melon had he left?
3. A miller wishes to put 39 bushels of wheat into bags, each bag to hold $2\frac{1}{6}$ bushels; how many bags would it require?
4. A man owned $\frac{2}{3}$ of a ship and sold $\frac{3}{8}$ of his share for $5475; what was the whole ship worth?
5. If $7\frac{1}{2}$ pounds of coffee cost $187\frac{1}{2}$ cents; what will a bag containing $63\frac{2}{5}$ pounds cost?

IV.

1. Before subtracting fractions, why is it necessary to change them to others having a common denominator?
2. Arrange the fractions $\frac{3}{7}$, $\frac{2}{5}$, $\frac{7}{15}$, $\frac{13}{25}$, $\frac{2}{3}$ of $\frac{9}{10}$ in order of magnitude.
3. If $2\frac{2}{3}$ will pay a woman's wages for $2\frac{1}{2}$ days, how much will pay for $5\frac{1}{3}$ days' work?
4. James by mistake subtracted $\frac{7}{8}$ instead of $\frac{5}{7}$; was his answer too large, or too small, and how much?
5. A man owning $\frac{3}{4}$ of a factory, sold $\frac{4}{7}$ of what he owned for $15750; what was the factory worth?

V.

1. State the principle involved when fractions are changed to others having a common denominator.
2. I bought $7\frac{6}{13}$ thousand feet of boards for $135.80; at the same rate, what would $19\frac{3}{4}$ thousand feet cost?
3. I paid $7888.30 for $83\frac{5}{16}$ acres of land; what would 7 acres cost at the same rate?
4. What is the least number that must be taken from 60 so that it may be exactly divisible by $7\frac{5}{12}$?
5. On $\frac{2}{3}$ of my field I planted corn; on $\frac{2}{3}$ of the remainder I sowed wheat; on $\frac{2}{3}$ of the remainder I planted potatoes; the rest, consisting of $\frac{1}{3}$ of an acre, was planted in beans; how large was my field?

CHAPTER V.

DECIMALS.

Section I.—Definitions.

Oral Exercises.

1. If an apple is divided into ten equal parts, what is one of the equal parts called? What are 7 of these called? 3 of them?

2. If a unit is divided into 10 equal parts, what are the parts called? What is the fractional unit?

3. If 1 tenth of an apple is divided into ten equal parts what part of the whole apple is 1 part? 3 parts? 9 parts?

4. How are hundredths got? How are they got from tenths?

5. What part of 1 tenth is 1 hundredth? How many 1 hundredths in 1 unit; in 1 tenth?

6. If 1 hundredth of an apple is divided into ten equal parts, what is the fractional unit called?

7. How many thousandths are equal to 1 hundredth? To 1 tenth? To 1 unit?

8. What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{10}$ or $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{1}{100}$?

122. A Decimal Fraction is one which has for its denominator 10, 100, 1000, or *some power* of ten.

The **Power** of a number is the product obtained by multiplying the number *by itself* one, or more times.

Thus 9 is the *second* power of 3, for $9 = 3 \times 3$.
27 " *third* " 3, for $27 = 3 \times 3 \times 3$.
81 " *fourth* " 3, for $81 = 3 \times 3 \times 3 \times 3$.

123. The *Denominator* of a decimal fraction is never expressed, but is always understood. For brevity decimal fractions are usually called *Decimals*. A decimal fraction is expressed by writing the *Numerator* with a point (.) before it.

Thus, $\frac{1}{10}$ is written ·1.
$\frac{3}{10}$ " ·3.
$\frac{1}{100}$ " ·01.
$\frac{79}{100}$ " ·79.
$\frac{1}{1000}$ " ·001.
$\frac{139}{1000}$ " ·139.

124. The **Point** placed before decimals is called the *decimal point.* It separates the fractional part from whole numbers.

125. The *first* place to the right of the decimal point is that of **tenths**; the *second* place is that of **hundredths**; the *third*, that of **thousandths**; the *fourth*, that of **ten-thousandths**; the *fifth*, that of **hundred-thousandths**; &c.

Thus, $23.045 = 2\times10+3+\frac{0}{10}+\frac{4}{100}+\frac{5}{1000}$.

Hence it appears that decimals are simply an extension of the ordinary system of notation and numeration.

126. Naughts *affixed* to a decimal have no effect on its value; that is ·9, ·90, ·900 are all equal;

for $\cdot9 = \frac{9}{10}$
$\cdot90 = \frac{90}{100} = \frac{9}{10}$.
$\cdot900 = \frac{900}{1000} = \frac{9}{10}$

127. To convert a decimal to a vulgar fraction

Since ·378 means 3 tenths, 7 hundredths, and 8 thousandths;

$$\therefore \cdot378 = \frac{3}{10}+\frac{7}{100}+\frac{8}{1000}$$
$$= \frac{300+70+8}{1000}$$
$$= \frac{378}{1000}.$$

Similarly ·00307 means 3 thousandths and 7 hundred-thousandths;

$$\therefore \cdot00307 = \frac{3}{1000}+\frac{7}{100000}$$
$$= \frac{300+7}{100000}$$
$$= \frac{307}{100000}.$$

Hence, *To express a decimal as a vulgar fraction write the given decimal as a whole number for the numerator of*

the vulgar fraction, and for the denominator write 1, followed by as many ciphers as there are decimal places in the given decimal.

Conversely a fraction having 10, 100, 1000, &c., for denominator may be expressed as a decimal *by writing the numerator and counting off from the right as many figures as there are ciphers in the denominator.*

Thus $3\frac{175}{1000}=3{\cdot}175$, and $\frac{75}{1000}={\cdot}075$.

Exercise lxx.

Express the following decimals as common fractions:

1. ·7.	6. ·4123.	11. ·00427.
2. ·36.	7. ·0614.	12. ·00036.
3. ·08.	8. ·0078.	13. ·02007.
4. ·784.	9. ·7614.	14. ·712465.
5. ·709.	10. ·3005.	15. ·000006.

Express the following fractions as decimals:

16. $\frac{8}{10}$.	20. $\frac{136}{1000}$.	24. $126\frac{367}{1000}$.
17. $\frac{77}{100}$.	21. $2\frac{7}{100}$.	25. $\frac{1845}{100000}$.
18. $\frac{27}{100}$.	22. $4\frac{16}{100}$.	26. $3\frac{7}{1000000}$.
19. $\frac{7}{100}$.	23. $16\frac{126}{1000}$.	27. $16\frac{163}{1000000}$.

Exercise lxxi.

Write the following decimals in words:

1. ·9.	5. 4·31.	9. 21·3601.
2. ·27.	6. 7·216.	10. 17·0064.
3. ·368.	7. 3·314.	11. 18·00081.
4. ·064.	8. 5·8167.	12. 20·01458.

Express in figures the following:

13. Eight tenths; two, and seven hundredths; nine thousandths.

14. Eight hundred and seven, and ninety-four thousandths; three thousand and seventeen, and seven hundred and nine ten-thousandths; three, and one thousand and eight millionths.

15. Six, and four ten-thousandths; eighty, and six hundred and nine ten-millionths; one hundred and one, and one thousand and one hundred-thousandths.

Section II.—Addition.

128. To add Decimals.

Ex. 1. What is the sum of 3·7, 14·035, 81·64 and 7165?

3·7000		3·7
14·0350		14·035
81·6400	or	81·64
·7165		·7165
100·0915		100·0915

Since we can add figures of the same name only we write the addends so that units will be under units, tenths under tenths, &c. This is always the case when the points range in the same straight line. Then, beginning at the lowest order, we add as if the figures were integers and place the decimal point in the sum before the tenths.

Exercise lxxii.

(1)	(2)	(3)	(4)
42·3	12·326	4031·06	·608242
13·06	204·00	108·304	·0315044
8·049	8·3024	9·001345	·8034
1·6	52·007	76·739	·086
·037	324·1	250·0007	·9108

Find the sum of

5. 4·5 + 70·63 + 1·079 + 25.
6. ·126 + 3·05 + ·07 + ·528 + 7·093.
7. 111·306 + ·0317 + 2·793 + ·007.
8. 470·05 + 72·701 + 3·0315 + 418·2658.
9. 12·3987 + 4·1462 + ·02063 + 13 + 10.962.
10. 210·7 + 14563·21 + ·0173 + 382·74156.
11. 9·127 + 17·72 + ·0041 + 2·31 + 170·96.
12. ·101285 + 17·061 + 3·2001 + 5·38706.
13. 2·325 + ·0012 + 5·086 + 219·6832 + ·407.

14. A merchant has 4 pieces of calico measuring respectively 25·5 yards, 29·125 yards, 34·25 yards, and 33·75 yards; how many yards are there in the 4 pieces?

15. Four fields contain as follows: 15·375 acres, 12·6125 acres, 14·003 acres, 16·5 acres; how many acres do the four fields contain?

Section III.—Subtraction.

129. To subtract decimals.

Ex. 1. From 17·013 take 1·90764.

17·01300		17·013
1·90764	or	1·90764
15·10536		15·10536

write the subtrahend under the minuend, placing tenths under tenths, hundredths under hundredths, &c. Then, as there are more figures in the subtrahend than in the minuend, we may annex as many ciphers as will render the number of decimal places in each the same. This will not affect the value of the minuend (Art. 126). We then subtract as in whole numbers and place the decimal point in the remainder immediately to the left of the tenths.

Exercise lxxiii.

	(1)	(2)	(3)	(4)
From	18·5	2·8706	·50376	·36
Take	2·3476	·49	·065	·12704

From

5. 1·869 take ·0374.
6. ·0061 " ·00089.
7. 6·723 " 2·7981.
8. 9·305 " 7·9.
9. 204·1 take 36·002.
10. 1000 " 999·99.
11. 2 " 1·3678.
12. 17·36 " 9·0184.

Find the value of

13. (7·2 − 2·75) − (1·9 − ·0027).
14. 36 + 7·07 − 24·896 − (3·164 − ·789).
15. (273.29 − 41·802) − (7·162 + 51·386 − ·09863).
16. The length of a seconds pendulum is 39·1392 inches, and that of a French metre 39·371 inches. Find the difference in length between them.
17. A sovereign weighs 123·274 grains, and a shilling 87·272 grains; find their difference in weight.
18. Take eleven thousandths from eleven hundredths.
19. Add together the sum and difference of seventy-three thousandths and one hundred and fifteen millionths.
20. From a piece of muslin containing 27·5 yards, a merchant sold 13·75 yards; how much was left?

Section IV.—Multiplication.

130. To multiply decimals.

Ex. 1. Multiply ·7 by ·9.

Since $\cdot 7 = \frac{7}{10}$ and $\cdot 9 = \frac{9}{10}$;

$\therefore \cdot 9 \times \cdot 7 = \frac{9}{10} \times \frac{7}{10} = \frac{63}{100} = \cdot 63$.

Ex. 2. Multiply ·731 by ·06.

Since $\cdot 731 = \frac{731}{1000}$ and $\cdot 06 = \frac{6}{100}$;

$\therefore \cdot 06 \times .731 = \frac{6}{100} \times \frac{731}{1000} = \frac{4386}{100000} = \cdot 04386$.

Ex. 3. Multiply 3·76 by 2·4.

Since $3 \cdot 76 = \frac{376}{100}$ and $2 \cdot 4 = \frac{24}{10}$;

$\therefore 2 \cdot 4 \times 3 \cdot 76 = \frac{24}{10} \times \frac{376}{100} = \frac{9024}{1000} = 9 \cdot 024$.

Hence, *To multiply decimals, multiply as in the case of integers and mark off from the right of the product as many decimal places as there are decimals in the factors.*

Exercise lxxiv.

	(1)	(2)	(3)	(4)
Multiply	4·64	53·062	·1346	675·1
By	3·35	4·53	·203	·008

Multiply

5. 713 by 3·47.
6. 3·96 by ·068.
7. 9·07 by 1.06.
8. ·008 by ·009.
9. 13·14 by ·0236.
10. 714·6 by 1·124.
11. 9·006 by ·0045.
12. 1·001 by 1·009.

13. A square link contains 62·726 square inches; what the area in inches of 5327 square links?

14. A pint of water weighs 1·25 pounds avoirdupois; what is the weight of 7·8 pints?

15. Gold is 19·26 times heavier than water; what weight of gold is of the same bulk as 17·342 pounds of water?

16. The circumference of a circle measures 3·14159 times its diameter; what will be the length of the circumference of a circle whose diameter measures 37·258 miles?

17. Find the product of the sum and difference of ·27 and 27.

18. What is the weight of five cubic feet of water, if a cubic foot weighs 62·455 pounds avoirdupois?

131. To multiply by 1 followed by ciphers.

Ex. 4. Multiply 71·134 by 10; by 100; by 100000.

71·134	71·134	71·134
10	100	100000
711·340	7113·400	7113400·000

From these examples, it will be seen that the decimal point has been moved to the *right* in the product as many places as there are ciphers in the multiplier.

Hence, *To multiply by 1 followed by ciphers, move the decimal point as many places to the right in the multiplicand as there are ciphers in the multiplier, and the result will be the product.*

Section V.—Division.

132. To divide one decimal by another.

Ex. 1. Divide 9 by ·3.

$$\cdot\frac{9}{3}=\frac{10\times 9}{10\times\cdot 3}=\frac{9\cdot 0}{3}=30.$$

In this example we multiply both divisor and dividend by 10. This makes the divisor a whole number. We then proceed as in ordinary division.

Ex. 2. Divide 97·92 by 9.

9)97·92 — As the divisor is already a whole number
10·88 — we proceed to divide as usual.

Ex. 3. Divide 3·24 by ·00081.

Multiplying both divisor and dividend by 100000 we get 324000÷81, which can easily be worked by ordinary division.

Ex. 4. Divide ·736644 by 234·

We multiply the divisor and dividend by 10; the divisor is now a whole number. The operation will then stand as follows:

2346)7·36644(·00314
7 038

3284
2346

9384
9384

We first bring down 3 tenths and put the decimal point in the quotient. The divisor is not contained in 73 tenths; we therefore put a 0 in the quotient and bring down 6 hundredths. Since the divisor is not contained in 736 hundredths, we put another 0 in

the quotient and bring down 6 thousandths. The divisor is now contained in 7366 thousandths. The rest of the work proceeds as in ordinary division.

Hence, *If the divisor is a decimal, we multiply both divisor and dividend by such a power of 10 as will make the divisor a whole number, and then we divide as in simple division, placing the decimal point in the quotient as soon as the tenths figure of the dividend is brought down.*

Exercise lxxv.

Divide

1. 16·578 by 5·4.
2. 48·591 by ·96.
3. 2·56 by ·0032.
4. 4·126 by 640.
5. 3·1 by ·0025.
6. ·0012 by 1·6.
7. ·0774 by 480.
8. 21·3 by 37·5.
9. 202 by ·01.
10. 406·8 by ·018.
11. 1·066 by 13.
12. 15·77 by ·19.

133. To divide by 1 followed by ciphers.

Ex. 4. Divide 136·15 by 10; by 100; by 10000.

10 | 136·15	100 | 136·15	10000 | 136·15
13·615	1·3615	·013615

From these examples, it will be seen that the decimal point has been moved to the *left* in the quotient as many places as there are ciphers in the divisor.

Hence, *To divide by 1 followed by ciphers, move the decimal point as many places to the left in the dividend as there are ciphers in the divisor, and the result will be the quotient.*

Section VI.—Reduction of Decimals.

134. To reduce a Vulgar Fraction to a Decimal.

Ex. 1.—Reduce $\frac{3}{40}$ to a decimal.

40)300(.075
280
—
200
200
—

$\frac{3}{40}$ equals $\frac{1}{40}$ of 3 (Art. 94). 3 equals 30 tenths, and $\frac{1}{40}$ of 30 tenths is 0 tenths. 30 tenths equals 300 hundredths, and $\frac{1}{40}$ of 300 hundredths is 7 hundredths, and 20 hundredths remaining. 20 hundredths equals 200 thousandths, and $\frac{1}{40}$ of 200 thousandths is 5 thousandths; hence $\frac{3}{40}$=·075.

Hence, *To reduce a vulgar fraction to a decimal, annex ciphers to the numerator and divide by the denominator of the fraction, and place the decimal point in the quotient as soon as the first cipher annexed is brought down.*

Exercise lxxvi.

Reduce the following to decimals:

1. $\frac{3}{10}$.
2. $\frac{3}{4}$.
3. $\frac{5}{8}$.
4. $\frac{9}{40}$.
5. $\frac{5}{32}$.
6. $\frac{1}{40}$.
7. $\frac{3}{80}$.
8. $\frac{7}{8}$.
9. $\frac{11}{160}$.
10. $\frac{5}{64}$.
11. $\frac{7}{125}$.
12. $6\frac{3}{5}$.
13. $24\frac{1}{125}$.
14. $32\frac{1}{40}$.
15. $46\frac{5}{16}$.

Section VII.—Circulating Decimals.

135. To reduce a circulating decimal to a vulgar fraction.

In reducing vulgar fractions to decimals we find that sometimes the division will not terminate, but the same figure or figures will be repeated over again continually.

Ex. 1. Reduce $\frac{1}{3}$ to a decimal.

$\frac{1}{3} = \cdot3333$, &c.

Ex. 2. Reduce $\frac{5}{11}$ to a decimal.

$\frac{5}{11} = \cdot4545$, &c.

136. Decimals of this kind are called **Repeating** or **Circulating Decimals.** The part repeated is called the **Period** or **Repetend.**

137. It is usual to express the repetend by writing it down and placing a dot over the first and last figures of the part repeated. When there is only one figure repeated the dot is placed over it.

Thus, $\cdot3333$, &c., is indicated $\cdot\dot{3}$.
$\cdot4545$, &c., " $\cdot\dot{4}\dot{5}$
$\cdot2333$, &c., " $\cdot2\dot{3}$

Ex. 3. Reduce $\frac{3}{22}$ to a decimal.

$\frac{3}{22} = \cdot1363636$, &c., $= \cdot1\dot{3}\dot{6}$.

138. A pure circulating decimal is one in which the figures that repeat begin immediately after the decimal point.

139. A mixed circulating decimal is one in which the figures that repeat do not begin immediately after the decimal point.

140. Since

$\frac{1}{9} = \cdot11111\ldots$ Also $\frac{1}{99} = \frac{1}{9} \div 11 = \cdot010101\ldots$

$\frac{2}{9} = \cdot22222\ldots$ $\frac{5}{99} = \cdot050505\ldots$

$\frac{5}{9} = \cdot55555\ldots$ $\frac{17}{99} = \cdot171717\ldots$

Similarly, $\frac{1}{999} = \frac{1}{9} \div 111 = \cdot001001\ldots$

$\frac{125}{999} = \cdot125125\ldots$

From the preceding examples it is evident that a *Pure Circulating Decimal may be expressed as a fraction by writing the figures that repeat as numerator, and as many nines as there are figures in the repetend for denominator of the fraction.*

Thus, $\cdot\dot{0}\dot{5} = \frac{5}{99}$. $\cdot\dot{3}\dot{7}\dot{8} = \frac{378}{999}$.

$\cdot\dot{5}\dot{4} = \frac{54}{99}$. $5\cdot\dot{4}\dot{3} = 5\frac{43}{99}$.

$3\cdot\dot{4} = 3\frac{4}{9}$. $\cdot0\dot{3}7\dot{8} = \frac{378}{9999}$.

141. Mixed Circulating Decimals may be reduced to vulgar fractions in the following manner:

Ex. 4. $\cdot034 = \cdot03\frac{4}{9} = \frac{3\frac{4}{9}}{100} = \frac{31}{900} = \frac{34-3}{900}$.

Ex. 5. $\cdot0543 = \cdot05\frac{43}{99} = \frac{5\frac{43}{99}}{100} = \frac{538}{9900} = \frac{543-5}{9900}$.

Ex. 6. $\cdot0136 = \cdot013\frac{6}{9} = \frac{13\frac{6}{9}}{1000} = \frac{123}{9000} = \frac{136-13}{9000}$.

From these examples it is evident that a *Mixed Circulating Decimal may be expressed as a fraction by subtracting the part of the decimal which does not repeat from the whole decimal and placing the remainder as numerator, and as many nines as there are figures in the repetend, followed by as many ciphers as there are figures in the part which does not repeat, as denominator of the fraction.*

Exercise lxxvii.

Reduce to vulgar fractions:

1. $\cdot\dot{3}$	5. $\cdot 0\dot{2}\dot{4}$.	9. $4\cdot 0\dot{5}3\dot{1}$.
2. $\cdot 5\dot{4}$.	6. $\cdot 3\dot{1}\dot{4}$.	10. $11\cdot 2\dot{8}\dot{7}$
3. $\cdot\dot{7}2\dot{9}$.	7. $\cdot 00\dot{6}7\dot{5}$.	11. $3\cdot 4\dot{1}\dot{8}$
4. $\cdot\dot{3}2\dot{9}$.	8. $\cdot 0\dot{4}4\dot{3}$.	12. $2\cdot 3\dot{4}\dot{5}$

142. The **Addition** or **Subtraction** of **Circulating Decimals** is generally performed by repeating the period as many times as seems sufficient to insure the required degree of accuracy, and then adding or subtracting.

143. Multiplication or **Division** of **Circulating Decimals** may also be performed by carrying out the repetend, but these operations are more usually performed by reducing the decimals to vulgar fractions, then multiplying or dividing these fractions, and reducing the results once more to decimals.

Ex. 7. Multiply $\cdot 2\dot{3}$ by $\cdot 3\dot{6}$.

$\cdot 3\dot{6} \times \cdot 2\dot{3} = \frac{3}{9}\frac{6}{9} \times \frac{2}{9}\frac{3}{0} = \frac{14}{165} = \cdot 08\dot{4}$.

Ex. 8. Divide $\cdot 1\dot{6}$ by $\cdot 00\dot{2}\dot{7}$.

$\cdot 1\dot{6} \div \cdot 00\dot{2}\dot{7} = \frac{15}{90} \div \frac{27}{999} = \frac{540}{9} = 61\cdot\dot{1}$

Exercise lxxviii.

Find the value of

1. $\cdot 3\dot{1}00\dot{7} + 21\cdot 003 + 41\cdot 607\dot{3}4\dot{2}$.
2. $\cdot\dot{3} - \cdot 0\dot{9}$ and $\cdot 0\dot{4} - \cdot 00769238$.
3. $37\cdot 2\dot{3} \times \cdot 2\dot{6}$ and $7\cdot 7\dot{2} \times \cdot\dot{2}9\dot{7}$.
4. $\cdot\dot{3} \div \cdot 0\dot{9}$ and $\cdot 042 \div \cdot 036$.

EXAMINATION PAPERS.

1. What are decimal fractions? How does the use of them facilitate calculation?

2. Represent as vulgar fractions 1·25, .0004. How does it affect the value of a decimal to place ciphers (1) after

the decimal places, (2) between the decimal places and the decimal point. Decimals may be multiplied and divided by 10, 100, 1000, &c., merely by shifting the decimal point; show this. Divide ·000121 by 11.

3. What are the advantages of decimal fractions? Express as a decimal, 17359 divided by one million. Divide ·00125 by 2·5. If the number of decimal places in the divisor exceeds the number in the dividend, how do you proceed? Explain this by making 2·5 the dividend and ·00125 the divisor.

4. Multiply 2·564 by ·047, and divide ·00169 by ·013. Verify the result by putting the decimals in the form of vulgar fractions.

5. What are recurring decimals? Find the recurring decimal equivalent to $\frac{5}{7}$, and find the vulgar fraction equivalent to the recurring decimal ·81246246....

II.

1. Explain the notation of decimal fractions, and show how the value of a decimal is affected by moving the decimal point two places to the right or left. Write $\frac{375}{1000}$ as a decimal, and express the one-millionth part of the same fraction as a decimal. Multiply 85·345 by 4·175. Divide 25·6 by ·00016.

2. Divide ·365 by 20. If in obtaining the quotient you cut off the cipher from the divisor and actually divide by 2, what corresponding change should be made in the dividend?

3. Prove that $\cdot\dot{3}33\dot{3} \times \cdot 2121\dot{2}1 = \cdot 0707\dot{0}7$.

4. Prove the rule for fixing the position of the decimal point, when one decimal fraction is multiplied by another.

Express as vulgar fractions in their lowest terms:

(1) ·0625 × ·0032; (2) ·016 ÷ ·64; ·4̇5 − ·45.

5. Simplify $\frac{1{\cdot}18}{{\cdot}152} \times \frac{3{\cdot}04}{2.95}$, and divide the result by ·00125.

III.

1. Prove the rule for dividing one decimal fraction by another, and find the value of $\frac{{\cdot}05 \times {\cdot}05 \times .05 + 1}{1{\cdot}05}$.

2. State and explain the rule for reducing a vulgar fraction to a decimal fraction.

Find the value of $\frac{1}{4} \div {\cdot}01001$ and of $10{\cdot}01 \div \frac{1}{20}$.

3. Reduce ·064 and 15·625 to vulgar fractions; multiply them together in that form. and then reduce the result to decimals. Prove by multiplying the decimals as they stand.

4. Which is the greater, $1\frac{2}{3}\times 2\frac{5}{8}$, or $\cdot 018\times 216$?

5. Suppose unity represents ·0012, what number represents ·0001?

IV.

1. Whether is 1·121472653 more accurately represented by 1·1214726 or 1·1214727, and why?

2. Express in decimal notation the value of $8\cdot 0625 - 6\frac{1}{2}\frac{1}{5} - \cdot 00375 + 1\cdot 09236 - \frac{25878}{10000}$.

3. *A* bought a house with ·25 of his money; he spent ·575 of it in buying a farm and had $2100 left; find the cost of the house and farm respectively.

4. What is the smallest number that can be exactly divided by the nine significant figures? Simplify $\frac{\frac{2}{3}\text{ of }\frac{4}{5}-\frac{7}{9}\text{ of }\frac{4}{21}}{\frac{2}{3}\text{ of }\frac{2}{15}-\frac{2}{3}\text{ of }\frac{4}{18}}$.

5. What number is that, from which if there be taken $\frac{2}{7}$ of ·375, and to the remainder ·5$\dot{3}$ of ·3125 be added, the sum is 10?

V.

1. Find the value of $\frac{8}{11}$ of $(\frac{7}{9}+1\frac{2}{3})$, and prove it equal to $\frac{1}{3}$ of $20\frac{3}{4}\div 10\frac{3}{8}$.

2. Prove the rule for finding the value of a circulating decimal; and reduce $1\div 99999$ and $1\div 10001$ to circulating decimals.

3. Prove that $46\cdot 2\div 92\cdot 4 = \cdot 75\times\cdot 6$.

4. Prove that $\cdot 02\times\cdot 02\times\cdot 005\times 005 = \cdot 0001\times\cdot 0001$.

5. Divide $\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{16}+\frac{1}{20}$ by $\frac{1}{6}+\frac{1}{10}+\frac{1}{18}+\frac{1}{30}$, and reduce the result to a decimal.

CHAPTER VI.

COMMERCIAL ARITHMETIC.

Section 1.—Tables and Reduction.

144. ENGLISH OR STERLING MONEY.

4 farthings (far.) . . . = 1 penny, or 1d.
12 pence . . . . . = 1 shilling, " 1s.
20 shillings . . . . . = 1 pound, " £1.

NOTE 1.—Farthings are usually written as a fraction of 1d. Thus 1 far. is written $\frac{1}{4}$d.; 2 far., $\frac{1}{2}$d.; 3 far., $\frac{3}{4}$d.

NOTE 2.—£1 sterling = 4.86\frac{2}{3}$, and 1s. = 24$\frac{1}{3}$ cents.

Oral Exercises.

Repeat the table of English money.
How many far. in 2d.? in 3d.? in 6d.? in 8d.?
How many pence in 12 far.? in 16 far.? in 20 far.?
How many pence in 2s.? in 3s.? in 5s.? in 6s.?
How many far. in 1s.? in 2s.? in 3s.? in 5s.?
How many shillings in £1 12s.? in £2 15s.?

145. There are two kinds of Reduction, **Reduction Descending** and **Reduction Ascending.**

146. Reduction Descending is the process of changing a number from a *higher* to a *lower* denomination.

147. Reduction Ascending is the process of changing a number from a *lower* to a *higher* denomination.

Ex. 1. Reduce £6 5s. 3$\frac{1}{4}$d. to farthings.

£6 5s. 3¼d.
20
—
125s.
12
—
1503d.
4
—
6013 far.

In 1 pound there are 20 shillings, and in £6 there are 6 times 20s., or 120s.; 120s. plus 5s. are 125s.; in 1 shilling there are 12 pence, and in 125s. there are 12 times 12d., or 1500d.; 1500d. plus 3d., are 1503d.; in 1d. there are 4 farthings, and in 1503d. there are 1503 times 4 far., or 6012 far.; 6012 far. plus 1 far. are 6013 far.

Ex. 2. How many £ s. d. in 3679 farthings?

far.
4 | 3679
12 | 919 3 far.
20 | 76 7d.
£3 16s.

Ans. £3 16s. 7¾d.

There are 4 far. in 1d.; hence, in 3679 far. there are as many pence as the number of times 4 is contained in 3679; 3679÷4=919 and 3 over. This 3 is 3 far. There are 12d. in 1s.; hence, in 919d. there are as many shillings as the number of times 12 is contained in 919; 919÷12=76 and 7 over. This 7 is 7d. There are 20s. in £1; hence, in 76s. there are as many pounds as the number of times 20 is contained in 76; 76÷20=3 and 16 over. This 16 is 16 shillings.

Exercise lxxix.

Reduce

1. 7s. 8d. to pence.
2. £1 3s. to farthings.
3. 7145d. to £, &c.
4. 6185s. to £, &c.
5. £10 0s. 6d. to pence.
6. £2 6s. 8d. to pence.
7. 3910 far. to £, &c.
8. 7163d. to £, &c.
9. £191 9s. 11¾d to far.
10. £3 6s. 10¼d. to far.
11. 78916d. to £, &c.
12. £100 7d. to far.

148. UNITED STATES MONEY.

10 mills (*m.*) . . . . . = 1 cent, or 1 c.
10 cents . . . . . = 1 dime, " 1 d.
10 dimes . . . . . = 1 dollar, " 1 $.
10 dollars . . . . . = 1 eagle, " 1 e.

149. AVOIRDUPOIS WEIGHT.

16 drams (dr.) . . . = 1 ounce . . . . or 1 oz.
16 ounces . . . . = 1 pound . . . . " 1 lb.
25 pounds . . . = 1 quarter . . . . " 1 qr.
4 quarters . . . . = 1 hundred-weight . " 1 cwt.
20 hundred-weight . = 1 ton . . . . . " 1 t.

NOTE 1.—Avoirdupois Weight is used for weighing everything except jewels, precious metals, and medicines, when dispensed.

NOTE 2.—28 pounds equal 1 quarter in Great Britain.

Oral Exercises.

Repeat the table of Avoirdupois Weight.
How many ounces in 2 lb.? in 3 lb. 4 oz.? in 4 lb.?
How many quarters in 28 lb.? in 49 lb.? in 100 lb.?
How many drams in 2 oz. 6 dr.? in 3 oz. 4 dr.?
How many tons in 58 cwt.? in 112 cwt.? in 200 cwt.?

Ex. 3. Reduce 2 cwt. oz. 11 dr. to drams.

cwt.	qrs.	lb.	oz.	dr.
2	0	0	4	11
4				
8 qr.				
25				
200 lb.				
16				
3204 oz.				
16				
51275 dr.				

Ex. 4 Reduce 147658 lbs. to tons, etc.

25 { 5 | 147658
5 | 29531 . . . 3 } 8 lb.
4 | 5906 . . . 1
20 | 1476 . . . 2 qr.
73 tons 16 cwt.

Ans. 73 t. 16 cwt. 2 qr. 8 lb.

Exercise lxxx.

Reduce

1. 2 t. 3 qr. 6 lb. to drams.
2. 5 lb. 6 oz. 14 dr. to drams.
3. 21645 oz. to cwt., &c.
4. 76385 qrs. to tons, &c.
5. 3 cwt. 8 lb. 5 oz. to ounces.
6. 51649 lb. to tons, &c.

150. TROY WEIGHT.

24 grains (gr.) . . . = 1 pennyweight, . or 1 dwt.
20 pennyweights . . = 1 ounce, ... " 1 oz.
12 ounces . . . . . = 1 pound, ... " 1 lb.

NOTE 1.—This is chiefly used for weighing gold, silver and jewels.

NOTE 2.—1 lb. avoirdupois = 7000 grains,
1 lb. troy . . = 5760 grains.

Oral Exercises.

How many oz. in 2 lb.? in 3 lb.? in 5 lb.?

How many lb. in 36 oz.? in 48 oz.? in 60 oz.? in 44 oz.? in 78 oz.?

How many dwt. in 2 oz.? in 3 oz.? in 4 oz.? in 48 gr.?

151. APOTHECARIES' WEIGHT.

20 grains (gr.)	= 1 scruple,	or 1 sc. or 1 ℈.
3 scruples,	= 1 dram,	" 1 dr. " 1 ʒ.
8 drams,	= 1 ounce,	" 1 oz. " 1 ℥.
12 ounces,	= 1 pound,	" 1 lb. " 1 ℔.

NOTE 1.—The ounce and pound of Apothecaries' Weight are the same as in Troy Weight.

NOTE 2.—Apothecaries' Weight is used in mixing medicines. These are bought and sold by Avoirdupois Weight.

How many

1. Grains in 7 ℈? 11 ℈?
2. Scruples in 9 ʒ? 16 ʒ?
3. Drams in 24 ℨ? 96 ℨ?
4. Drams in 5 ℥? 7 ℥?
5. Ounces in 88 ʒ? 96 ʒ?
6. Pounds in 108 ℥? 168 ℥?

Exercise lxxxi.

Reduce

1. 1 lb. 4 oz. to ounces.
2. 7163 sc. to lb. &c.
3. 7685 dwt. to lb. &c.
4. 11 oz. 3 drs. to grains.
5. 3 oz 6 dwt. to grains.
6. 73564 grains to lb. (Troy &c.

152. LONG MEASURE.

12 inches (in.)	= 1 foot,	or 1 ft.
3 feet	= 1 yard,	" 1 yd.
5½ yards	= 1 rod,	" 1 rd.
40 rods	= 1 furlong,	" 1 fur
8 furlongs	= 1 mile,	" 1 mi
3 miles	= 1 league	" 1 l.

NOTE 1.—*Cloth Measure* is not now used. Cloth is bought by the yard, half-yard, quarter-yard, etc.

NOTE 2.—Gunt Chain is used in measuring land. is 22 yards and is divided into 100 links.

NOTE 3.—Mariners use the following:

6 feet	= 1 fathom.
120 fathoms	= 1 cable length.
880 fathoms	= 1 mile.

Oral Exercises.

Repeat the table of Lineal Measure.

How many feet in 4 yd.? in 6 yd. 1 ft.?

How many miles in 17 fur.? in 320 rods? in 59 fur.?
How many feet in 9 fath.? in 2 rd.? in 12 yd.?

Ex. 5. How many feet in 12 rd. 3 yd. 2 ft.?

rd.	yd.	ft.
12	3	2
$5\frac{1}{2}$		
63		
6		
69 yd.		
3		
209 ft. *Ans.*		

NOTE.—We multiply by 5, and add to the product the 3 yds., and then multiplying by $\frac{1}{2}$ we have 69 yd.

Ex. 6. How many rods in 209 ft.?

feet
3)209
$5\frac{1}{2}$)69 yd....2 ft.
2 2
11)138
12... 6 half yd. = 3 yd.

Ans. 12 rd. 3 yd. 2 ft.

NOTE.—To divide by $5\frac{1}{2}$, we reduce both to halves, then the remainder is *halves*, which we reduce to *wholes*, by dividing by 2.

Exercise lxxxii.

Reduce

1. 1 mi. 1. fur 1 rd., to inches.
2. 76452 in. to mi., &c.
3. 7568 feet to mi., &c.
4. 2 rd. 1 yd. to feet.
5. 7 chains to feet.
6. 16752 in. to fathoms.

153. SURFACE OR SQUARE MEASURE.

144 square inches	= 1 square foot,	or 1 sq. ft.
9 square feet	= 1 square yard,	" 1 sq. yd.
$30\frac{1}{4}$ square yards	= 1 square rod.	" 1 sq. rd.
40 square rods	= 1 rood	" 1 r.
4 roods . .	= 1 acre . . .	" 1 a.
640 acres . .	= 1 square mile	" 1 sq. m.

NOTE 1.—A surface is that which has length and breadth without thickness.

NOTE 2.—A square is a plane surface which has four equal sides and four equal angles.

NOTE 3.—A square inch is a square each of whose sides is an inch long.

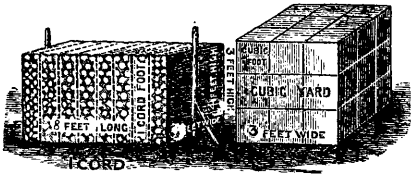
NOTE 4.—10,000 square links = 1 square chain.
10 square chains = 1 acre.

154. CUBIC OR SOLID MEASURE.

1728 cubic inches (*cu. in.*) = 1 cubic foot, or 1 cu. ft.
27 cubic feet . . . . . = 1 cubic yard, " 1 cu. yd.
128 cubic feet . . . . . = 1 cord, . . " 1 cd.

NOTE.—A cubic is a solid bounded by 6 equal squares.

NOTE.—A cord is a pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long.



Oral Exercises.

Repeat the table of square measure.
How many inches in 2 sq. ft. ?
How many acres in 16 roods ? in 320 sq.. rods ?
How many feet in 3 cu. yd. ? in 4 cu. yd. 20 cu. ft. ?
What is the difference between 3 sq. in. and 3 in. square?
3 in. square is a square each side of which is 3 in. long, and hence = 9 sq. in.

Exercise lxxxiii.

Reduce

1. 1997 sq rd. to acres, &c.
2. 3 sq. rds. 2 ft. to inches.
3. 8469 cu. in to feet.
4. 7689 cu. ft. to cords.
5. 12. a. 6 rd. to inches.
6. 78 cu. ft. 640 in. to inches.

155. DRY MEASURE.

2 pints (pt.) . . . . . = 1 quart, or 1 qt.
4 quarts . . . . . = 1 gallon, " 1 gal.
3 gallons . . . . . = 1 peck, " 1 pk.
4 pecks . . . . . = 1 bushel, " 1 bu.

NOTE 1.—Dry Measure is used in measuring grain, fruit, &c.

NOTE 2.—By the "Weights and Measures" Act of 1873, the "Imperial Bushel," containing eight "Imperial Gallons," of 277·274 cubic inches in each gallon, is the standard bushel in Canada.

NOTE 3.—By the same Act the following articles are to be estimated by the *Cental* of 100 lbs.: Barley, beans, charcoal, corn, oats, peas, potatoes, rye, salt, seeds and wheat.

NOTE 4.—8 bushels=1 quarter in Great Britain.

NOTE 5.—The following table shows the weight of a bushel of the article named, as determined by the same Act:

Wheat . . 60 lb.	Beans . . . 60 lb.	Flax Seed . . 50 lb.
Rye . . . 56 lb.	Oats . . . . 34 lb.	Clover Seed. . 60 lb.
Corn . . . 56 lb.	Peas . . . . 60 lb.	Timothy Seed 48 lb.
Barley . . 48 lb.	Buckwheat. 48 lb.	Potatoes . . . 60 lb.

156 LIQUID MEASURE.

4 gills (gi.)	= 1 pint,	or 1 pt.
2 pints	= 1 quart	" 1 qt.
4 quarts	= 1 gallon,	" 1 gal.

NOTE 1.—A barrel of beer . . contains 36 gals.
A hogshead of beer . " 54 gals.
A hogshead of wine . " 63 gals.

NOTE 2.—The *wine gallon* contains 231 cubic inches; the beer gallon contains 282 cubic inches, and the Imperial or standard gallon, 277·274 cubic inches.

NOTE 3.—6 wine gals. = 5 standard gals.

157. MEASURE OF TIME.

60 seconds (sec)	= 1 minute,	or 1 min.
60 minutes	= 1 hour,	" 1 h.
24 hours	= 1 day,	" 1 da.
7 days	= 1 week,	" 1 wk.
12 calendar months or 365 days	= 1 year,	" 1 yr.
366 days	= 1 leap year.	

NOTE 1.—The number of days in each month may be remembered by means of the following lines:

Thirty days has September,
April, June, and November;
February has twenty-eight alone—
All the rest have thirty-one;
But leap year coming once in four,
February then has one day more.

NOTE 2.—The leap years are those that can be divided by 4 without a remainder: as, 1864, 1863, 1872, &c. But of the even hundreds, only those that can be divided by 400 are leap years. The year 1900 will not be a leap year, but 2000 will be.

158. MISCELLANEOUS TABLE.

12 units = 1 dozen.	24 sheets . . = 1 quire
12 dozen = 1 gross.	20 quires . . = 1 ream.
12 gross = 1 great gross.	196 lb. flour . = 1 barrel.
20 units = 1 score.	200 lb. pork . = 1 barrel.

Oral Exercises.

Repeat Time Measure.

How many days in 3 weeks? in 5 weeks and 3 days?

How many dozen in 84? in 132? in 150?

Was 1600 a leap year? 1876? 1854?

How many hours in 360 min.? in 788 min.? 600 min.?

Exercise lxxxiv.

Reduce

1. 7 da. 16 hr. to seconds.
2. 7684 pints to bushels, &c.
3. 84 gal. 3 gills to gills
4. 36 bu 3 qt. 1 pt. to pints.
5. 2685 gills to gallons.
6. 17 qr. 3 bu. to pecks.
7. 3685 lb. of wheat to bu
8. 785693 sec. to weeks, &c.
9. 3586 lbs. Timothy seed to bu., &c.
10. 78 da 9 min. to seconds.
11. 1576 cu. ft. to cords.

Section II.—Compound Addition.

159. To add compound numbers.

160. A compound number is one composed of 2 or more numbers of different denominations which can be reduced to the same denomination.

The sum of £6 and £4 is found by *simple* addition.

The sum of £6 12s. and £4 9s. is found by *compound* addition.

Ex. 1. Find the sum of £7 6s. 8d., £5 9s. 3d., £8 9s. 7d., and £9 7s. 9d.

£	s.	d.
7	6	8
5	9	3
8	9	7
9	7	9
29	31	27

£	s.	d
7	6	8
5	9	3
8	9	7
9	7	9
30	13	3

We write the numbers so that units of the same name will be in the same column. Then we add the pence column as in simple addition and find the sum to be 27. similarly with the other columns. Hence the correct

sum is £29 31s. 27d. But it is usual in writing denominate numbers not to have more units of any denomination than 1 less than the number required to make 1 of the next higher denomination; thus, a rod 12 in. long is said to be 1 ft. in length. We do not say 20 cwt. of hay, but 1 ton, &c. We therefore change the 27d. to 2s 3d We set down the 3d. under the pence column and add the 2s. to 31s.; 31s. + 2s. = 33s.; 33s. = £1 13s. We set down the 13s. under the shillings column and add the £1 to £29; £29 + £1 = £30.

Exercise lxxxv.

(1)

lb.	oz.	dwt.
17	9	16
25	6	12
72	11	13
57	10	19

(2)

cwt.	qr.	lb.	oz.
20	3	12	11
16	2	16	12
17	0	22	15
19	1	18	13

(3)

rd.	yd.	ft.	in.
17	4	2	6
21	2	1	7
23	3	0	8
25	5	2	9

(4)

£	s.	d.
5	5	5
8	1	$7\frac{3}{4}$
2	0	$1\frac{1}{2}$
13	0	$11\frac{3}{4}$
6	6	6

(5)

bu.	pk.	qt.	pt.
10	1	1	1
2	3	6	0
5	2	3	1
8	3	1	1
15	2	4	0

(6)

rd.	yd.	ft.	in.
37	4	1	9
30	5	2	2
	3	2	7
1	0	2	10
25	1	1	11

7. Find the sum of 1 wk. 2 da. 13 h. 40 min. 30 sec.; 2 wk. 6 da. 10 h. 8 min. 3 sec.; 5 da. 22 h. 55 min. 45 sec.; 4 h. 1 min. 15 sec.; and 1 wk. 2 da. 4 h. 5 min

8. Add together 10 rd. 4 yd. 2 ft. 8 in.; 1 rd. 3 yd. 5 in.; 8 rd. 2 yd. 1 ft. 6 in.; 1 rd. 4 in.; and 2 yd. 1 ft. 9 in.

Section III.—Compound Subtraction.

161. To subtract Compound Numbers.

Ex. 1. From 16 lb. 8 oz. 6 dwt. take 7 lb. 4 oz. 12 dwt.

lb.	oz.	dwt.
16	8	6
7	4	12
9	3	14

We write the subtrahend under the minuend, so that units of the same name will be in the same column, and begin at the right to subtract.

Since we cannot take 12 dwt. from 6 dwt., we take 1 oz. or 20 dwt. from the 8 oz. and add it to the 6 dwt., making 26 dwt. 26 dwt. − 12 dwt. = 14 dwt.

Since we took 1 oz. from 8 oz. we left only 7 oz.; 7 oz. − 4 oz. = 3 oz. 16 lb. − 7 lb. = 9 lb.

Exercise lxxxvi.

(1)

lb	ʒ	ʒ	ϑ	gr.
24	7	2	1	16
16	10	3	2	17

(2)

mi.	fur.	rd.
60	0	0
40	7	39

(3)

a.	r	sq rd.
69	3	25
10	0	38

(4)

fur.	rd.	yd.	ft.	in.
7	31	1	1	3
1	39	1	2	7

(5)

£	s.	d.
43	11	10
15	14	6

(6)

r.	p.	yd.
3	17	18
2	18	30

7. A farmer had 200 bu. of wheat, and sold 28 bu. 2 pk. 5 qt. 1 pt. to one men, and as much to another; how much remained?

8 A miner having 112 lb. of gold sent his mother 17 lb. 10 oz. 15 dwt 10 gr., and 3 lb. 16 dwt. less to his father; how much did he retain?

9. From a barrel of beer containing 54 gallons, a person drew 12 gal 3 qt. one day, and 9 gal. 2 qt. 1 pt. another; how much was left?

10 From 39 sq. rd. 29 sq. yd. 128 sq. in., subtract 17 sq. rd. 16 sq. yd. 5 sq. ft.

11. A grocer has 1 cwt. 18 lb. of sugar in one barrel, 3 qr. 21 lb. in another, and 1 cwt. 2 qr. 11 lb. in a third. After selling 1 cwt. 3 qr. 15 lb, how much will he have left?

Section IV.—Compound Multiplication.

162. To multiply a Compound Number.

Ex. 1. Multiply 3 da. 19 hrs. 59 min. by 97.

da.	hrs.	min.
3	19	59
		97
291	1843	5723

da.	hrs.	min.
3	19	59
		97
371	18	23

We multiply each denomination separately, as in simple multiplication, and obtain as product 291 da. 1843 hrs. 5723 min. But as 5723 min. = 95 hrs. 23 min., we write down 23 min., and add the 95 hrs to 1843 hrs.; 1843 hrs. + 95 hrs. = 1938 hrs. = 80 da. 18 hrs., &c.

NOTE.—The usual method of working this example is to multiply first by 10, this product by 9, then to multiply the 3 da. 19 hrs. 59 min. by 7, and add the result to the

last product. We recommend the method in the example as being on the whole easier and more convenient.

Exercise lxxxvii.

(1)			(2)				(3)			
cwt.	lb.	oz.	lb.	oz.	dwt.	gr.	da.	h.	min.	sec.
18	16	9	16	8	15	17	10	20	30	40
		5				3				7

4. What is the value of 39 oxen at £15 7s. 11¼d. each?

5. What is the weight of 345 hogsheads of sugar, each weighing 14 cwt. 1 qr. 20 lb.?

6. What is the weight of 1 doz. spoons, each weighing 1 oz. 2 dwt. 16 gr.?

7. If a man owning 5 farms, of 120 a. 1 r. 12 sq. rd. each, sells 450 a. 3 r. 25 sq. rd, how much land has he left?

8. If 2 gal. 2 qt. 1 pt. 1 gi. leak out of a water-pipe in 1 hour, what will be the waste in 1 leap year?

9. Suppose a person to walk, on an average, 3 mi. 2 fur. every morning, and 3 mi. 20 rd. 1 yd. every afternoon, how far will he walk in 2 weeks?

10. If from 2 lb. of silver enough is taken to make a dozen spoons, weighing 1 oz. 10 dwt. 2 gr. each how much will be left?

11. What cost 97 tons of lead at £2 17s. 9½d. per ton?

12. If a man travel 17 mi. 3 fur. 19 rd 3 yd. 2 ft. 7 in. in 1 day, how far would he travel in 38 days?

13. If 1 acre will produce 27 bu. 3 pk. 6 qt. 1 pt. of corn, what will 98 acres produce?

Section V.—Compound Division.

163. To divide a Compound Number.

Ex. 1. Divide 80 da. 6 h. 40 min. by 17.

da. h. m. da. h. m.
17) 80 6 40 (4 17 20.
68

12 da.
24
294 h.
17
124
119
5 h.
60
340 min.
340

We write the divisor at the left of the dividend. 17 is contained 4 times in 80 da. and 12 da. over; 12 da. = 288 h.; 288 h. + 6 h. = 294 h. 17 is contained 17 times in 294 h. and 5 h. over; 5 h. = 300 min.; 300 min. + 40 min. = 340 min. 17 is contained 20 times in 340 min.

Ex. 2. Divide £12 1s. 6d. by £1 6s. 10d.

$$\frac{\text{£12 1s. 6d.}}{\text{£1 6s. 10d.}} = \frac{2898\text{d.}}{322\text{d.}} = 9.$$

Ex. 3. *A* divided a field of 11 a. into lots of 1 r. 4 per. each; how many lots were there?

$$\frac{11\text{a}}{1\text{r. 4 per.}} = \frac{1760\text{ per.}}{44\text{ per.}} = 40.$$

When we divide one compound number by another, we reduce each to the lowest denomination named in either, and divide as in simple division.

Exercise lxxxviii.

(1)	(2)	(3)
£ s. d.	lb. oz. dwt. gr.	t. cwt. qr. lb.
4) 61 18 4	6) 76 10 14 12	7) 112 16 2 16

4. Divide 4 gal. 2 qt. by 144.
5. Divide 40 cu. yd. 10 cu. ft. by 18.
6. Divide £48 7s. 4d. by £6 11d.
7. Divide 69 bu. 3 pk. 6 qt. by 6 bu. 3 pk. 6 qt.
8. Divide 697 lb. 7 oz. 5 dr. by 60 lb. 10 oz. 6 dr.
9. Divide 80 bu 2 pk. 4 qt. by 13 bu. 3 pk. 5 qt
10. A farmer put up 1000 bushels of apples in 350 barrels of uniform size; how many bushels, &c., did each barrel contain?
11. How many demijohns, each containing 2 gal. 3 qt. 1 pt., can be filled from a tank holding 71 gal. 3 qt. 1 pt. of wine?
12. A drove of cattle ate 6 t. 19 cwt. 87 lb. of hay in a week; how long will 34 t. 19 cwt. 35 lb. last them?

Section VI.—Denominate Fractions.

164. To find the value of a Fraction of a Denominate Number.

Ex. 1. How many shillings, &c., are there in $\frac{3}{8}$ of a pound?

£	s.	d.
8) 3	0	0
	7	6

Since $£\frac{3}{8} = \frac{1}{8}$ of £3 (Art. 94), we divide £3 by 8 as in compound division.

Ex. 2. Find the value of $3\frac{1}{4}$ of $\frac{2}{15}$ of 2 t. 3 cwt.

$3\frac{1}{4}$ of $\frac{3}{13}$ of 2 t. 3 cwt. $= \frac{13}{4}$ of $\frac{3}{13}$ of 2 t. 3 cwt.
$= \frac{3}{4}$ of 2 t. 3 cwt.
$= \frac{6\text{ t. }9\text{ cwt.}}{4} = 1$ t. 12 cwt. 1 qr.

Exercise lxxxix.

What is the value

1. Of $\frac{4}{5}$ of a bushel?
2. Of $\frac{2}{3}$ of a mile?
3. Of $\frac{7}{8}$ of a rod?
4. Of $\frac{3}{10}$ of a mile?
5. Of $\frac{7}{8}$ of a ton?
6. Of $\frac{5}{9}$ of an acre?
7. Of $\frac{2}{7}$ of £3 16s. $8\frac{3}{4}$d.? of £18 16s. $7\frac{1}{2}$d. $\div 3\frac{2}{3}$?
8. Of $\frac{3}{5}$ of a week + $\frac{3}{4}$ of a day + $\frac{2}{3}$ of an hour?
9. Of $\frac{33}{1100}$ cwt. $- \frac{6}{13}$ of 2 lbs. 8 oz. 10 drs.?

165. To express one number as the fraction of another.

Ex. 3. Express 4 rd. 2 yd. 1 ft. 4 in. as the fraction of 1 mile.

4 rd. 2 yd. 1 ft. 4 in. = 880 in. and 1 m. = 63360 in.

Now 1 in. $= \frac{1}{63360}$ of 63360 in.;

$\therefore$ 880 in. $= \frac{880}{63360}$ of 63360 in.

Hence the fraction required is $\frac{880}{63360} = \frac{1}{72}$.

NOTE.—The example, *Express 4 lbs. as the fraction of 8 lbs.* may be written in any of the following ways:

1. Reduce 4 lb. to the fraction of 8 lb.
2. What fraction of 8 lb. is 4 lb.?
3. What part of 8 lb. is 4 lb.?
4. If 8 lb. is the unit, what is the measure of 4 lb.?

Exercise xc.

1. What part of an ounce is $\frac{3}{10}$ of a scruple?
2. What part of a ton is $\frac{4}{5}$ of an ounce?
3. What part of a mile is $\frac{8}{9}$ of a rod?
4. What part of an acre is $\frac{2}{3}$ of a square foot?
5. Reduce $\frac{8}{9}$ of a gill to the fraction of a gallon.
6. Reduce $\frac{2}{7}$ of an inch to the fraction of a rod.
7. Reduce $\frac{4}{5}$ of a lb. to the fraction of a ton.
8. What fraction of £3 2s. $6\frac{3}{4}$d. is 14s. $10\frac{1}{4}$d.?
9. Express 13s. $10\frac{1}{2}\frac{1}{5}$d as a fraction of £2 9s. 7d.
10. Express 2 a. 31 per. as a fraction of 4 a. 2 r. 17 per.
11. Reduce $\frac{3}{4400}$ of a ton to the fraction of an ounce.
12. Reduce $\frac{1}{15720}$ of a mile to the fraction of an inch.

166. To find the value of a Decimal of a Denominate Number.

Ex. 4. What is the value of ·7875 of £1?

£ ·7875
20

·7875 of £1 = ·7875 of 20s.
= 15·75s.

s. 15·7500
12

·75 of 1s. = ·75 of 12d.
= 9d.

d. 9·0000

Hence ·7875 of £1 = 15s. 9d.

Ex. 5. Find the value of $2\cdot1\dot{6}$ of 1 yd.

$2\cdot1\dot{6}$ of 1 yd. $= 2\frac{1}{9}\frac{3}{9}$ of 1 yd $= \frac{1}{6}^{3}$ of 1 yd. = 2 yd. 6 in.

Exercise xci.

Find the value of

1. ·94375 of 1 acre
2. ·815625 of 1 lb. Troy.
3. ·875 of 1s.
4. ·785 of 1 hr.
5. ·497 of 1 day.
6. ·4375 of £1.
7. ·965625 of 1 mile.
8. ·778125 of 1 ton.
9. ·628125 of £1.
10. 3·4583 of 1s
11. 2·5384375 of 1 day.
12. ·020833 of £1.

167. To express a Compound Number as a Decimal of a Higher Denomination.

Ex 6. Reduce 3 r. 16 per. to the decimal of 1 a.; and express 5 a. 3 r. 16 per. in acres only.

40) 16 per.

4) 3·4 r.

·85 a.

16 per. $= \frac{16}{40}$ r. = ·4 r;

∴ 3 r. 16 per. = 3·4 r.

3·4 r. $= \frac{3\cdot4}{4}$ a. = ·85 a.

Hence 5 a 3 r. 16 per. = 5·85 a.

Exercise xcii.

Reduce

1. 10s 6d to the decimal of £1.
2. 5 cwt 2 qr. 14 lb to the decimal of 1 ton.
3. 15 dwt 15 gr. to the decimal of 1 oz. troy.
4. 6 fur. 8 rd to the decimal of 1 mile.
5. 2 qt. 1 pt. to the decimal of 1 peck.
6. Express £9 5s. 4½d. in pounds only.

7. Express 17 cwt. 3 qr. 14 lb. 8 oz. in cwt. only.
8. Express 7 bu. 3 pk. 1 gal. in bushels only.
9. Express $3\frac{3}{4}$ ft. as the decimal of 1 fathom.
10. What decimal of 4 oz. is 2 oz. 16 dwt. 19·2 gr.
11. Express 5 da. 9 hr. 46 min. 48 sec. in hours only.
12. Express $\frac{2}{7}$ of $\frac{1}{2}$ of $22\frac{3}{4}$ lb. as the decimal of 1 ton.

Section VII.—Practice.

168. Practice is a convenient method of solving many examples in Multiplication of Compound Numbers.

Ex. 1. Find the cost of 364 articles at $33\frac{1}{3}$ cents each.

$33\frac{1}{3}$c. = $\$\frac{1}{3}$ | \$364 = cost at \$1 each.

\$121.33$\frac{1}{3}$ = " $33\frac{1}{3}$c. each.

Ex. 2. Find the price of 2 a. 3 r. 14 per. of land at \$160 per acre.

4 | 2 × \$160 = \$320 = price of 2 a.

40 | 3 × \$40 = 120 = " 3 r.

14 × \$1 = 14 = " 14 per.

\$454 = entire cost.

Ex. 3. Find the cost of 7 t. 6 cwt. 3 qr. 5 lb. of iron at \$60 per ton.

20 | 7 × \$60 = \$420 = cost of 7 t.

4 | 6 × \$3 = 18 = " 6 cwt.

25 | 3 × \$·75 = 2·25 = " 3 qr.

5 × \$·03 = ·15 = " 5 lb.

\$440.40 = entire cost.

Exercise xciii.

Find the price of

1. 768 articles at 25c.
2. 297 " 50c.
3. 364 " \$1.20.
4. 291 " \$1.33$\frac{1}{3}$.
5. 485 articles at \$5.50.
6. 328 " \$1 87$\frac{1}{2}$.
7. 147 " \$3.37$\frac{1}{2}$.
8. 264 " \$1.16$\frac{2}{3}$.

9. 15 a. 3 r. 25 per. of land at $24 per acre
10. 9 gal. 3 qt. 1 pt of wine at $3 60 per gallon.
11. 84 bu. 3 pk. 1 gal. of wheat at $1.20 per bushel.
12. 7 oz. 15 dwt. 6 gr. of gold at $16 per ounce.
13. 29 a 3 r. 17 per. of land at $80 per acre.
14. 3 t. 13 cwt. 1 qr. 15 lb. of hay at $12 per ton.
15. What is the cost of constructing a road 17 mi. 3 fur. 15 rd long at $1880 per mile.

Exercise xciv.

Problems Involving the Previous Rules.

1. What is the value of a silver pitcher weighing 2 lb. 10 oz. avoirdupois, at $2.24 per ounce Troy?

1 oz. Troy. $= 480$ gr.

1 lb. Avoird. $= 7000$ gr.

2 lb. 10 oz. " $= 2\frac{5}{8} \times 7000$ gr. $= \frac{21}{8} \times \frac{7000}{480}$ oz. Troy.

Price 1 oz. Troy $= \$2.24$.

Price of $\frac{21}{8} \times \frac{7000}{480}$ oz. Troy $= \frac{21}{8} \times \frac{7000}{480} \times \$2.24 = \$85.75$

2. How many pounds of gold are actually as heavy as 10 lb of iron?

3. If a druggist buys 25 lb. avoirdupois of drugs at $\$8\frac{1}{3}$ a pound, and sells them in prescriptions at 75 cents an ounce apothecaries' weight, what is the gain?

4. How many sovereigns will weigh one ounce avoirdupois, if 1869 weigh 40 pounds troy?

5. If $\frac{3}{8}$ of an inch on a map corresponds to seven miles of a country, what distance on the map represents 20 miles?

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6. The value of 1 lb. troy of standard gold is £46 14s. 6d.; calculate the value of a vase of the same material whose weight is 39 oz. 18 dwt.

1 lb. $= 240$ dwt; 39 oz. 18 dwt. $= 798$ dwt.

£46 14s. 6d. $= 11214$d.

Cost of 240 dwt. $= 11214$d.;

" 1 dwt. $= \frac{11214}{240}$d. $= \frac{5607}{120}$d.;

" 798 dwt. $= \frac{798 \times 5607}{120}$d. $= \frac{133 \times 5607}{20}$d.

$= 372861\frac{13}{20}$d. $= £155$ 7s. $2\frac{13}{20}$d.

7. If 31 cwt. of cheese cost £69 4s. 8d., what will 15 cwt. 2 qr. cost?

8. Bought 2 oz. of tea for $7\frac{3}{4}$d., what is that per lb.?

9. If 3 qr. 24 lb. cost £4 16s. 8d., how much is that per lb?

10. If when flour is $5 barrel the five-cent loaf of bread weighs 10 oz., what ought to be its weight when flour is $8 a barrel?

11. If $1\frac{1}{4}$ acres of land sell for $34.50, what will 20 a. 2 r 10 per. cost, at the same rate?

12. If 18 a. 3 r 20 per. cost $900, what will 150 acres cost at the same rate?

13. If $1\frac{1}{2}$ bushels of wheat cost $1.68\frac{3}{4}$, what will 154 bu 1 pk. 6 qt. cost?

14. If a train travels 300 miles in 9 hr. 40 min, how long will it be in travelling 223 miles?

15. If 7 gal 1 qt of wine cost $17.40, what will 3 qt. 1 pt. cost at the same rate?

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16. If 15 yards, $\frac{3}{4}$ of a yard wide, will make a dress, how many yards, $\frac{5}{8}$ of a yard wide, will make another dress of the same size?

Yards required $\frac{3}{4}$ or $\frac{6}{8}$ yd. wide $= 15$ yards;
" $\frac{1}{8}$ yd. " $= 6 \times 15$ yds.;
" $\frac{5}{8}$ yd. " $= \frac{6 \times 15}{5}$
$= 18$ yds.

17. How many yards of cloth, $\frac{5}{8}$ yd. wide, will be required to line 35 yards, $1\frac{1}{4}$ yards wide?

18. If it requires 36 yards of carpeting, $\frac{3}{4}$ yd. wide, to cover a floor, how many yards, $\frac{7}{8}$ yd. wide, will be required to cover the same floor?

19. A regiment of 1000 men are to have new coats; each coat is to contain $2\frac{1}{2}$ yards of cloth, $1\frac{1}{4}$ yards wide, and to be lined with shalloon $\frac{3}{4}$ yd. wide; how many yards of shalloon will be required?

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20. A bankrupt owes $4000, and his assets—that is, his whole property—amount to no more than $840; what dividend will his creditors receive in the dollar?

Assets paid on $4000 $= \$840$;
" " $\$1 = \$\frac{840}{4000} = \$\frac{21}{100} = 21$ cents.

21. A merchant became insolvent, owing $6850, and had only $4932 with which to pay his creditors; how much should a creditor, whose claim is $1540, receive?

22. What does a bankrupt pay in the pound if his creditors receive £376 5s. out of £2076?

23. How much will a creditor lose on a debt of \$5342.25 if he receives only $67\frac{1}{2}$ cents in the dollar?

24. A creditor loses $37\frac{1}{2}$ cents in the dollar of what was due to him, and thereby loses \$330; what was the sum due?

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25. The people of a school section wish to build a new school-house, which will cost \$2850. The taxable property of the section is valued at \$190000; what will be the tax in the dollar, and what will be a man's tax whose property is valued at \$7500?

Tax on \$190000 = \$2850;

" \$1 = $\$\frac{2850}{190000}$ = \$·015 = $1\frac{1}{2}$ cents;

" \$7500 = \$7500 × \$·015 = \$112.50.

26. In a school section a tax of \$800 is to be raised. If the amount of taxable property is \$250000, what will be the tax in the dollar, and what is *A*'s tax, whose property is valued at \$1800?

27. What is the assessed value of property taxed \$37.80 at the rate of $4\frac{1}{2}$ mills in the dollar?

28. A person, after paying an income tax of 22 mills in the dollar, has \$2934 left; what is his income?

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29. A merchant buys a chest of tea containing 2 qr. 10 lb. at 60 cents per lb, and two chests containing 3 qr. 15 lb. at 70 cents per lb.; what will he gain by selling the mixture at 80 cents per lb.?

2 qr. 10 lb. = 60 lb.; 3 qr. 15 lb. = 90 lb.

60 lb. at 60 cents per lb. = \$36;

90 " 70 " " = \$63;

∴ cost of 150 = \$99.

Cost of 150 " 80 " " = \$120;

∴ gain = \$21.

30. A grocer buys coffee at \$34 per cwt., and chicory at \$10 per cwt., and mixes them in the proportion of 5 parts of chicory to 7 of coffee; he sells the mixture at 30 cents per lb.; what does he gain on each pound?

31. If I mix 20 lb of tea at 70 cents per lb with 15 lb. at 60 cents per lb. and 40 lb. at $62\frac{1}{2}$ cents per lb., what is 1 lb. of the mixture worth?

CHAPTER VII.

AVERAGES AND PERCENTAGES.

Section I.—Averages.

169. The **Average** of several numbers is that number which substituted for each of them will produce a sum equal to that of the given numbers.

Ex. 1. Find the average of 30, 35, 42, 80 and 100.

$$30 + 35 + 42 + 80 + 100 = 287.$$

There are 5 numbers; therefore $\frac{1}{5}$ of 287 will be the number which substituted for each of the given numbers will produce the sum 287 : $\frac{287}{5} = 57{\cdot}4$.

Exercise xcv.

Find the average of

1. 16, 18, 26, 30, 36, 42, 50 and 56.
2. 17, 0, 20, 30, 70, 100, 27, 9 and 17.
3. 120, 340, 560, 780, 320 and 840.
4. Five pupils obtained the following marks at an examination, 60, 36, 75, 21, and 80, respectively; what was their average mark?
5. There were 45 pupils at school on Monday, 43 on Tuesday, 47 on Wednesday, 45 on Thursday, and 40 on Friday. What was the average attendance for the week?
6. The average temperature of the different months during the past 37 years at Toronto was, of Jan. 22°·94, Feb. 22°·58, March 29°·05, April 40°·63, May 51°·68, June 61°·84, July 67°·43, Aug. 66°·32, Sept. 58°·10, Oct. 45°·74, Nov. 36°·03, Dec. 25°·56. What was the average yearly temperature during that period?

Section II.—Percentage.

170. The term, *per cent.* means *by* or *on a hundred*, thus, 3 per cent. on anything means 3 on every hundred of it. Hence 1 per cent. of a number is $\frac{1}{100}$ of it; 2 per cent. is $\frac{2}{100}$ of it; 7 per cent. is $\frac{7}{100}$ of it, &c.

171. The sign, %, is generally used to represent the words per cent. Thus, 3% is read 3 per cent.

Ex. 1. Find 5 per cent of \$360.

Since \$100 yields \$5;

\$1 " $\$\frac{5}{100}$;

\$360 " $\$\frac{360 \times 5}{100}$ or \$18.

Exercise xcvi.

Find

1. 16 per cent. of 450.
2. 20 " of \$75.
3. $33\frac{1}{3}$ " of 69 sheep.
4. $5\frac{1}{4}$ per cent. of \$200.
5. $2\frac{1}{3}$ " of 600 men.
6. $7\frac{1}{7}$ " of 630.

Ex. 2. A merchant sold 80 yd. of cloth from a web containing 250 yd.; what per cent of the web did he sell?

From 250 yd. he sold 80 yd.;

1 yd. " $\frac{80}{250}$ yd.;

100 yd. " $\frac{100 \times 80}{250}$ yd. or 32 yd

∴ he sold 32%.

7. A farmer who had 800 bu. of wheat sold 320 bu.; what per cent. of his wheat did he sell?

8. A fourth of a field has been ploughed; what per cent. of the field remains to be ploughed?

9. 780 is what per cent. of 1200? of 2145?

Ex. 3. Of what number is 60, 8%?

Since 8 = 8 % of 100;

1 = 8 % of $\frac{100}{8}$;

60 = 8 % of $\frac{60 \times 100}{8} = 750$.

10. Find the number of which 275 is 25 %.

11. How much must be a clerk's salary in order that 17 % of it may be \$204?

Section III.—Insurance.

172. Insurance is security guaranteed by one party on being paid a certain sum, to another against any loss.

173. The Premium is the sum paid for the insurance. It is always a certain per cent. of the sum insured.

174. The **Policy** is the written contract of insurance.

Ex 1. What is the premium for insuring a house valued at \$5000 at $1\frac{1}{4}$ per cent.?

Premium on \$100 = \$$1\frac{1}{4}$;
" \$1 = \$$\frac{1\frac{1}{4}}{100}$;
" \$5000 = \$$\frac{5000 \times 1\frac{1}{4}}{100}$ = \$62.50.

Exercise xcvii.

Find the premium on

1. \$600 at 3 %.
2. \$840 at $1\frac{1}{2}$%.
3. \$760 at 2 %.
4. \$375 at 3 %.
5. \$8000 at $1\frac{1}{4}$ %.
6. \$7360 at $1\frac{1}{2}$ %.
7. \$3500 at $1\frac{3}{4}$ %.
8. \$4890 at $1\frac{1}{3}$ %.

Ex. 2. For what sum should goods worth \$4900 be insured at 2% so that, in case of loss, the owner may recover both the value of the goods and premium paid?

Premium on \$100 at 2 % is \$2.

Insurance on goods worth \$98 = \$100;
" " " \$1 = \$$\frac{100}{98}$;
" " " \$4900 = \$$\frac{4900 \times 100}{98}$ = \$5000.

9. For what sum must a house worth \$2400 be insured at 4 % so that, in case it is burned, the owner may recover both its value and the premium paid?

10. What sum should be insured at 3 %, on goods worth \$6790, that the owner may receive both their value and the premium, in case of loss?

11. The premium at $2\frac{1}{2}$ % on a cargo of goods amounted to \$1750; what was the value of the cargo?

12. The premium for insuring a house at $\frac{1}{4}$ % is \$24; what is the value of the house?

13. The premium for insuring a house and furniture at $1\frac{1}{2}$ %, is \$79.14; what is the value of the property insured?

Section IV.—Commission and Brokerage.

175. Commission is the charge made by an agent for buying or selling goods, and is generally a percentage on the *money employed in the transaction.*

176. Brokerage is the charge made by a broker for buying or selling stocks, bills of exchange, etc.

Ex. 1. My agent has bought tea, on my account, to the amount of $750; what is his commission at 2 %?

The commission on $100 = $2;

" $1 = $\frac{2}{100}$;

" $750 = $\frac{750 \times 2}{100}$ = $15.

Exercise xcviii.

Find the commission on

1. $360 at 4 %.	4. $1200 at 2½ %.
2. $790 at 2 %.	5. $7600 at 3¼ %.
3. $800 at 1¼ %.	6. $4800 at 2⅓ %.

Ex. 2. I send my agent $1470 with instructions to deduct his commission at 5 % and invest the balance in wheat; how much does he invest?

Commission on $100 at 5 % is $5.

Sum invested out of $105 = $100;

" $1 = $\frac{100}{105}$;

" $1470 = $\frac{1470 \times 100}{105}$

= $1400.

7. Sent $2600 to my agent to invest after deducting his commission at 4 %; what sum did he invest?

8. I sent my agent $9180 with instructions to deduct his commission at 2 % and invest the balance in wheat; how much wheat did he purchase at $1.20 per bushel?

9. An agent receives $31.55 as his compensation for purchasing goods at 4 % commission; what is the value of the goods purchased?

10. A broker sells a bill of exchange worth $700; what is his brokerage at ¼ per cent.?

11. If a commission of $106.47 is paid for selling $3276 worth of goods, what is the rate per cent.?

Section V.—Interest.

1. If I lend you \$500, and you have to pay me \$1 for the use of each \$100 per year, how much will I receive for 1 year?

2. How much must you pay for the use of \$600 for 1 year, if you have to pay \$2 for the use of each \$100 per year, or 2 cents for each dollar? If you have to pay \$3? \$4? \$8?

177. The sum paid for the *use* of money is called **Interest.**

178. The money on which the *interest* is paid is called the **Principal.**

179. The number of dollars paid for the use of \$100 for one year is called the **Rate per cent.**

NOTE 1. When the *rate per cent.* is stated without the mention of any length of time, the time is understood to be one year.

Ex. 1. What is the interest on \$2750 for 1 year at 8 per cent.?

Interest on \$100 for 1 year $= \$8$;
" \$1 " $= \$\frac{8}{100}$;
" \$2750 " $= \$\frac{2750 \times 8}{100}$;
$= \$220.$

Exercise xcix.

1. What is the interest on \$600 for 1 year at 8 %?
2. What is the interest on \$550 for 1 year at 7 %?
3. What is the interest on \$3152.16 for 1 year at $7\frac{1}{2}$ %?
4. A man borrowed \$7200 for 1 year, viz., \$1250 at 7 %; \$1340 at $7\frac{1}{2}$ %; \$2360 at 5 %; and the remainder at $8\frac{1}{4}$ %; how much interest has he to pay at the end of the year?
5. Four brothers have to divide equally the interest of \$25800 at 7 %; how much does each receive each year?

Ex. 2. What is the interest on \$575 for 5 years at 7 %?

Interest on \$100 for 1 year $= \$7$;
" \$1 " $= \$\frac{7}{100}$;
" \$1 for 5 years $= \$\frac{5 \times 7}{100}$;
" \$575 " $= \$\frac{575 \times 5 \times 7}{100}$;
$= \$201.25.$

6. What is the interest on $986 for 4 years at 6 % ?
7. What is the interest on $1573 for 4 years at 8 % ?
8. What is the interest on $500 for 2 years at 8½ % ?
9. What is the interest on $2245.85 for 5 years at 7½ % ?

Ex. 3. What is the interest on $672 for 4 yr. 8 mo. at 9 % ?

4 yr. 8 mo. $= 4\frac{8}{12}$ yr. $= 4\frac{2}{3}$ years.

Interest on $100 for 1 year $= \$9$;

" $1 " $= \$\frac{9}{100}$;

" $1 for $4\frac{2}{3}$ years $= \$\frac{41\times9}{100}$;

" $672 " $= \$\frac{672\times41\times9}{100}$

$= \$282.24.$

10. What is the interest on $924 for 3 yr. 7 mo. at 6 % ?
11. What is the interest on $954 for 4 yr. 8 mo. at 7 % ?
12. What is the interest on $504.72 for 3 yr. 10 mo. at 8 % ?
13. What is the interest on $640.75 for 3 yr. 4 mo. at 9 % ?

180. From the preceding examples we have the following rule for finding the interest on a given sum of money at a given rate per cent. for any number of years :

Multiply the Principal by the Rate per cent., the product by the number of years, and divide this result by 100.

181. The **Amount** is the name given to the sum of the principal and interest together.

Ex. 4. If a man borrows $480 for 8 months at 8 %, what amount should he return at the end of that period ?

Interest on $480 for 12 months $= \$\frac{480\times8}{100}$;

" " 1 month $= \$\frac{480\times8}{12\times100}$;

" " 8 months $= \$\frac{8\times480\times8}{12\times100}$

$= \$25.60.$

Interest	= $ 25.60
Principal	= $480.00
Amount	= $505.60

14. What is the amount of $840 for 10 months at 6% ?
15. What is the amount of $1573 for 4 years at 8% ?
16. To what sum will $784 amount in 2yr. 9mos. at 7% ?

In the preceding examples we have expressed the months as a fraction of a year, but in actual practice more accuracy is generally required, and we must express the given parts of a year in days.

When interest is required from one date to another, the day of the first date is to be left out, because it is not until the day following that one day's interest will have accrued.

Ex. 5. Find the interest on $1200 from March 1, 1875, to May 31, 1878, at 7 per cent.

Time from March 1, 1875, to May 31, 1878 = 3 yr. 91 days

Interest on $100 for 365 days = $7 ;

" $100 " 1 day = $\$\frac{7}{365}$;

" $1 " 91 days = $\$\frac{91 \times 7}{100 \times 365}$;

" $1200 " " = $\$\frac{1200 \times 91 \times 7}{100 \times 365}$;

$= \$\frac{2400 \times 1 \times 7}{100 \times 730}$

$= \$20.942\ldots$

Interest on $1200 for 3 yr. at 7% = $252.00.

" " for 3 yr. 91 days at 7 % = $272.94.

17. Find the interest on $500 for 156 days at 7 %.
18. Find the interest on $7500 from May 5 to Oct. 27, at 8 %.
19. Find the interest on $8000 from Jan. 26, 1876, to March 31, 1878, at 7½ %.

Ex. 6. At what rate per cent. must $756 be put at interest for 4 years to yield $241.92 ?

Interest on $756 for 1 year = $\$\frac{241.92}{4}$ = $60.48 ;

" $1 " = $\$\frac{60.48}{756}$;

" $100 " = $\$\frac{100 \times 60.48}{756}$

= $8, or 8 per cent.

20. A man pays $72 for the use of $900 for 1 year, what is the rate per cent. ?
21. A men lent $484 for 5 years, and received $181.50 for the interest ; what was the rate per cent. ?

22. If $103.68 interest is received on a principal of $432 for 4 years, what is the rate per cent.?

Ex. 7. What principal will bring $200 interest in 146 days at 5 per cent.?

Principal to give $5 in 365 days = $100;
" $1 " " = $\frac{100}{5}$ = $20;
" $200 " " = 200 × $20 = $4000;
" " " 1 day = 365 × $4000;
" " " 146 days = $\frac{365 \times 4000}{146}$;
= $10000.

23. A man borrowed money at 7 per cent. and paid $245 interest a year; how much money did he borrow?

24. A man bequeathed his wife $875 a year, his daughter $770 a year, and his son $630 a year; what sum must be invested at 7 per cent. to produce these amounts?

25. Suppose a gentleman's interest on money, at 6 per cent., is $45 per month; how much is he worth?

Ex. 8. In what time will $800 amount to $880 at 8 per cent.?

Interest = $880 − $800 = $80.

The interest of $800 for 1 year at 8 per cent. is $64.

Time to produce $64 = 1 year;
" " $1 = $\frac{1}{64}$ year;
" " $80 = $\frac{80}{64}$ = $1\frac{1}{4}$ years;
= 1 yr. 3 mo.

26. How long a time would be required for $525 to gain $110.25 at 7 per cent.?

27. How long a time would it require for $625 to amount to $756.25 at 7 per cent.?

28. A principal of $600 was loaned May 20th, 1873, at $7\frac{1}{2}$ per cent. At what date did it amount to 796.87\frac{1}{2}$?

29. A note given for $273.25 at 7 per cent, remained unpaid until the interest equalled the principal. How long did it run?

Section VI.—Present Worth and Discount.

John Smith owes me a debt of $108 to be paid at the end of a year, without interest; how much is the debt worth at present, and how much should be allowed for the immediate payment of the debt, money being worth 8 per cent.?

If I receive \$100, and put it out to interest at 8 % for one year, it will amount to \$108; hence, the present worth of the debt is \$100. Evidently \$8 should be allowed for immediate payment.

182. The **Present Worth** of a note or debt, payable at some future time, without interest, is such a sum as, being put out to interest, will amount to the given debt when it becomes due.

183. The allowance or deduction made for the payment of the debt before it becomes due is called **Discount.**

Ex. 1. What is the present worth of \$535, payable in 1 year, the rate of interest being 7 per cent.?

Amount of \$100 in 1 yr. at 7 % = \$107.
Present worth of \$107 = \$100;
" " \$1 = \$$\frac{100}{107}$;
" " \$535 = \$$\frac{535 \times 100}{107}$
= \$500.

Exercise c.

1. What is the present worth of \$1250.509, payable in 1 year, the rate of interest being 7 %?
2. What is the present worth of \$512.40, payable in 1 year, when money is worth 12%?

Ex. 2. What is the present worth of \$787.75 due in 2 yr 6 mo., when money is worth 6 %?

Amount of \$100 for 2 yr. 6 mo. at 6 % = \$115.
Present worth of \$115 = \$100;
" " \$1 = \$$\frac{100}{115}$;
" " \$787.75 = \$$\frac{787.75 \times 100}{115}$
= \$685.

3. What must be paid now to cancel a debt of \$994.50 due 1 yr. 9 mos. hence, at 6 %?
4. Which is the more profitable, to buy lumber at \$25 a thousand on 9 months' credit, or at \$24.50, on 6 months' credit, money being worth 6 %?
5. Bought two lots for \$2541, on 3 years' time, without interest; what is the cash value, money being worth 7 %?
6. I buy goods for \$1150 cash and sell them for \$1224 on a credit of 4 months; do I gain or lose, and how much, interest being 6 %?

Ex. 3. A note for $1380.06 becomes due in 15 months; what deduction should be made for the immediate payment of the money, supposing money to be worth 8 %?

The interest on $100 for 15 months = $10.
Discount on $110 " = $10;
" $1 " = $\frac{10}{110}$;
" $1380.06 " = $\frac{1380.06 \times 10}{110}$;
= $125.46.

7. What is the discount on $897.82 payable in 3 years, when money is worth 7 %?

8. What is the discount on a note for $1174.32 due in 3 yr. 3 mo., money being worth 8 %?

9. What is the difference between the interest and the discount on $525 due 10 mos. hence, at 6 %?

184. The discount found in Ex. 3 is called *True Discount.* There is another kind of discount called *Bankers' Discount,* or *Bank Discount.*

The difference between the two kinds of discount is this —the *true* discount is the interest of the present value of the bill for the time, while the *bankers' discount* is the interest of the amount of the bill itself, not only for the specified time but for three days additional called *days of grace.* The bankers' discount is thus always in excess of the true discount.

Ex. 4. What is the discount and present worth of a note of $584, drawn Jan. 8 at 11 months, discounted at the Bank May 10, at 5 per cent.

11 mo. from Jan. 8 = Dec. 8,
which with 3 days of grace = Dec. 11.
From May 10 to Dec. 11 is 215 days.
Discount or interest of $584 for 215 days at 5 % = $17.20
Principal = $584.

Present worth = $566.80.

10. What is the bank discount on a note for $730 at 6 % for 30 days, days of grace included?

11. Suppose a bill for $1200 is drawn on the 12th of August at 6 months, and paid by a banker on the 1st of January, find the money he takes off at 7 %.

CHAPTER VIII.

SQUARE ROOT.

1. What is the *second* power of 5? of 9? of 12?

2. What number multiplied by itself will produce 36? 49? 121? 81?

3. Find the number whose *second* power is 9? is 25? is 64? is 144?

4. Resolve each of the following numbers into *two equal* factors: 16, 25, 81, 49, 100.

185. The *Second Power* of a number is called its **Square.**

NOTE.—The square of a number is indicated by writing 2 to the right and above the given number; thus, 5^2 is read 5 squared.

186. The **Square Root** of a number is one of its *two equal factors.*

187. Principle. The square of a number of two digits is equal to the square of the tens, plus twice the tens multiplied by the units, plus the square of the units. Thus,

$$45^2 = 40^2 + 2 \times 40 \times 5 + 5^2 = 1600 + 400 + 25 = 2025.$$

Ex. 1. Find the square root of 2025.

```
20 25 ( 45
16
---
85 | 425
     425
     ---
```

We separate the number into periods of two figures each, by means of a line, counting from the decimal point. We then find the largest number whose square does not exceed 20. This is 4. We write 4 as the first figure of the square root and place its square, 16, under 20 and subtract, and to the remainder, 4, we annex the next period, 25, to make a dividend. We double the figure 4, placed in the root to form the first figure of a divisor. As we have to annex another figure to 8, we call the 8, 8 tens or 80. 80 is contained in 425 5 times. We write 5 as the second figure of the root and annex it to the 8. We next multiply 85 by 5 and write the product under 425 and subtract. As there is no remainder the square root is 45.

Exercise ci.

Find the square root of

1. 289.	5. 1296.	9. 4096.
2. 361.	6. 5625.	10. 1369.
3. 576.	7. 9025.	11. 2209.
4. 625.	8. 2401.	12. 3136.

Ex. 2. Find the square root of 4124961.

```
4|12|49|61 (2031
 4
403 | 1249
      1209
4061 | 4061
       4061
```

After finding the first figure of the root and subtracting its square from the left hand period and bringing down the next period, 12, we find that 40 is not contained in 12. We therefore, put a 0 in the root and bring down the next period. We then double the part of the root already found and write 40 as a divisor. We call it 400, and find that it goes into 1249, 3 times. We put 3 in the root and annex it to the 40. We now multiply 403 by 3 and write the product, 1209, under the 1249, and subtract, &c.

Find the square root of

13. 390625.	16. 5764801.
14. 262144.	17. 40005625.
15. 117649.	18. 25080064.

Ex. 3. Extract the square root of ·7 to four places of decimals.

```
·70|00|00|00| (·8366.
 64
 ---
163 | 600
       489
 ----
1666 | 11100
       9996
 -----
16726 | 110400
        100356
 ------
  10044
```

In finding the square root of a decimal fraction care must be taken to make the decimal consist of an *even* number of figures. This is done so that the denominator of the equivalent vulgar fraction may be a complete square, which is the case in $\frac{70}{100}$, $\frac{7000}{10000}$, $\frac{700000}{1000000}$, &c., but not in $\frac{7}{10}$ $\frac{700}{1000}$, $\frac{70000}{100000}$, &c.

Find the square root of

19. ·2209.	22. ·714.	25. 71.
20. ·0729.	23. ·895.	26. 6·4.
21. ·1024	24. ·9.	27. 14·4.

CHAPTER IX.

MEASUREMENTS OF SURFACES AND SOLIDS.

Section I.—Area of a Rectangle.

188. A Rectangle is a plane surface having four sides and four equal angles. A slate, a door, &c., are examples of a rectangle.

Ex. 1. A room is 18 feet long and 15 feet wide; what is its area?

Area of surface 1 ft. long by 1 ft. wide = 1 sq. ft.
" 18 ft. long by 1 ft. wide = 18 sq. ft.
" 18 ft. long by 15 ft. wide = 15 × 18 sq. ft.
= 270 sq. ft.

Hence, *To find the area of a rectangle, multiply its length by its width.*

Exercise cii.

Find the area of the rectangles having the following dimensions:

1. 8 ft. by 12 ft.
2. 6½ ft. by 14 ft.
3. 21 ft. by 25 ft.
4. 2 yd. 2 ft. by 7 yd.
5. 17 yd. by 20 yd. 2 ft.
6. 19 ft. 7 in. by 24 ft.

Section II.—Carpeting Rooms.

189. Carpets are sold in strips, and when the width of a strip is known, we can ascertain what *length* of carpet will be required to cover a given surface.

Ex. 1. How many yards of carpet 2 ft. 3 in. wide will be required for a room 21 ft. by 18 ft.?

Area of surface to be covered . . = 18 × 21 sq. ft.
Length of carpet 1 ft. wide, required to cover given area . . . . = 18 × 21 feet;

Length of carpet, $2\frac{1}{4}$ ft. wide, required to cover given area . . . . . $= \frac{18 \times 21}{2\frac{1}{4}}$ ft.
$= 56$ yards.

Exercise ciii.

How many yards of carpet 27 in. wide will be required for rooms whose dimensions are:

1. 27 ft. by 21 ft.?
2. 15 ft. by 12 ft.?
3. 18 ft. by 24 ft.?
4. 26 ft. by 36 ft.?

Find the cost of carpeting rooms whose dimensions are

5. 18 ft. by 20 ft. with carpet 3 ft. wide, at \$1.20 a yd.
6. 20 ft. by 24 ft., with carpet 30 in. wide, at 90 cts. a yd.
7. 15 ft. by $17\frac{1}{2}$ ft., with carpet 3 ft. wide, at \$1 a yd.
8. The cost of carpeting a room 18 ft. long by 16 ft. wide, with carpet worth \$1.20 a yd.. is \$51.20; how wide is the carpet?

Section III.—Papering a Room.

190. Room papers, like carpets, are sold in strips, and we ascertain the quantity that will cover a wall in the same manner as we ascertained the quantity of carpet required to cover a floor.

Ex. 1. How many yards of paper 16 in. wide will be required for a room 18 ft. long, 14 ft. wide, and 8 ft. high, which contains 1 door 7 ft. high by $3\frac{1}{2}$ ft. wide and 3 windows each 5 ft. high by $2\frac{1}{2}$ ft. wide?

Length of surface to be covered . . . . . $= (18+14+18+14)$ ft. $= 64$ ft.
Area of entire walls . . $= (8 \times 64)$ sq. ft. $= 512$ sq. ft.
Area of door . . . . . $= (3\frac{1}{2} \times 7)$ sq. ft. $= 24\frac{1}{2}$ sq. ft.
Area of 3 windows . . . $= (3 \times 2\frac{1}{2} \times 5)$ sq. ft. $= 37\frac{1}{2}$ sq. ft.
Area of door and windows $= (24\frac{1}{2}+37\frac{1}{2})$ sq. ft. $= 62$ sq. ft.
Area to be papered . . . $= (512 - 62)$ sq. ft. $= 450$ sq. ft.
450 sq. ft. . . . . $= 450 \times 144$ sq. in.

∴ length of paper required $= \frac{450 \times 144}{16} = 4050$ in.
$= 112\frac{1}{2}$ yards.

Exercise civ.

1. How many yards of paper 20 in. wide will be required for a room 20 ft. long, 15 ft. wide, and 9 ft. high?
2. How many sq. ft. of paper will be required for a room 18 ft. 9 in. long, 15 ft. 3 in. wide and $8\frac{1}{2}$ ft. high?

3. A room 24 ft. long, 20 ft. wide and 10 ft. high contains 2 doors each 7 ft. by 4 ft. and 6 windows each $5\frac{1}{2}$ t. by 4 ft.; find how many yards of paper 2 ft. wide will be required to paper it.

4. How many yards of paper 30 in. wide will it require to cover the walls of a room 15 ft. long, 12 ft. wide and 8 ft. high?

5. William Benson has agreed to plaster the walls and ceiling of the room in the last example, at 10 cents per sq. yd.; what will his bill amount to?

Section IV.—Measurement of Solidity.

Ex. 1. Find the number of cubic feet in a rectangular piece of timber 24 ft. long, 3 ft. wide, and 2 ft. thick.

If this piece of timber be cut into blocks 1 ft. long there will be 24 such blocks.

Number of cu. ft. in 1 block $= 6$ cu. ft.

" 24 blocks $= 24 \times 6$ cu. ft. $= 144$ cu. ft.

Hence, *To find the cubic content of a rectangular solid, take the product of its length, breadth, and thickness.*

Exercise cv.

Find the cubic content of the rectangular solids whose dimensions are

1. 8 ft., 6 ft., 5 ft.
2. $2\frac{1}{2}$ ft., $5\frac{1}{3}$ ft., $7\frac{1}{8}$ ft.
3. 3 ft., $7\frac{1}{2}$ ft., $8\frac{1}{3}$ ft.
4. 2·6 ft., 3·5 ft., 5 ft.

Ex. 2. How many bricks will be required to build a wall 20 ft. long, 15 ft. high, and 18 in. thick, each brick being 8 in. long, 4 in. wide, and 3 in. thick?

Cubic content of wall . . $= (20 \times 12 \times 15 \times 12 \times 18)$ cu. in.

" brick . . $= (8 \times 4 \times 3)$ cu. in.;

$\therefore$ number of bricks required $= \frac{20 \times 12 \times 15 \times 12 \times 18}{8 \times 4 \times 3}$

$= 8100.$

5. How many bricks will be required to build a wall 45 ft. long, 20 ft. high, and 15 in. thick, each brick being 9 in. long, $4\frac{1}{2}$ in. wide, and 3 in. thick?

6. What will it cost to put a stone foundation under a barn 36 ft. long by 24 ft. wide at 2 cents a cubic yard, the wall being 7 feet high and 2 ft. thick?

Miscellaneous Problems.

1. A garrison of 800 men had provisions to last for 60 days, but 15 days afterwards 80 men were killed ; how long will they last the remainder ?

They would last 800 men 45 days.

" 1 man 800×45 days.

" 720 men $\frac{800 \times 45}{720}$ days $= 50$ days.

2. 28 shanty men have provisions for 20 days, but 7 men more arrived ; how long will the provisions now last ?

3. A garrison of 1000 men was victualled for 28 days ; after 11 days it was reinforced by 2400 men ; how long will the provisions last ?

4. A garrison of 450 men had provisions for 5 months, but 200 men were sent away ; how long will the provisions last the remainder ?

5. A garrison of 1000 men was victualled for 30 days ; after 10 days it was reinforced by 3000 men ; in what time would the provisions be exhausted ?

6. A can do a piece of work in 8 days, and B can do it in 9 days ; how long will it require A and B working together to do it ?

The part A does daily $= \frac{1}{8}$

" B " " $= \frac{1}{9}$

" A and B do " $= \frac{1}{8} + \frac{1}{9} = \frac{17}{72}$;

$\therefore$ they do $\frac{17}{72}$ in $\frac{1}{17}$ day ;

$\therefore$ they do the whole work in $4\frac{4}{17}$ days, or $4\frac{4}{17}$ days.

7. A can do a piece of work in 12 hours, and B can do it in 15 hours ; in what time can both working together do the work ?

8. A can do a piece of work in 20 days, B can do it in 24 days, and C can do it in 30 days ; in what time will they all do it working together ?

9. A can build a wall in 8 days, B in 12 days, and C in 15 days ; in what time can they all build it working together ?

10. A quantity of flour lasts a man and wife 9 days, and the wife alone 27 days ; how long would it last the man alone ?

11. A can do a piece of work in 20 days; after working at it for 8 days B comes to help him and they finish the work in 5 days; how long would it take B by himself to do the work?

12. A can do $\frac{2}{3}$ of a piece of work in 8 days; B can do $\frac{3}{4}$ of the same work in 12 days; in what time could both working together do 2 such pieces of work?

13. A and B can mow a field in 12 days; A and C in 15 days; B and C in 20 days; in what time could A mow it by himself?

A and B can do $\frac{1}{12}$ of work in 1 day;
A and C " $\frac{1}{15}$ "
B and C " $\frac{1}{20}$ "
$\therefore$ 2 A's and 2 B's and 2 C's " $\frac{1}{12} + \frac{1}{15} + \frac{1}{20}$ " $= \frac{1}{5}$;
$\therefore$ A and B and C " $\frac{1}{2}$ of $\frac{1}{5}$ " $= \frac{1}{10}$;
$\therefore$ A " $(\frac{1}{10} - \frac{1}{20})$ " $= \frac{1}{20}$;
$\therefore$ A can do the work in 20 days.

14. A and B can do a piece of work in 8 days; A and C can do it in 9 days, and B and C in 10 days; in what time can all three working together do it?

15. A and C can dig a garden in 10 days; B and C can dig $\frac{1}{3}$ of the same garden in 4 days, and B alone can dig it in 20 days; in what time can A do it by himself?

16. A piece of work has been half done by A, B, and C working together, in 8 days; if A and B together can finish it in 12 days, in what time could C have finished it?

17. A can do a piece of work in 6 days of 10 hours each, and B can do it in 8 days of 9 hours each; for how many hours a day should A and B be engaged together, that the work may be done in 4 days?

18. If 6 men or 9 women can do a piece of work in 12 days, in what time will 4 men and 7 women do it?

6 men do the work in 12 days $\therefore$ 1 man does $\frac{1}{72}$ of it in 1 day
9 women do the work in 12 days $\therefore$ 1 woman does $\frac{1}{108}$ of it in 1 day;
$\therefore$ 4 men and 7 women do $\frac{4}{72} + \frac{7}{108}$ or $\frac{13}{108}$ of it in 1 day;
$\therefore$ " " do $\frac{1}{108}$ of it in $\frac{1}{13}$ day;
$\therefore$ " " do it in $\frac{108}{13}$ day, or $8\frac{4}{13}$ days.

19. If 7 boys or 4 men can do a piece of work in 9 days, in what time can 4 boys and 7 men do it?

20. If 3 men or 5 women do a piece of work in 12 days, in what time can 2 men and 1 woman do it?

21. If 1 man and 2 women can do a piece of work in 8 days, and 3 men and 4 women can do it in 3 days, in what time can 1 man or 1 woman do it?

Since 1 man and 2 women do $\frac{1}{8}$ of it in 1 day;
$\therefore$ 2 men and 4 women do $\frac{1}{4}$ " "
But 3 men and 4 women do $\frac{1}{3}$ " "
$\therefore$ 1 man does $\frac{1}{3}-\frac{1}{4}$, or $\frac{1}{12}$ " "
$\therefore$ 1 man will do it in 12 days.
Now 1 man and 2 women do $\frac{1}{8}$ of it in 1 day;
$\therefore$ 2 women do $\frac{1}{8}-\frac{1}{12}$, or $\frac{1}{24}$ " "
$\therefore$ 1 woman will do it in 48 days.

22. If 3 men and 2 boys do a piece of work in 8 days, and 3 men and 7 boys can do it in 6 days, in what time can 1 man or 1 boy do it?

23. If 2 men and 5 boys can do a piece of work in 20 days, and 1 man and 8 boys can do it in 18 days, in what time can 1 man or 1 boy do it?

24. If 7 men and 5 women can do a piece of work in $2\frac{2}{3}$ days, and 3 men and 8 women can do it in $3\frac{11}{13}$ days, in what time can 1 man or 1 woman do the work?

25. 3 women and 2 boys can do a work in $6\frac{2}{3}$ days, and 2 women and 3 boys can do it in $7\frac{1}{17}$ days; in what time can 1 woman or 1 boy do it?

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26. A cistern is filled by 2 pipes in 8 and 10 hours respectively, in what time will they fill it when they both run at the same time?

They fill $\frac{1}{8}+\frac{1}{10}$ of the vessel in 1 hour;
$\therefore$ " $\frac{5+4}{40}$, or $\frac{9}{40}$ " "
$\therefore$ " $\frac{1}{40}$ in $\frac{1}{9}$ hour;
$\therefore$ they fill the vessel in $\frac{40}{9}$ or $4\frac{4}{9}$ hours.

27. A vessel is filled by 3 taps, running separately, in 60, 75 and 90 minutes respectively; in what time will they fill it when they all run at the same time?

28. Two pipes running together can empty a cistern in 8 hours, and one by itself can do it in 12 hours; in what time can the other empty it?

29. Two pipes running together can empty a vessel in 50 minutes; one of them can empty $\frac{2}{5}$ of the vessel in 40 minutes; in what time can the other empty $\frac{1}{3}$ of it?

30 A cistern is filled by two pipes, *A* and *B*, in 20 and 24 minutes respectively, and is emptied by a tap, *C*, in 30 minutes; in what time will it be filled by all running together?

31. A bath is filled by a pipe in 60 minutes; it is emptied by a waste pipe in 40 minutes; in what time will the bath be emptied if both pipes are opened at once?

One pipe empties $\frac{1}{40}$ of vessel in 1 minute;

the other fills $\frac{1}{60}$ of vessel in 1 minute;

$\therefore$ when both are running $(\frac{1}{40} - \frac{1}{60})$, or $\frac{1}{120}$ of the vessel is emptied in 1 minute;

$\therefore$ the vessel is emptied in 120 minutes.

32. A vessel can be filled by 2 taps running separately in 30 and 36 minutes respectively, and emptied by a third in 15 min.; if the vessel is full and all 3 taps running at once, in what time will it be emptied?

33. A bath can be filled by two taps running separately in 20 and 30 minutes respectively, and emptied by two others in 24 and 18 min. respectively; if the bath is full and all four taps opened, in what time will the bath be emptied?

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34. *A* spent $\frac{1}{3}$ of his money on Monday; he spent $\frac{1}{4}$ of the remainder on Tuesday, and on Wednesday he spent $\frac{1}{5}$ of what he had left; he had still \$10; how much had he at first?

Remainder after 1st spending $= \frac{2}{3}$ of money.

" 2nd " $= \frac{3}{4}$ of $\frac{2}{3}$ of money,

" 3rd " $= \frac{4}{5}$ of $\frac{3}{4}$ of $\frac{2}{3}$ of money

$= \frac{2}{5}$ of money;

$\therefore$ $\frac{2}{5}$ of money $=$ \$10;

$\therefore$ $\frac{1}{5}$ " $=$ $\frac{\$10}{2}$;

$\therefore$ $\frac{5}{5}$ of his money $= \frac{5 \times 10}{2}$

$=$ \$25.

35. A father willed to his eldest son $\frac{2}{3}$ of his property; to his second son $\frac{2}{7}$ of it, and to his youngest son the rest amounting to \$7238; what was the property worth?

36. A post is $\frac{1}{3}$ in the earth, $\frac{3}{7}$ in the water, and 13 feet above the water; what is the length of the post?

37. A man devotes ·12 of his income to charity, ·25 for educating his children, ·45 for household expenses, and saves the remainder, which is \$284.76; what is his income?

38. A ship whose cargo was worth \$25000 being disabled, ·45$\frac{1}{2}$ of the whole cargo was thrown overboard; what would a merchant lose who owned ·25 of the cargo?

61. A grocer bought a quantity of sugar for \$115; for what must he sell it to gain 18 per cent.?

62. A grocer sells a quantity of sugar for \$324, and thereby loses 10 per cent.; what did the sugar cost?

That which sold for \$90 cost \$100;
" " \$1 " $\$\frac{100}{90}$;
" " \$324 " $\$\frac{324 \times 100}{90}$
$= \$360.$

63. A man sells a piece of cloth for \$52.67 and thereby gains 15 per cent.; what was the cost of the cloth?

64. Sold salt at \$1.37½ per bushel, which was 5 per cent. less than cost; what was the cost

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65. Divide \$200 between A and B, so that for every \$3 that A gets, B shall get \$2.

Sum of shares $= \$3 + \$2 = \$5.$
A's share of $\$5 = \$3$;
" $\$1 = \$\frac{3}{5}$;
" $\$200 = \$\frac{200 \times 3}{5} = \$120.$

B's share may be found in a similar manner, or by simply subtracting A's share from the whole sum to be divided.

NOTE.—In the above case the shares of A and B are said to be in the ratio of 3 to 2, or in the proportion of 3 to 2.

66. The sum of two numbers is 1260, and they are to each other as 57 and 48; what are the numbers?

67. Divide \$500 among three persons, A, B, and C, so that the three portions may be to each other as the numbers 5, 9 and 6, respectively.

68. A bankrupt has three creditors, to whom the sums due are as the numbers 3, 4, 5; if his assets are valued at \$600, find the sums they will respectively receive.

69. At an election the number of votes cast was 510, and ⅔ of the votes for one candidate equalled ¾ of the votes for another; how many votes were cast for each?

Let A and B be the candidates.
⅔ of A's votes $=$ ¾ of B's votes;
⅓ " " $= \frac{1}{2}$ of ¾ of B's votes $= \frac{3}{8}$ of B's votes;
" " $= \frac{3 \times 3}{8} = \frac{9}{8}$ of B's votes.
$\frac{17}{8}$ of B's " $+ \frac{8}{8}$ of B's votes $= 510.$
$\frac{17}{8}$ " " $= 510;$
" " $= \frac{8 \times 510}{17} = 240;$
A's " $= 510 - 240 = 270.$

70. A and B have 210 acres of land, and $\frac{3}{4}$ of A's share equals $\frac{2}{3}$ of B's; how many acres has each?

71. Two neighbours raised 3800 bushels of wheat, and $\frac{2}{5}$ of what one raised equalled $\frac{1}{9}$ of what the other raised; how much did each raise?

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72. A and B engage in trade; A furnishes \$6000 and B \$4000; they gain \$1200; what is each one's share of the gain?

The total sum in trade is \$6000 + \$4000 = \$10000;
with which they gain \$1200.
Gain on \$10000 = \$1200;
" \$1 = $\$\frac{1200}{10000}$ = $\$\frac{12}{100}$;
" \$6000 = $\$\frac{6000 \times 12}{100}$ = \$720 = A's share.
" \$4000 = $\$\frac{4000 \times 12}{100}$ = \$480 = B's share.

73. A, B and C buy a house for \$2500; A pays \$500; B \$1200; C \$800; they rent it for \$300; what is each one's share of the rent?

74. A man dying, willed to his son \$6500, to his widow \$8000, and to his daughter \$5500; but his estate amounted to only \$12000; how much did each get?

75. A and B jointly rented a pasture for \$24; A put in 36 cows and B 24 cows; how much of the rent ought each to pay?

76. A, B and C hired a carriage for \$15.75, each agreeing to pay in proportion to the number of miles he rode. A rode 90 miles, B 75, and C 60 miles; what part of the hire ought each to pay?

77. A and B engaged in trade; A put in \$560 for 6 months, and B \$450 for 8 months; they gained \$513; what was each man's share of the gain?

\$540 for 6 months = 6 × \$540 for 1 month.
\$450 for 8 months = 8 × \$450 "
Total sum in trade for 1 month = \$3240 + \$3600 = \$6840
Gain on \$6840 = \$513
" \$1 = $\$\frac{513}{6840}$ = $\$\frac{3}{40}$;
" \$3240 = $\$\frac{3240 \times 3}{40}$ = \$243 = A's gain.
" \$3600 = $\$\frac{3600 \times 3}{40}$ = \$270 = B's gain.

78. Three men, A, B and C, rented a pasture for \$70.00; A put in 36 cows for 5 months; B 48 cows for 4 months; and C 72 cows for 3 months; what part of the rent ought each to pay?

79. Bowman, Johnston and Reed agreed to do a piece of work for $1600; Bowman furnished 7 men for 30 days; Johnston 5 men for 40 days; and Reed 6 men for 32 days; how much should each receive if they paid $35 clerk hire?

80. Two persons are in partnership 2 years; *A* at first put in $2500 and *B* $3000; at the end of nine months *A* took out $800 and *B* put in $500; they lost in two years $3825; what was each one's share of the loss?

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81. What is the compound interest of $400 for 3 years at 6 per cent.?

NOTE.—Compound interest is interest, not only for the use of the sum borrowed, but also for the use of the interest if it be not paid when it falls due.

Amt. of $100 for 1 yr. $= \$106$;

" $1 " $= \$\frac{106}{100} = \$1.06$.

" $1 for 2 yr. $= \$1.06 + $ int. of $1.06

$= \$1.06 + \frac{6}{100} \times \$1.06 = \$(1.06)^2$;

" $1 for 3 yr. $= \$(1.06)^2 +$ int. of $\$(1.06)^2$

$= \$(1.06)^2 + \frac{6}{100} \times \$(1.06)^2 = \$(1.06)^3$;

" $400 for 3 yr. $= 400 \times \$(1.06)^3 = \$476.4064$.

Amount . . . . $= \$476.406$

Principal . . . . $= 400.00$

Compound Interest . $= \$76.406$

82. What is the compound interest of $650 for 3 years at 6 per cent.?

83. Find the amount of $1000 for 4 years at 5 per cent.

84. Find the difference between the simple and compound interest of $350 for 3 years at 8 per cent.

85. A sum of money put out at simple interest for 2 years at 8 per cent. amounted to $464: to what sum would it have amounted had it been lent at compound interest?

86. The true discount on a sum of money for 3 years at 8 per cent. is $120; what is the compound interest of the same for the same time?

87. A man deposits in the savings bank $500, on which the interest at 6 per cent. per annum is to be added to the principal every 6 months; how much money has the man in the bank at the end of two years?

EXAMINATION PAPERS.

JULY EXAMINATIONS, 1877.

ADMISSION TO HIGH SCHOOLS.

TIME—TWO HOURS.

Examiner—J. A. McLELLAN, LL.D.

1. What is the least number that must be added to five millions to make the sum exactly divisible by seven thousand and nineteen?

2. Simplify $\frac{20}{21} - \left(\frac{48\frac{1}{2} + 7\frac{2}{3} - 16\frac{2}{4}}{16\frac{1}{4} \times 14\frac{1}{3} \times 12\frac{1}{4}} \div \frac{5\frac{2}{7}}{7\frac{2}{3}}\right)$.

3. Simplify $\frac{£14\ 12s.\ 11d.}{10\frac{2}{3} - 3\frac{2}{5}} \times \frac{£10\ 10s.\ 10d}{10s.\ 9\frac{1}{2}d.}$

4. A man bought a quantity of hay at $15 for 20 cwt. He sold it at 85 cents per cwt., gaining $22.25. How many cwt. did he buy?

5. $3\frac{1}{8}$ yards of cloth cost $12.50; what will $23\frac{1}{8}$ yards cost?

6. A person having an annual income of $1400, spends a sum equal to $625 50 more than he saves. Find his daily expenditure (year = 365 days).

7. A lady had in her purse just money enough to buy a certain quantity of silk; but she spent $\frac{5}{11}$ of the money in flannel, $\frac{2}{3}$ of the remainder in calico, and had then only enough money left to buy $10\frac{1}{2}$ yards of silk. How many yards of silk could she have bought at first?

8. A room 15 feet wide and 18 feet long is covered with matting at a cost of $25; what would be the expense of covering, with the same quality of matting, a room a yard longer and a yard wider?

9. The average of four quantities is $18\frac{5}{8}\frac{5}{7}$; the first is 26·207, the second 3·592, and the third is 38·06. Find the fourth.

10. A bankrupt owes to A $1033.84, and to B $612.80; if A receives $357.44½, what will B receive?

Note.—10 marks to each question.

JULY EXAMINATIONS, 1878.

ADMISSION TO HIGH SCHOOLS.

TIME—TWO HOURS.

Examiner—JOHN J. TILLEY.

Values.	
12	1. Define prime number, multiple of a number, highest common factor of two or more numbers. Find the prime factors of 1260.
12	2. The quotient is equal to six times the divisor; the divisor is equal to six times the remainder, and the three together, plus 45, amount to 561. Find the dividend.
12	3. I sell 12½ tons of coal for $80, which is one-seventh more than the cost. Find the gain per cwt
12	4. ·001 × ·001 ÷ ·0001.
12	5. A cistern is two-thirds full; one pipe runs out and two run in. The first pipe can empty it in eight hours, the second can fill it in twelve hours, and the third can fill it in sixteen hours. There is also a leak half as large as the second pipe, in how many hours will the cistern be half full?
12	6. Ten men can do a piece of work in twelve days. After they have worked four days, three boys join them in the work, by which means the whole is done in ten days. What part of the work is done by one boy in one day?
16	7. I buy a number of boxes of oranges for $600, of which twelve boxes are unsaleable. I sell two-thirds of the remainder for $400, and gain on them $40. How many boxes did I buy?
12	8. Find the total cost of the following:—Cutting a pile of wood 90 ft. long, 6 feet high and 4 ft. wide, at 60c. per cord.—Digging a cellar 44 ft. long, 30 ft. wide, and 8 ft. deep, at 18c. per cubic yard.—Plastering a room 24 ft. long, 16 feet wide and 10 ft. high, at 15c. per square yd.—Sawing 6800 shingles at 40c. per 1000.

DECEMBER EXAMINATIONS, 1879.

ADMISSION TO HIGH SCHOOLS.

TIME—TWO HOURS.

Examiner—J. J. TILLEY.

Values.	
14	1. A man has 703 ac. 3 roods 22 sq. rods 14¼ sq. yds.; after selling 19 ac. 1 rood 30 sq. rods 2½ sq. yds., among how many persons can he divide the remainder so that each person may receive 45 ac. 2 roods 20 sq. rods 25 sq. yards?
14	2. Find the price of digging a cellar 41 ft. 3 in. long, 24 feet wide and 6 feet deep at 20 cents per cubic yard.
14	3. The fore wheel of a waggon is 10½ ft. in circumference, and turns 440 times more than the hind wheel, which is 11⅔ ft. in circumference; find the distance travelled over in feet.
14	4. $\dfrac{3\frac{1}{8} - 1\frac{1}{8} \text{ of } \frac{9}{10} + 8}{1\frac{1}{2}(8\frac{5}{12} + 3\frac{3}{8} - \frac{7}{12} + 3\frac{3}{5})} \div \dfrac{\cdot 05 - \cdot 005}{\cdot 25 \div \cdot 5}$.
14	5. Find the total cost of the following:— 2745 lbs. of wheat at $1.20 per bush. 867 " " oats " 35c. " 1936 " " barley " 60c. " 165 " " hay " $8 per ton. 2675 feet of lumber at $10 per 1000 feet.
14	6. If, when wheat sells at 90 cents per bush., a 4 lb. loaf of bread sells at 10 cents, what should be the price of a 3 lb. loaf when wheat has advanced 45 cents in price?
16	7. At what price must I mark cloth which cost me $2 40 per yard, so that after throwing off ⅛ of the marked price I may sell it at ⅛ more than the cost price?

UNIVERSITY OF HALIFAX.

SELECTED FROM MATRICULATION EXAMINATIONS,

1877 AND 1878.

1. A person fails to the amount of £9000 and his effects are worth only £3515 12s. 6d.; what will be the dividend in the £? and what the loss upon a debt of £750?

2. What fraction of a ton is equal to the difference between 1 oz. and $\frac{1}{8}$ of 1 lb. avoirdupois?

3. Reduce the circulating decimals $4\cdot51\dot{3}6\dot{3}$, ·07 and $\cdot\dot{4}6\dot{2}$ to vulgar fractions in their lowest terms.

4. Find the simple interest on $600.80 for $4\frac{1}{2}$ years at $4\frac{1}{2}$ per cent. Find the compound interest on $200 for 2 years at 5 per cent., interest compounded annually.

5. Show how to reduce a mixed circulating decimal to a vulgar fraction. Divide 234·6 by 7 7.

6. Calculate the simple and also the compound interest on £1576 14s. 3d. for 3 years at 8 per cent.

7. Simplify $\frac{3\frac{1}{7} - \frac{1}{8} \text{ of } \frac{3}{5}}{[illegible] + \frac{4}{5} - \frac{1}{6}}$ and reduce it to a decimal.

8. Extract the square root of 2 and also of 77 to six decimal places.

9. If [illegible] men and 1 boy can do a piece of work in [illegible] days, and if 1 man and two boys can do the same work in [illegible] days, how long would [illegible] men and 7 boys require to do the work?

CITY OF TORONTO.

COMBINED EXAMINATION.

1. Divide 540 tons, 10 cwt., 3 qrs., 15 lbs., by 575.

2. How many seconds are there from 9 o'clock a.m., on 13th January, 1876, to 16 minutes past 10 o'clock p.m., on 27th June, 1877?

3. Reduce 3s. 6d. to the decimal of £7 10s.

4. Simplify $\cfrac{2}{3+\cfrac{1}{4+\cfrac{1}{5+\frac{1}{3}}}}$

5. Reduce 1 ton to lbs., oz., &c., Apothecaries' weight.

6. Simplify $\cfrac{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}{\frac{1}{3\frac{1}{3}}+\frac{1}{4\frac{1}{4}}+\frac{1}{5\frac{1}{5}}}$

7. If 6 men will dig a trench 30 yds. long and 8 yds. broad in six days of 16 hours each, in how many days of 12 hours each will 8 men dig a trench 40 yds. long and 16 yds. broad?

8. Divide the difference of 100 and $\frac{1}{100}$ by their sum; and also their sum by their difference, and find the sum of the quotients.

9. What is the interest on $757.60 for four years and four months, at 6½ per cent?

10. Find the difference in the expense of carpeting a room 24 ft. 9 in. long and 15 ft. 6 in. broad, with carpet ¾ of a yard wide, at $1.50 per yard, and with carpet ⅞ of a yard wide, at 70 cents per yard.

11. A man sells a house for $437.50, and loses 12½ per cent. on the original cost. What was the original cost?

12. How many flag stones, each 2·88 ft. wide and 8·30 ft. long, are required to pave a walk round a grass plot 137·31 ft. long and 125·79 ft. wide, the walk being 4.15 yds. wide?

COUNTY OF WATERLOO.

COMPETITIVE EXAMINATION

Promotion from 5th to 6th Form.

1. Divide 480 into two parts, one of which shall be three-fifths of the other.

2. The figures in the units and millions places are 8 and 5 respectively; what will they become when 999,999 is added to the number?

3. Find the price of 9 tons, 19 cwt., 3 qrs., 12 lbs. 8 oz., at $86.24 a ton. Reduce the answer to £ s. d., taking the value of the £ to be $4·867.

4. A car is exactly filled by barrows that hold each 9 cwt., and emptied by sacks that hold each 5 cwt.; given that it holds between 8 and 10 tons, find the exact amount.

5. Simplify $\frac{\cdot 004 \div \cdot 0005}{2\cdot 42\dot{3} + 3\cdot 57\dot{6} + 2\cdot 0001\dot{9}\dot{1}}$

6. A man borrowed $500 on the 21st January, 1874, at 8 per cent. interest, payable half-yearly; every half-year he pays $80. How much will he owe on the 21st July, 1876, whatever he pays each half-year over and above the interest due being deducted from the principal?

7. A, who owes B $1000, sends him 725 M ft. of lumber, which B sells at $16 per M, charging 3 per cent. commission, and paying out of the proceeds expenses amounting to $161.50. How much is coming to A?

8. If B buys for A, with the sum you found due to him from No. 7, flour at $4.50 a barrel, charging 3½ per cent. commission, how many barrels will A receive?

9. If I buy broadcloth at $3.60 a yard, what must I ask for it a yard that I may be able to throw off 10 per cent. from my price and yet make 25 per cent. profit?

10. How much Dominion 6 per cents at 108 can I buy for $16,200, and what yearly income shall I have therefrom?

11. The population of a town has increased 5 per cent. since last year. It has now 3780 inhabitants; how many had it then?

12. I borrowed $250 on the 1st October, 1875, and paid back $260 on the 24th February, 1876; what rate of interest per annum had I been charged?

COUNTY OF DURHAM.

COMPETITIVE EXAMINATION.

Senior.

Values.	
15	1. A can do $\frac{1}{4}$ of a piece of work in one hour, B can do $\frac{2}{3}$ of the remainder in 1 hour, and C can finish it in 20 minutes. How long will it take A, B and C together to do the work?
15	2. If it costs \$70.40 to carpet a room 22 feet long with carpet $2\frac{1}{4}$ feet wide, at \$1.10 per yard, find the width of the room.
15	3. By selling tea at 96 cts. per lb., a merchant gains $\frac{1}{7}$ of the cost; he then raises the price to \$1.05 per lb.; what does he clear on every \$8.40 of his outlay by the latter price?
18	4. A bankrupt who is paying $37\frac{1}{2}$ cts. in the \$ divides among his creditors \$6300, and secretly retains \$2100. What do his debts amount to, and how much in the \$ would his creditors receive had they obtained all his assets?
18	5. Find the price of 423 lbs. of peas and oats mixed equally by measure, when peas are 70 cts. and oats 40 cts. per bush.
18	6. A merchant sells 60 lbs. of tea and coffee for \$43.50—the tea at 90 cts. and the coffee at 40 cts. per lb. How many lbs. of each did he sell?
18	7. John McCromb rents a house, the cash value of which is \$2400, and which is kept in repair by the owner at an annual outlay of \$20 paid at the end of the year. He pays in advance an annual rent of \$200. How much a year would he save by buying the house and paying for it in cash, money being worth 7 per cent. per annum?
18	8. Simplify (*a*) $\frac{2{\cdot}2\dot{3} + 3{\cdot}7\dot{6}}{5{\cdot}5 \div {\cdot}69\dot{4}} + \frac{3\,2\dot{1} - 1{\cdot}2\dot{1}}{1{\cdot}3\dot{4} + 2{\cdot}45\dot{6} - {\cdot}8 \times \frac{1}{2}} - \frac{{\cdot}\dot{6}\dot{7} \div 1{\cdot}3\dot{5}}{{\cdot}51 \times 3\frac{1}{3}\frac{1}{3}}$ (*b*) $(\frac{1}{3} - \frac{1}{4}$ of $\frac{1}{3})$ of £4 11s. 4d. is what fraction of $(\frac{1}{2}$ of $\frac{3}{4} - \frac{1}{2})$ of \$164.40, £1 being worth \$4$\frac{1}{2}$.

120 marks a full paper.

ANSWERS.

Exercise I.—Page 2.

1. 5; 5; 9. 2. 1; 1 book; 1 ball.
3. 6, 7, 3, 4, 2 are abstract; 8 books, 9 men, 5 apples, 1 cent are concrete. 4. 1 mile; 1; 1 cent.
5. 3 apples, 7 apples, and 6 apples; 4 boys and 9 boys; 7, 9 and 8; 2 cents and 5 cents; 4 girls and 5 girls.

Exercise II.—Page 3.

1. 7; 9; 4; 2. 2. 36; 84; 20; 60.
3. 44; 70; 96; 16. 4. 11; 12; 39; 56. 5. 48; 97; 36; 60.
6. Seven; eleven; fifteen; nineteen; fifty-nine; eighty-four; ninety-six; ninety-eight.
7. Seventy-one; twelve; twenty-eight; ninety-one; forty-four; seventeen; twenty-two; thirty-four.
8. Twenty; thirty seven; forty-eight; seventy-six; ninety-nine; sixty-nine; seventy; eighty-seven.
9. Fourteen; thirty-five; eighty-nine; seventy-eight; fifty-four; forty-nine; fifty; thirteen.
10. Ninety; eighty; thirty-nine; twenty-eight; eleven; nineteen; twenty-seven; thirty-one.

Exercise III.—Page 4.

1. 149; 303; 974. 2. 209; 420; 694.
3. 560; 9 8, 444. 4. 735; 960; 406.
5. 309; 687; 572.
6. Two hundred and seven; three hundred and seventy-one; one hundred and eighty-five; one hundred and ninety; three hundred and sixty-eight.
7. Five hundred and seventy, four hundred and seventy-two; eight hundred and seven; nine hundred and nine; nine hundred and ninety.
8. Three hundred and sixty-eight; five hundred and eighty-four; seven hundred and sixty; three hundred and twenty-one; nine hundred and ninety nine.
9. Three hundred and ninety-four; seven hundred and eighty-six; four hundred and seventy-five; seven hundred and eighty-two; seven hundred.
10. Five hundred and six; three hundred; four hundred and seven; seven hundred and forty; three hundred and ninety-seven.

Exercise IV.—Page 5.

1. 6006; 4300; 9080.
2. 3700; 7906; 3084.
3. 64009; 807068; 700316.
4. 4030097; 809007039; 586000007.
5. 8000000000; 6400700024; 4004000004.
6. 408003009; 74000074004; 500000000500.
7. 80070000000; 800000008; 300000300090.
8. 57700000080; 11000011; 19000014000.
9. 70000000070; 400000001; 600600000000600.
10. 99000000000608; 700070007000; 16016000000016.
11. Seven thousand and seventy-seven; eighty-five thousand and seventy-nine; fifty-six thousand nine hundred and fifty; four hundred and seventy-three thousand, six hundred and twenty-eight.
12. Fifty-six thousand four hundred and eighteen; seven hundred and eighty-four thousand and six; four hundred thousand five hundred and seven; three hundred and sixty thousand and four.
13. Three hundred thousand and seventy-one; nine hundred and one thousand and seven; seven hundred and twenty thousand and nine; one hundred and eighty-two thousand and ten.
14. Three millions one hundred and forty thousand and six; fifty millions and six hundred; three billions six hundred millions ten thousand and seventy.
15. Fifty-one billions six hundred and thirty-six millions two hundred and seven thousand six hundred and forty; seventy billions and one hundred; nine hundred and twenty billions seventy millions seventy thousand and seventy.

Exercise V.—Page 6.

1. XIX; XXIV; XLIX; LXXXIV; XCIX.
2. CLXXXVII; CCVIII DCCLXXXI; CMLXII; CMXCIX.
3. MCCCI; MCCCXC; MDCLXXXIV; MDCCCXV; MDCCCLXXVIII.
4. 44; 69; 94; 71.
5. 99; 129; 177.
6. 555; 1604; 1819; 1090.

Exercise VI.—Page 10.

1. 46 horses.	2. 98 boys.	3. 39 girls.	4. 978.
5. 956.	6. 898.	7. 889.	8. 879.
9. 979.	10. 697.	11. 798.	12. 998.
13. 898.	14. 879.	15. 889.	16. 8589.
17. 9879.	18. 8989.	19. 9989.	20. 98978.
21. 87968.	22. 88998.	23. 79988.	24. 797898.
25. 768989.	26. 789689.	27. 988989.	

Exercise VII.—Page 11.

1. 79 cents. 2. 88 trees. 3. 968 acres.
4. 796 dollars. 5. 989 miles. 6. 969 yards.
7. 878 bales. 8. 8989 dollars. 9. 97989 dollars.
10. 899898 persons.

Exercise VIII.—Page 13.

1. 113 dollars	2. 78 cents.	3. 152 boys.	4. 145 girls.
5. 146.	6. 247.	7. 162.	8. 161.
9. 217.	10. 213.	11. 1861.	12. 1357.
13. 1015.	14. 1954.	15. 1931.	16. 1759.
17. 2704.	18. 1656.	19. 1951.	20. 1976.
21. 1842.	22. 2141.	23. 23878.	24. 18294.
25. 16954.	26. 14978.	27. 15113.	28. 16046.
29. 24459.	30. 31405.	31. 29377.	32. 21232.
33. 22825.	34. 29165.	35. 256648.	36. 226871.
37. 166581.	38. 338306.	39. 2033781.	40. 19989286.
41. 24692.	42. 25879.	43. 27265.	44. 24447.
45. 23378.	46. 238390.	47. 246818.	48. 81148.
49. 75953.	50. 103618.	51. 41121.	

Exercise IX.—Page 14.

1. 222 dollars. 2. 1661 acres. 3. 120 days.
4. 1061 miles. 5. 936 pounds. 6. 7428 bushels.
7. 3411 acres. 8. 633 dollars. 9. 2104 pages.
10. 2237 dollars. 11. 1173 dollars.
12. B, 601 dollars; C, 1066 dollars; 2132 dollars.

Exercise X.—Page 18.

1. 313.	2. 241.	3. 251.	4. 402.
5. 644.	6. 464.	7. 143.	8. 305.
9. 344.	10. 304.	11. 733.	12. 530.
13. 4442.	14. 5022.	15. 2223.	16. 2001.
17. 2530.	18. 4422.	19. 5512.	20. 2734.
21. 5024.	22. 6257.	23 1361.	24. 4623.
25. 423.	26 60224.	27 86275.	28. 31216.
29. 5082.	30. 43262.	31. 86425.	32. 35137.
33. 66243.	34. 75331.	34. 61161.	36. 40063.
37. 260265.	38. 64153.	39. 35432.	40. 77443.
41. 161116.	42. 741551.	43. 21353.	44. 44516.
45. 57234.	46. 364.	47. 233.	48. 228.
49. 322.	50. 432.	51. 2533.	52. 1243.
53. 6216.	54. 83136.	55. 56454.	

Exercise XI.—Page 19.

1. 43 girls. 2. 44 cents. 3. 16 dollars.
4. 34 runs. 5. 44 questions. 6 43 dollars
7. 88 dollars. 8. 2112 dollars. 9. 14442 dolls.

Exercise XII.—Page 21.

1. 325.	2. 373.	3. 262.	4. 293.
5. 255.	6. 144.	7. 184.	8. 256.
9. 263.	10. 286.	11. 362.	12. 309.
13. 168.	14. 365.	15. 266.	16. 169.
17. 69.	18. 66.	19. 458.	20. 178.
21. 298.	22. 175.	23. 197.	24. 118.
25. 339.	26. 468.	27. 177.	28. 497.
29. 479.	30. 293.	31. 1497.	32. 2858.
33. 4188.	34. 948.	35. 1933.	36. 2919.
37. 4944.	38. 2857.	39. 5339.	40. 1299.
41. 1359.	42. 5247.	43. 2279.	44. 5263.
45. 3784.	46. 5682.	47. 4469.	48. 1789.
49. 11844.	50. 19528.	51. 52888.	52. 35499.
53. 49289.	54. 25012.		

Exercise XIII.—Page 22.

1. 8 dollars. 2. 77 yards. 3. 560 dollars. 4. 1803.
5. 3251. 6. 344 cents. 7. 37 quarts. 8. 175 dollars.
9. 506 dollars. 10. 375 acres; 12021 dollars.

Exercise XIV.—Page 22.

1. 177.	2. 739.	3. 1811.	4. 691.
5. 2262.	6. 520.	7. 2152.	8. 22326.
9. 77865.	10. 3598.		

Exercise XV.—Page 23.

1. 357 dollars.	2. Lost 632 dollars.	3. 853288.
4. 10534.	5. 171 dollars.	6. 956.
7. 811 feet.	8. John 28, James 32.	9. 41265.
10. 6628.	11. 5211 and 3553.	

Exercise XVI.—Page 27.

1. 14864.	2. 16864.	3. 216936.	4. 368492.
5. 195 boys.	6. 282 cents.	7. 959 cows.	8. 1488 apples
9. 2106 girls.	10. 1890.	11. 3360.	12. 3070.
13. 23526.	14. 47901.	15. 43710.	16. 78112.
17. 53838.	18. 70340.	19. 72028.	20. 661672.
21. 153132.	22. 630855.	23. 352794.	24. 646857.
25. 53936.	26. 54360.	27. 432481.	28. 364704.
29. 428215.	30. 1023024.	31. 3417355.	32. 6044346.
33. 6283784.	34. 2217177.	35. 7865490.	36. 9162527.
37. 10860916.	38. 9091656.	39. 10743888.	

Exercise XVII.—Page 27.

1. 16280 cents. 2. 185430 cents. 3. 2709 dollars.
4. 15215 dollars. 5. 336 sheep; 392 dollars.
6. 94 pounds; 2610 cents; 752 cents; 658 cents.

7. 3402 dollars; 224 dollars; 3626 dollars 2237 dollars.
8. 1320 paragraphs; 11880 lines; 95040 words; 47520 letters.
9 296 cents; 414 cents; 710 cents; 118 cents.
10. 1096 cents; 2214 cents; 3310 cents; 1118 cents.

Exercise XVIII.—Page 29.

1. 11950.	2 40992.	3. 118377.
4. 66738.	5. 482514.	6. 340488.
7. 189945.	8. 240896.	9. 134010.
10. 62550.	11. 183576.	12. 1348560.
13 98560 yards.	14. 68520 feet.	15. 8496 cents.
16. 459 days.	17. 3000 dollars.	18. 8505 dollars.
19. 19845 dollars.	20. 40320 min.	

Exercise XIX.—Page 30.

1. 472140.	2. 300720.	3. 226196.
4. 562650.	5. 724885.	6. 6608822
7. 6586169.	8. 6509916.	9. 1194372.
10. 4127874.	11 9781440.	12. 11961586.
13. 14821755.	14. 25581580.	15. 23120856.
16. 81362385.	17. 29455710.	18. 31259069.
19. 70132632.	20. 14069499.	21. 41316048.
22 26514000.	23. 42741832.	24 16765686.
25. 66693951.	26. 217702278.	27. 163588743.
28. 307551216.	29. 276010344.	30. 114297351.
31. 348112165.	32. 283046032.	33. 671374392.
34. 2139927997.	35. 1627916724.	36. 1244124564.
37. 341614192.	38. 3481804952.	39. 4141417504.
40. 90355098.		

Exercise XX.—Page 31.

1. 127405.	2. 6317698.	3. 6825456.	4. 1960452.
5. 6486216.	6. 6749472.	7. 8214296.	8. 25596104.
9. 80071992.	10. 738110274.		

Exercise XXI.—Page 31.

1. 445800.	2. 592900.	3. 60744600.	4. 11887500.
5. 2430 00.	6. 258030.	7. 11214000.	8. 4096000.
9. 422500.	10. 627000000.	11. 64610000.	12. 488300000.

Exercise XXII.—Page 31.

1. 454560 sheets.	2. 195559 yards.	3. 6125 dollars.
4. 1653 yards.	5. 3015648 pounds	6. 1228275 dollars
7. 263952 apples.	8. 915750 pages.	9. 61275 yards.
10. 193662 yards.	11. 44100 dollars.	12. 12180 dollars.
13. 1216420 dollars.	14. 3026000 dollars.	15. 292800 barrels.
16 7080520 dollars.	17. 277107850 dollars	18. 372480 hats.
19. 89784 yards.	20. 7344 miles.	

Exercise XXIII.—Page 32.

1. 608 dollars. 2. 162764 men. 3. 530229 gal.
4. 2773 dollars. 5. 12652 cents. 6. 7455 dollars; 9940 dollars. 7. The horses; 2532 dollars.
8. A's 706800; B's 1126125 letters. 9. 18750 cents.
10. 116550 cents. 11. 457047. 12. 944 days.
13. 1441 dollars. 14. House, 2800 dollars; Farm, 2975 dollars. 15. Loss 254 dollars.
16. Gain 3100 dollars 17. 1752 dollars. 18. 9000 dollars.
19. 8110 dollars. 20. 10690 dollars.

Exercise XXIV.—Page 39.

1. 18.	2. 29.	3. 27.	4. 46.	5. 48.	6. 192.
7. 155.	8. 241.	9. 291.	10. 325.	11. 213.	12. 191.
13. 233.	14. 144.	15. 18[illegible].	16. 147.	17. 170.	18. 195.
19. 149.	20. 167.	21. 122.	22. 141.	23. 154.	24. 162.
25. 139.	26. 112.	27. 114.	28. 119.	29. 138.	30. 137.
31. 112.	32. 117.	33. 118.	34. 122.	35. 124.	36. 52.
37. 64.	38. 96.	39. 82.	40. 74.		

Exercise XXV.—Page 39.

1. 228.	2. 368.	3. 274.	4. 187.	5. 269.
6. 245.	7. 272.	8. 174.	9. 138.	10. 246.
11. 223.	12. 171.	13. 182.	14. 255.	15. 275.
16. 128.	17. 156.	18. 183.	19. 144.	20. 206.
21. 184.	22. 204.	23. 243.	24. 152.	25. 109.
26. 123.	27. 147.	28. 129.	29. 157.	30. 168.
31. 163.	32. 187.	33. 156.	34. 153.	35. 176.
36. 139.	37. 108.	38. 109.	39. 129.	40. 144.
41. 246.	42. 556.	43. 419.	44. 609.	45. 1223.
46. 367.	47. 676.	48. 1208.	49. 1337.	50. 1410.
51. 907.	52. 457.	53. 947.	54. 3669.	55. 13879.
56. 988.	57. 442.	58. 285.	59. 7032.	60. 7484.

Exercise XXVI.—Page 40.

1. 59 oranges. 2. 173 days' work. 3. 918 pounds.
4. 231 yards. 5. 96 rods. 6. 91 cents.
7. 15 bushels. 8. 123. 9. 6052 bushels.
10. 8 cords.

Exercise XXVII.—Page 41.

1. $2177\frac{1}{3}$.	2. $1248\frac{4}{9}$.	3. 9606.
4. $1640\frac{5}{7}$.	5. $12317\frac{1}{8}$.	6. $69048\frac{1}{5}$.
7. $420061\frac{1}{4}$.	8. $572004\frac{2}{3}$.	9. $777070\frac{3}{8}$.
10. $729584\frac{9}{10}$.	11. 1398260.	12. 7400061.
13. 823956	14. $6273804\frac{1}{3}$.	15. $4238753\frac{6}{11}$.
16. 420560.	17. 20073842.	18. 37037048.
19. $6470853\frac{11}{12}$.	20. 7298426.	21. 7480093.
22. $2730956\frac{9}{11}$.	23. $8844326\frac{11}{12}$.	24. 92506025.
25. $4560387\frac{4}{10}$.	26. 87376008.	

Exercise XXVIII.—Page 41.

1. 432 barrels. 2. 1256 pounds. 3. 6226 dollars.
4. $11\frac{1}{4}$ dollars. 5. 8 dollars. 6. 2423 minutes.
8. $130\frac{3}{4}$ pounds. 9. $125\frac{1}{4}$ dollars. 10. $52\frac{1}{4}$ weeks.
11. 252. 12. $689\frac{3}{4}$ acres. 13. $15745\frac{7}{9}$ pounds.
14. 548 bricks.

Exercise XXIX.—Page 44.

1. $24\frac{30}{81}$. 2. $8\frac{39}{41}$. 3. $18\frac{38}{53}$.
4. $74\frac{53}{61}$. 5. $39\frac{50}{74}$. 6. $588\frac{51}{65}$.
7. $531\frac{45}{56}$. 8. $945\frac{27}{76}$. 9. $554\frac{86}{122}$.
10. $49\frac{49}{801}$. 11. $231\frac{231}{346}$. 12. $498\frac{158}{936}$.
13. $375\frac{375}{425}$. 14. $2259\frac{1}{239}$. 15. $5050\frac{99}{198}$.
16. 2831. 17. $6205\frac{162}{1419}$. 18. $6200\frac{160}{1860}$.
19. 3746. 20. $2025\frac{372}{1736}$. 21. $4998\frac{432}{1976}$.
22. $9710\frac{140}{2492}$. 23. 4321. 24. 4671.
25. 3180. 26. 3615. 27. 1142.
28. $7277\frac{9467}{78817}$. 29. 2567. 30. 50444.

Exercise XXX.—Page 44.

1. 43 days. 2. 38 days. 3. 1090 feet.
4. 32 dollars. 5. 129 years. 6. 123 dollars.
7. 46 dollars. 8. 545 bales. 9. $313\frac{5}{17}$ miles.
10. 2075 barrels.

Exercise XXXI.—Page 45.

1. $1734\frac{9}{27}$. 2. $1366\frac{44}{54}$. 3. $1549\frac{61}{63}$. 4. $307\frac{38}{45}$.
5. $246925\frac{21}{35}$. 6. $110147\frac{18}{49}$. 7. $549\frac{17}{25}$. 8. $800\frac{43}{96}$.
9. $1032\frac{6}{81}$. 10. $4236\frac{57}{99}$. 11. 2570. 12. $5599\frac{89}{143}$.

Exercise XXXII.—Page 46.

1. $24\frac{5}{30}$. 2. $127\frac{23}{63}$. 3. $12\frac{242}{753}$. 4. $326\frac{61}{306}$.
5. $804\frac{9}{90}$. 6. $118\frac{33}{83}$. 7. $2\frac{346}{1706}$. 8. $5\frac{3509}{3800}$.
9. $30\frac{9751}{123100}$. 10. $153\frac{5611}{6400}$. 11. $673\frac{1773}{2500}$. 12. $632\frac{6479}{15000}$

Exercise XXXIII.—Page 46.

1. 108 yards. 2. 65 hours. 3. 123 pounds.
4. 30 pounds. 5. 42 bushels. 6. 235 dollars.
7. 1378 quarters. 8. 237 bushels. 9. 43 bushels.
10. 38 miles.

Exercise XXXIV.—Page 47.

1. 8314. 2. 129. 3. 233289. 4. 348.
5. 186. 6. 272. 7. 10005100. 8. 19052.
9. 194 and 86. 10. 784623.

Exercise XXXV.—Page 48.

1. 367 acres. 2. 2310 dollars. 3. 845 dollars.
4. 15 weeks. 5. 44 dollars. 6. 56 cents.
7. 1650 barrels. 8. 24 months. 9. 551 dollars.

10. 1210 dollars.
11. 41600 c. ft.
12. 217 sheep.
13. 520 dollars.
14. 30 hours.
15. 240 cents.
16. 54 cents.
17. 56 dollars.
18. 96 dollars.
19. 100 dollars.
20. 40 cents.
21. 12 dollars.
22. 6880 dollars.
23. 23725 days.
24. 7 56 pounds.
25. 5640 yards.
26. 365 acres.
27. 31250 dollars.
28. 954 dollars.
29. 1971 bushels.
30. 15 days.
31. 12 days.
32. 32 days.
33. 361 days.
34. 50 days.
35. 90 days.
36. 48 days.
37. 6 days.
38. 10 days.
39. 119 days.
40. 28 men.
41. 84 men.
42. 21 men.
43. 108 men.
44. 25 men.
45. 114 men.
46. 56 men.
47. 72 men.
48. 150 men.
49. 15 beggars.

Examination Papers.—Page 51.

I.

2. 488979. 3. 944813. 4. 7706307429. 5. 1116 dollars.

II.

2. 29900000. Twenty nine millions, nine hundred thousand.
3. 846055. 4. 2699; 329; $\overline{\text{D}}\text{CCCLXXXVII}\overline{\text{C}}\text{MLXXI}$.
5. 16 bushels.

III.

3. 54365636568. 5. 580 acres; 61 dollars.

IV.

4. 228 dollars. 5. 608 sheep.

V.

3. 86 4. 86 5. 53 dollars.

VI.

2. 3571 dollars. 3. 112. 4. 180 acres; 36 dollars.
5. 24 days.

VII.

2. 40831 dollars. 3. 663 miles. 4. 247. 5. 6525 dollars.

VIII.

1. 4700 dollars. 2. 973. 3. 31 dollars.
4. 201 cents. 5. 964 miles; 1181 miles.

Exercise XXXVIII.—Page 58.

1. $1163.55 2. $1834.07. 3. $3220.65. 4. $1624.90.
5. $99.05 6. $82.28. 7. $5232.74. 8. $22.

Exercise XXXIX.—Page 59.

1. $94.58. 2. $58.75. 3. $43.19. 4. $592.61.
5. $5170.64. 6. $23.79. 7. $261.07. 8. $7915.80.
9. $4.08. 10. $50.37. 11. $1790.63. 12. $48.45.

Exercise XL.—Page 60.

1. $391.85. 2. $1482.96. 3. $926.25. 4. $376.70.
5. $6562.50. 6. $3522.75. 7. $157.50. 8. $27.75.
9. $3364.20. 10. $16.80. 11. $247. 12. $169.
13. $17 4. 14. $722 16. 15. $360. 16. $3.51.
17. $261.25. 18. $425.25.

Exercise XLI.—Page 62.

1. $12.72. 2 $21.37 3. $18 17. 4. $26 34.
5. $7.89. 6. $60.50½. 7. $10.40. 8. 73 sheep.
9. $2.22. 10. 6. 11. 365 days. 12. 16 pieces.

Exercise XLII.—Page 64.

1. $32.20. 2. $1196. 3. $4 35. 4. $27.76. 5. $47 02.
6. $189.77. 7. $14.24. 8. $771.51. 9. $18 78. 10. $3.31.

Examination Papers.—Page 65.

I.

2. 70 cents 3. $5.15. 4. 100. 5. 15 times more.

II.

2. $414.80. 3. $231.52. 4. 50 tons. 5. $1.10.

III.

2. 45 yards. 3. 85 votes. 4. $1191.75. 5. 1760.

IV.

1. 476 yards; 30 cents. 2. 400 bushels. 3. 1560 pair.
4. 100 days. 5. 1100; 430.

Exercise XLIII.—Page 68.

1. 2, 2, 2, 2, 3. 2. 2, 2, 2, 3, 3. 3. 3, 3, 3, 3.
4. 2, 2, 3, 3, 3. 5. 5, 5, 7. 6 2, 3, 3, 3, 5.
7. 2, 2, 2, 2, 2, 5. 8. 5, 5, 13. 9. 3, 11, 13.
10. 2, 2, 3, 23. 11. 2, 2, 2, 2, 2, 5.5.
12. 2, 2, 3, 3, 5. 13. No prime factors. 14. 2, 2, 2, 2, 3, 7,
15. 3, 3, 5, 19. 16. 3, 5, 7, 11. 17. 2 and 5.
18. 3. 19. 7 and 3. 20. 2, 2, 2, 3 and 5.

Exercise XLIV.—Page 68.

1. 2. 2. 2. 3. 12.
4. 18. 5. 30. 6. 72.
7. 20 yards. 8. 72 bushels. 9. $22.
10. 75 yards. 11. $2. 12. 1440.

Exercise XLV.—Page 70.

1. 5. 2. 4. 3. 8. 4. 14. 5 10.
6. 42. 7. 24. 8. 11. 9. 75. 10. 144.
11. 8 feet. 12. 21 feet. 13. 16 feet. 14. 8 quarts. 15. 45 pears.
16. 3, 11, or 33 pupils in each section.

Exercise XLVI.—Page 71.

1. 23. 2. 37. 3. 41. 4. 56.
5. 45. 6. 61. 7. 42. 8. 11.
9. 813. 10. 630. 11. Prime. 12. 21.
13. 184 lbs. 14. 7 and 12.

Exercise XLVII.—Page 73.

1. 30. 2. 60. 3. 36. 4. 150.
5. 360. 6. 180. 7. 360. 8. 770.
9. 2520. 10. 1512. 11. 1680. 12. 16800.
13. 1800. 14. 720720. 15. 50702925. 16. 173.
17. $2100. 18. 360 bushels. 19. 240 cents.
20. 84 bushels. 21. 120 days.

Examination Papers.—Page 74.

I.

1. 611, 707, and 1083 are comp.; 643, 757, and 991 are prime. 2. 8. 3. $3048. 4. 643. 5. 25 acres.

II.

2. 25. 3. 46. 4. 1680 marbles. 5. 47400 holes.

III.

1. 15, 16, 17, and 18. 2. 900 acres. 3. 9 cents.
4. 356. 5. 9672 rails.

IV.

1. 77 cents. 2. $1080. 3. 3600. 4. 10565999.
5. 1267994828100.

V.

1. 10296. 2. 240. 3 257. 4. 5 and 4.

Exercise XLVIII.—Page 80.

1. $\frac{35}{7}$. 2. $\frac{21}{5}$. 3. $\frac{65}{7}$. 4. $\frac{41}{9}$.
5. $\frac{133}{12}$. 6. $\frac{99}{8}$. 7. $\frac{431}{12}$. 8. $\frac{1402}{17}$.
9. $\frac{1100}{23}$. 10. $\frac{1032}{19}$. 11. $\frac{1090}{11}$. 12. $\frac{7741}{60}$.
13. $\frac{30037}{152}$. 14. $\frac{71810}{236}$. 15. $\frac{232477}{178}$. 16. $\frac{2432984}{673}$.
17. $\frac{101112}{101}$. 18. $\frac{2834810}{999}$.

Exercise XLIX.—Page 80.

1. $3\frac{4}{5}$. 2. $5\frac{1}{3}$. 3. $6\frac{2}{7}$. 4. $7\frac{3}{10}$.
5. 33. 6. 17. 7. $12\frac{4}{13}$. 8. $65\frac{7}{15}$.
9. $16\frac{5}{48}$. 10. 13. 11. 28. 12. $51\frac{18}{19}$.
13. $32\frac{55}{128}$. 14. $100\frac{19}{58}$. 15. $515\frac{13}{23}$. 16. $676\frac{19}{48}$.
17. $36\frac{666}{888}$. 18. $522\frac{90}{125}$.

Exercise L.—Page 81.

1. $\frac{3}{4}$. 2. $\frac{3}{5}$. 3. [illegible]. 4. $\frac{2}{3}$. 5. $\frac{5}{6}$. 6. $\frac{6}{7}$.
7. $\frac{5}{7}$. 8. $\frac{9}{10}$. 9. $\frac{1}{3}$. 10. $\frac{11}{12}$. 11. $\frac{16}{77}$. 12. $\frac{7}{9}$.
13. $\frac{61}{71}$. 14. $\frac{3}{4}$. 15. $\frac{162}{323}$. 16. $\frac{316}{417}$. 17. $\frac{19}{87}$. 18. $\frac{4}{7}$.
19. $\frac{7}{11}$. 20. $\frac{4}{15}$. 21. $\frac{11}{19}$. 22. $\frac{2}{3}$. 23. $\frac{19}{23}$. 24. $\frac{2419}{2597}$.

Exercise LI.—Page 83.

1. $\frac{29}{77}$. 2. $\frac{3}{11}$. 3. $3\frac{3}{10}$. 4. $1\frac{19}{36}$. 5. $\frac{8}{21}$. 6. $\frac{5}{66}$.
7. $\frac{10}{21}$. 8. $\frac{5}{9}$. 9. $\frac{9}{30}$. 10. $\frac{10}{33}$. 11. $\frac{1}{9}$. 12. $\frac{9}{82}$.
13. $\frac{21}{55}$. 14. 3. 15. $2\frac{19}{28}$. 16. $\frac{17}{72}$. 17. $1\frac{1}{4}$. 18. $1\frac{2}{5}$.

Exercise LII.—Page 83.

1. $\frac{1}{2}$. 2. $\frac{5}{8}$. 3. $\frac{9}{25}$. 4. $\frac{2}{15}$.
6. $\frac{1}{3}$. 7. $12\frac{1}{2}$ acres. 8. $8750.

Exercise LIII.—Page 84.

1. $\frac{27}{45}$, $\frac{35}{45}$. 2. $\frac{60}{70}$, $\frac{63}{70}$. 3. $\frac{88}{99}$, $\frac{63}{99}$.
4. $\frac{70}{105}$, $\frac{63}{105}$, $\frac{75}{105}$. 5. $\frac{75}{105}$, $\frac{70}{105}$, $\frac{84}{105}$. 6. $\frac{36}{60}$, $\frac{40}{60}$, $\frac{45}{60}$.
7. $\frac{252}{420}$, $\frac{140}{420}$, $\frac{120}{420}$, $\frac{105}{420}$. 8. $\frac{4680}{6552}$, $\frac{2457}{6552}$, $\frac{2912}{6552}$, $\frac{5544}{6552}$.
9. $\frac{2376}{3960}$, $\frac{2475}{3960}$, $\frac{1760}{3960}$, $\frac{2880}{3960}$.

Exercise LIV.—Page 85.

1. $\frac{20}{30}$, $\frac{25}{30}$, $\frac{18}{30}$. 2. $\frac{4}{8}$, $\frac{6}{8}$, $\frac{7}{8}$. 3. $\frac{35}{40}$, $\frac{36}{40}$, $\frac{14}{40}$.
4. $\frac{20}{33}$, $\frac{21}{33}$, $\frac{22}{33}$. 5. $\frac{30}{36}$, $\frac{16}{36}$, $\frac{21}{36}$. 6. $\frac{45}{60}$, $\frac{42}{60}$, $\frac{50}{60}$.
7. $\frac{15}{80}$, $\frac{35}{80}$, $\frac{38}{80}$. 8. $\frac{25}{30}$, $\frac{24}{30}$, $\frac{20}{30}$. 9. $\frac{35}{40}$, $\frac{30}{40}$, $\frac{32}{40}$.
10. $\frac{20}{10}$, $\frac{3}{10}$, $\frac{8}{10}$. 11. $\frac{15}{6}$, $\frac{20}{6}$, $\frac{5}{6}$. 12. $\frac{7}{9}$, $\frac{39}{9}$, $\frac{45}{9}$.
13. $\frac{48}{16}$, $\frac{72}{16}$, $\frac{1}{16}$. 14. $\frac{147}{168}$, $\frac{60}{168}$, $\frac{70}{168}$, $\frac{52}{168}$.
15. $\frac{44}{77}$, $\frac{35}{77}$, $\frac{9}{77}$, $\frac{402}{77}$. 16. $\frac{28}{105}$, $\frac{455}{105}$, $\frac{60}{105}$, $\frac{210}{105}$.
17. $\frac{336}{576}$, $\frac{408}{576}$, $\frac{178}{576}$, $\frac{495}{576}$. 18. $\frac{405}{180}$, $\frac{1200}{180}$, $\frac{580}{180}$, $\frac{828}{180}$.

Exercise LV.—Page 85.

1. $\frac{4}{7}$. 2. $\frac{25}{27}$. 3. $\frac{10}{21}$. 4. $\frac{11}{18}$.
5. $\frac{17}{20}$. 6. $\frac{17}{21}$. 7. $\frac{5}{34}$. 8. $\frac{7}{40}$,
9. $\frac{13}{15}$. 10. $\frac{10}{20}$; $\frac{16}{35}$. 11. $\frac{16}{33}$; $\frac{9}{22}$. 12. $\frac{2}{9}$; $\frac{3}{16}$.
13. $\frac{9}{10}$; $\frac{21}{25}$. 14. $\frac{13}{15}$; $\frac{17}{21}$. 15. $\frac{8}{9}$; $\frac{5}{8}$.
16. $\frac{4}{84}$, $\frac{8}{15}$, $\frac{5}{27}$, $\frac{17}{81}$, $\frac{2}{9}$. 17. $\frac{1}{2}$, $\frac{13}{20}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$

Exercise LVI.—Page 87.

1. $1\frac{5}{12}$. 2. $1\frac{27}{40}$. 3. $\frac{29}{36}$. 4. $1\frac{7}{12}$.
5. $1\frac{29}{40}$. 6. $\frac{187}{200}$. 7. $2\frac{2}{7}$. 8. $2\frac{7}{18}$.
9. $2\frac{9}{25}$. 10. $1\frac{91}{160}$. 11. $3\frac{31}{420}$. 12. $\frac{27}{31}$.

Exercise LVII.—Page 87.

1. $10\frac{11}{30}$. 2. $10\frac{7}{18}$. 3. $10\frac{29}{42}$. 4. $5\frac{29}{36}$.
5. $4\frac{109}{288}$. 6. $46\frac{9}{15}$. 7. $22\frac{89}{180}$. 8. $29\frac{7}{18}$.
9. $12\frac{4}{5}$. 10. $21\frac{17}{80}$. 11. $12\frac{4}{5}$. 12. $16\frac{2}{3}$.

Exercise LVIII.—Page 88.

1. $\frac{4}{35}$. 2. $\frac{17}{45}$. 3. $\frac{13}{36}$. 4. $\frac{1}{20}$. 5. $\frac{1}{18}$. 6. $\frac{9}{105}$.
7. $\frac{7}{80}$. 8. $\frac{409}{700}$. 9. $\frac{77}{171}$. 10. $\frac{7}{18}$. 11. $\frac{1}{13}$. 12. $\frac{1}{42}$.

Exercise LIX.—Page 89.

1. $1\frac{22}{35}$. 2. $1\frac{1}{90}$. 3. $3\frac{7}{12}$, 4. $2\frac{62}{63}$
5. $1\frac{7}{12}$ 6. $1\frac{1}{4}$. 7. $2\frac{11}{42}$. 8. $2\frac{53}{56}$
9. $4\frac{4}{7}$. 10. $12\frac{7}{18}$. 11. $24\frac{11}{18}$. 12. $10\frac{5}{8}$

Exercise LX.—Page 89.

1. $18\frac{13}{28}$. 2. $16\frac{9}{16}$ gallons. 3. $\frac{49}{90}$.
4. $20\frac{11}{15}$ yards. 5. $\$3\frac{1}{8}$. 6. $101\frac{1}{15}$ acres.
7. $14\frac{13}{30}$ reams. 8. $34\frac{13}{21}$ pounds. 9. $33\frac{1}{30}$ miles.
10. $\$38\frac{1}{15}$. 11. $\$98\frac{29}{35}$. 12. $\frac{43}{63}$, $\frac{29}{40}$, $\frac{107}{2520}$.

Exercise LXI.—Page 90.

1. $\$383\frac{7}{22}$. 2. $\$1\frac{33}{40}$. 3. $49\frac{7}{16}$ pounds.
4. $\$191\frac{1}{12}$. 5. $106\frac{31}{48}$ gallons. 6. $8\frac{31}{36}$.
7. $145\frac{17}{64}$ yards; $\$403\frac{13}{16}$. 8. $44\frac{9}{16}$ pounds. 9. \$3.
10. $10\frac{71}{84}$ gallons. 11. $\$147\frac{7}{36}$. 12. $774\frac{1}{83}$ acres.

Exercise LXII.—Page 92.

1. $7\frac{1}{2}$. 2. $6\frac{2}{5}$. 3. $2\frac{1}{4}$. 4. $9\frac{1}{3}$.
5. $37\frac{1}{3}$. 6. $71\frac{1}{2}$. 7. $26\frac{1}{4}$. 8. $10\frac{2}{19}$.
9. $4\frac{23}{28}$. 10. \$30. 11. \$13.14. 12. \$201.
13. \$1.77. 14. \$8.75.

Exercise LXIII.—Page 93.

1. $\frac{8}{17}$. 2. $\frac{1}{8}$. 3. $\frac{7}{84}$. 4. $\frac{7}{130}$. 5. $1\frac{1}{5}$.
6. $\frac{2}{3}$. 7. $\frac{3}{7}$. 8. $8\frac{1}{10}$. 9. $23\frac{31}{32}$. 10. $3\frac{1}{4}$ acres.
11. $2\frac{1}{4}$ cords. 12. $3\frac{5}{6}$ miles. 13. $\$8\frac{2}{3}$.

Exercise LXIV.—Page 95.

1. 15. 2. 40. 3. $35\frac{5}{6}$. 4. $54\frac{1}{4}$. 5. $\frac{18}{25}$.
6. $\frac{9}{80}$. 7. $\frac{3}{5}$. 8. $\frac{7}{44}$. 9. 1. 10. $\frac{10}{33}$.
11. $\frac{7}{15}$. 12. $\frac{20}{63}$. 13. $\$\frac{91}{250}$. 14. $\$\frac{5}{8}$. 15. *A*, $\$281\frac{1}{4}$; *B*, \$225; *C*, $\$303\frac{3}{4}$; \$810.

Exercise LXV.—Page 95.

1. $17\frac{5}{28}$. 2. $49\frac{1}{3}$. 3. 290. 4. 1320. 5. 8789.
6. $61\frac{1}{2}$. 8. $42\frac{3}{4}$ cents. 9. 10 acres. 10. $\$351\frac{9}{16}$. 11. $\$67\frac{3}{8}$.
12. $\$1667\frac{1}{4}$. 13. $\$227\frac{31}{240}$.

Exercise LXVI.—Page 97.

1. 14. 2. 21. 3. $34\frac{6}{7}$. 4. $11\frac{3}{7}$. 5. 3. 6. $\frac{40}{47}$.
7. $1\frac{8}{35}$. 8. $\frac{11}{17}$. 9. $10\frac{4}{5}$. 10. $\frac{885}{1208}$. 11. $1\frac{13}{35}$. 12. $17\frac{1}{14}$.

Exercise LXVII.—Page 97.

2. 24 bushels. 3. 14 tons. 4. 27 bushels.
5. $2\frac{1}{8}$ weeks. 6. 11 persons. 7. $\$4\frac{1}{2}$.
8. $2\frac{1}{3}$ weeks. 9. $11\frac{1}{2}$ bushels. 10. $190\frac{3}{8}$ days.
11 $\$67\frac{1}{2}$. 12. $10\frac{1}{4}\frac{1}{8}$.

Exercise LXVIII.—Page 99.

1. 16. 2. $\frac{3}{4}$. 3. $\frac{5}{27}$. 4. $\frac{5}{16}$. 5. $1\frac{7}{9}$.
6. $4\frac{1}{4}$. 7. 2. 8. $1\frac{1}{5}$. 9. 2. 10 $2\frac{1}{8}$.
11. $7\frac{1}{4}$. 12. $3\frac{1}{4}$. 13. $\frac{4}{8}\frac{3}{2}$. 14. $1\frac{7}{10}\frac{3}{2}$. 15. $\frac{5}{11}$.
16. $2\frac{1}{8}$. 17. $38\frac{5}{11}$. 18. 16. 19. $1\frac{1}{3}\frac{1}{2}$. 20. $1\frac{1}{4}\frac{1}{5}$.

Exercise LXIX.—Page 102.

1. $\frac{1}{4}$. 2. $3\frac{1}{3}\frac{1}{7}$. 3. 1. 4. $22\frac{1}{1}\frac{3}{7}$.
5. $3\frac{1}{4}\frac{7}{0}$. 6. $14\frac{2}{3}\frac{3}{5}$. 7. 4. 8. 1.
9. $\frac{1}{3}\frac{1}{0}$. 10. $1\frac{3}{4}$. 11. $\frac{5}{1}\frac{5}{2}$. 12. 15.
13. $1\frac{3}{5}\frac{1}{3}$. 14. 1. 15. $\frac{7}{2}\frac{5}{1}\frac{5}{0}$. 16. $3\frac{5}{2}\frac{1}{1}\frac{1}{0}$.
17. $\frac{1}{1}\frac{0}{0}\frac{0}{8}$. 18. $\frac{5}{1}\frac{5}{3}$. 19. $23\frac{5}{1}\frac{5}{1}$. 20. $\frac{5}{1}\frac{3}{3}$.

Examination Papers.—Page 103.

II.

8. $\frac{5}{1}\frac{5}{1}$. 4. $1\frac{1}{3}$; $\frac{1}{3}\frac{1}{3}$. 5. 24 days.

III.

2. $\frac{1}{3}$. 3. 18 bags. 4. $21900. 5. $15.85.

IV.

2. $\frac{3}{4}$ of $\frac{1}{1}\frac{0}{6}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{1}\frac{7}{6}$, $\frac{1}{1}\frac{3}{6}$. 3. $5.60. 4. Too large by $\frac{1}{3}\frac{1}{6}$.
5. $330.75.

V.

2. $350.45. 3. $660.80. 4. $\frac{3}{4}$. 5. 9 acres.

Exercise LXX.—Page 107.

1. $\frac{7}{10}$. 2. $\frac{36}{100}$. 3. $\frac{8}{100}$. 4. $\frac{784}{1000}$.
5. $\frac{700}{1000}$. 6. $\frac{4423}{10000}$. 7. $\frac{814}{10000}$. 8. $\frac{73}{100}$.
9. $\frac{7614}{10000}$. 10. $\frac{095}{1000}$. 11. $\frac{427}{10000}$. 12. $\frac{31}{10000}$.
13. $\frac{7000}{100000}$. 14. $\frac{712405}{1000000}$. 15. $\frac{6}{1000}$. 16. 8.
17. ·71. 18. ·27. 19. ·07. 20. 136.
21. 2·07. 22. 4·16. 23. 16·126. 24. 126·367.
25. ·18496. 26. 3·00007. 27. 16·00163.

Exercise LXXI.—Page 107.

1. Nine-tenths. 2. Twenty-seven hundredths. 3. Three hundred and sixty-eight thousandths. 4 Sixty-four thousandths. 5. Four, and thirty-one hundredths. 6. Seven, and two hundred and sixteen thousandths. 7. Three, and three hundred and fourteen thousandths. 8. Five, and eight thou-

sand one hundred and sixty-seven ten thousandths. 9. Twenty-one, and three thousand six hundred and one ten-thousandths. 10. Seventeen, and sixty-four ten-thousandths. 11. Eighteen, and eighty-one hundred thousandths. 12. Twenty, and one thousand four hundred and fifty-eight hundred-thousandths. 13. ·8; 2·07; ·009. 14 807·094; 3017·0709; 3·001008. 15. 6·0004; 80·0000609; 101·01001.

Exercise LXXII.—Page 108.

1. 65·046. 2. 600·7354. 3. 4475·105045.
4. 2·4397464. 5. 101·309. 6. 10·867.
7. 114·1377. 8. 959·0488. 9. 40·52753.
10. 15156·66886. 11. 200·1211. 12. 25·749445.
13. 227·5024. 14. 122·625 yds. 15. 58·4905 acres.

Exercise LXXIII.—Page 109.

1. 16·1524. 2. 2·3806. 3. ·43876.
4. ·23296. 5. 1·8316. 6. ·00521.
7. 3·9249. 8. 1·405. 9. 168·098.
10. ·01. 11. ·6322. 12. 8·3416.
13. 2·5527. 14. 15·799. 15. 173·03863.
16. ·2318 inches. 17. 36·002 grains. 18. ·099.
19. ·146. 20. 13·75 yards.

Exercise LXXIV.—Page 110.

1. 15·544. 2. 240·37086. 3. ·0273238.
4. 5·4008. 5. 2474·11. 6. ·26928.
7. 9·6142. 8. ·000072. 9. ·310104.
10. 803·2101. 11. ·040527. 12. 1·010009.
13. 334141·402 sq. in. 14. 9·75 pounds. 15. 334·00692 pounds
16. 117·04936022 mi. 17. 728·9271. 18. 312·275 pounds.

Exercise LXXV.—Page 112.

1. 8·07. 2. 50·615625 3. 800. 4. ·006446875.
5. 1240. 6. ·00075. 7. ·00016125. 8. ·568.
9. 20200. 10. 22600. 11. ·082. 12. 83.

Exercise LXXVI.—Page 113.

1. ·1875. 2. ·75. 3. ·625. 4. ·225.
5. ·15625. 6. ·025. 7. ·0375. 8. ·875.
9. ·06875. 10. ·078125. 11. ·056. 12. 6·6.
13. 24·008. 14. 9·525. 15. 46·3125.

Exercise LXXVII.—Page 115.

1. [illegible] 2. [illegible] 3. [illegible] 4. [illegible] 5. [illegible] 6. [illegible]
7. [illegible] 8. [illegible] 9. 4[illegible] 10. 11[illegible] 11. 8[illegible] 12. 2[illegible]

Exercise LXXVIII.—Page 115.

62·9204$\dot{1}$3349443052$\dot{4}$. 2. ·$\dot{2}\dot{4}$; ·03$\dot{2}$7116$\dot{5}$.
9·92$\dot{8}$; 2·$\dot{2}$9$\dot{7}$. 4. 8·$\dot{6}$; 1·$\dot{1}$4$\dot{5}$.

Examination Papers.—Page 115.

I.

2. $1\frac{1}{4}$; $\frac{1}{2500}$; ·000011.   3. ·017359 ; ·0005.
4. ·120508 ; ·13.   5. $\cdot\dot{7}1428\dot{5}$; $\frac{8411}{6660}$.

II.

1. ·375 ; ·000000375 ; 356·315375 ; 160000.   2. ·01825.
4. $\frac{1}{5000}$; $\frac{1}{40}$; $\frac{1}{250}$.   5. 8 ; 6400.

III.

1. ·9525.   2. $24\cdot\dot{9}7502\dot{4}$ · 500·5 ;   3. $\frac{8}{125}$; $15\frac{7}{8}$; 1.
4. $1\frac{2}{3}\times 2\frac{5}{8}$.   5. $\frac{1}{12}$.

IV.

1. 1·1214727.   2. ·54321.   3. \$3000, \$6000.
4. \$520 ; $3\frac{2}{3}$.   5. $9\frac{79}{84}$.

V.

1. $\frac{2}{3}$.   2. $\cdot\dot{0}000\dot{1}$; $\cdot\dot{0}00099\dot{9}$.   5. 1·60546875.

Exercise LXXIX.—Page 119.

1. 92d.   2. 1104 far.   3. £29 15s. 5d.
4. £309 5s.   5. 2406d.   6. 500d.
7. £[illegible] s. $5\frac{1}{2}$d.   8. £29 16s. 11d.   9. 183839 far.
10. 8209 far.   11. £328 16s. 4d.   12. 96028 far.

Exercise LXXX.—Page 120.

1. 1044736 dr.   2. 13[illegible]0 dr.   3. 13 cwt. 2 qr. 2lb. 13 oz.
4. 954 t. 16 cwt. 1 qr.   5. 49[illegible] oz.   6. 25 t. 16 cwt. 1 qr. 24 lb.

Exercise LXXXI.—Page 121.

1. 16 oz.   2. 24 lb. 10 oz. 3 dr. 1 scr.   3. 32 lb. 5 dwt.
4. 5460 gr.   5. 1584 gr.   6. 12 lb. 9 oz. 5 dwt. 4 gr.

Exercise LXXXII.—Page 122.

1. 71478 in.   2. 1 mi. 1 fur. 26 per. 2 ft.
3. 1 mi. 3 fur. 18 per. 3 yd. 2 ft.   4. 36 ft.
5. 462 ft.   6. 232 fath. 4 ft.

Exercise LXXXIII.—Page 123.

1. 12 a. 1 r. 37 rd.   2. 117900 in.   3. 4 cu. ft. 1557 in.
4. 60 c. 9 ft.   5. 75506904 cu. in.   6. 135424 cu. in.

Exercise LXXXIV.—Page 125.

1. 662400 sec.   2. 120 bu. 2 qt.   3. 2691 gi.
4. 2311 pt.   5. 83 gal. 3 qt. 1 pt. 1 gi.   6. 556 pk.
7. 61 bu. 25 lb.   8. 1 wk. 2 da. 2 hr. 14 min. 53 sec.
9. 74 bu. 34 lb.   10. 6739740 sec.   11. 12 c. 40 ft.

Exercise LXXXV.—Page 126.

1. 174 lb. 3 oz.
2. 74 cwt. 21 lb. 3 oz.
3. 88 rd. 5 yd. 1 ft. 6 in.
4. £34 14s. 8d.
5. 43 bu. 1 pk. 1 pt.
6. 95 rd. 5 yd. 2 ft. 3 in.
7. 6 wk. 3 da. 6 h. 50 min. 33 sec.
8. 22 rd. 2 yd. 8 in.

Exercise LXXXVI.—Page 127.

1. 7 lb. 8 oz. 6 dr. 1 scr. 19 gr.
2. 19 mi. 1 rd.
3. 59 a. 2 r. 27 rd.
4. 5 fur. 31 rd. 5 yd. 2 in.
5. £27 17s. 4d.
6. 38 per. 18 yd. 2 ft. 36 in.
7. 142 bu. 2 pk. 5 qt.
8. 79 lb. 3 oz. 5 dwt. 4 gr.
9. 31 gal. 2 qt. 1 pt.
10. 22 sq. rd. 12 yd. 4 ft. 128 in.
11. 1 cwt. 3 qr. 10 lb.

Exercise LXXXVII.—Page 128.

1. 90 cwt. 3 qr. 7 lb. 13 oz.
2. 50 lb. 2 oz. 7 dwt. 3 gr.
3. 75 da. 23 h. 34 min. 40 sec.
4. £600 9s. $6\frac{3}{4}$d.
5. 4985 cwt. 1 qr.
6. 1 lb. 1 oz. 12 dwt.
7. 150 a. 2 r. 35 sq. rd.
8. 23332 gal. 2 qt.
9. 88 mi. 3 fur. 2 rd. 3 yd.
10. 5 oz. 19 dwt.
11. £289. 5s. $9\frac{1}{2}$d.
12. 662 mi. 4 fur. 28 rd. 3 yd. 2 ft. 2 in.
13. 2739 bu. 1 pk. 5 qt.

Exercise LXXXVIII.—Page 129.

1. £15 9s. 7d.
2. 12 lb. 9 oz. 15 dwt. 18 gr.
3. 16 t. 2 cwt. 1 qr. 13 lb.
4. 1 gi.
5. 2 cu. yd. 6 ft. 960 in.
6. 8.
7. $10\frac{3}{37}$.
8. $11\frac{2667}{5843}$.
9. $5\frac{71}{89}$.
10. 2 bu. 3 pk. $3\frac{3}{7}$ qt.
11. 25 demijohns.
12. 5 weeks.

Exercise LXXXIX.—Page 130.

1. 3 pk. 1 qt. $1\frac{1}{5}$ pt.
2. 5 fur. 13 rd. 1 yd. 2 ft. 6 in.
3. 4 yd. 2 ft. $5\frac{1}{4}$ in.
4. 2 fur. 16 rd.
5. 17 cwt. 2 qr.
6. 2 r. 8 rd. 26 yd. 8 ft.
7. £1 12s. $10\frac{1}{2}\frac{7}{8}$d.; £5 2s. $8\frac{1}{2}\frac{3}{2}$d.
8. 4 da. 23 h. 28 min.
9. 1 lb. 7 oz.

Exercise XC.—Page 130.

1. $\frac{1}{80}$.
2. $\frac{1}{40000}$.
3. $\frac{1}{360}$.
4. $\frac{1}{63360}$.
5. $\frac{1}{36}$.
6. $\frac{5}{13440}$.
7. $\frac{7}{18000}$.
8. $\frac{713}{30003}$.
9. $\frac{4161}{14875}$.
10. $\frac{851}{721}$.
11. $\frac{40}{7}$.
12. $\frac{44}{13}$.

Exercise XCI.—Page 131.

1. 3 r. 13rd.
2. 9 oz. 15 dwt. 18 gr.
3. $10\frac{1}{2}$d.
4. 47 min. 6 sec.
5. 11 h. 55 min. $40\frac{1}{3}$ sec.
6. 8s. 9d.
7. 7 fur. 29 per.
8. 15 cwt. 2 qr. 6 lb. 4 oz.
9. 12s. $6\frac{3}{4}$d.
10. 3s. $5\frac{1}{2}$d.
11. 2 da. 12 h. 55 min. 21 sec.
12. $\frac{1}{2}$d.

Exercise XCII.—Page 131.

1. £·525.
2. ·282 t.
3. ·78125 oz.
4. ·775 mi.
5. ·3125 pk.
6. £0·26875.
7. 17·895 cwt.
8. 7·875 bu.
9. ·625 fath.
10. ·71.
11. 129·78 hr.
12. ·0625 t.

Exercise XCIII.—Page 132.

1. $192.
2. $148.50.
3. $436.80.
4. $588.
5. $2617.50.
6. $615.
7. $496.12½
8. $308.
9. $381.75.
10. $35.55.
11. $101.85.
12. $124.20.
13. $2338.50.
14. $44.04.
15. $32753.12½

Exercise XCIV.—Page 133.

2. 12½ lb.
3. 65.10\frac{5}{12}$.
4. 3$\frac{13}{14}$$\frac{3}{4}$.
5. 1$\frac{1}{4}$ in.
7. £34 12s. 4d.
8. 54. 2d.
9. 11$\frac{7}{8}$d.
10. 6$\frac{1}{2}$ oz.
11. $56·525.
12. 152.31\frac{1}{4}$$\frac{1}{8}$.
13. 173.7\frac{1}{2}$$\frac{7}{8}$.
14. 7 hr. 11 min. 8 sec.
15. $2.10.
17. 98 yd.
18. 30$\frac{3}{4}$ yd.
19. 4166$\frac{2}{3}$ yd.
21. $1108.80.
22. 3s. 7$\frac{5}{11}$$\frac{5}{11}$d.
21. $1736 23$\frac{1}{3}$.
24. $880.
26. $\frac{8}{11}$ ct.; $5.76.
27. $8400.
28. $3000.
30. 6 cents.
31. 64 cents.

Exercise XCV.—Page 136.

1. 34·25.
2. 32.22.
3. 493.33.
4. 54.40.
5. 44.
6. 43°.99.

Exercise XCVI.—Page 137.

1. 72.
2. $15.
3. 23 sheep.
4. $10.50.
5. 14 men.
6. 45.
7. 40%.
8. 75%.
9. 60%; 36$\frac{4}{11}$%.
10. 1100.
11. $1200.

Exercise XCVII.—Page 138.

1. $18.
2. $11.20.
3. $15.20.
4. $11 25.
5. $100.
6. $110.40.
7. $166.25.
8. $65.20.
9. $2500.
10. $7000.
11. $70000.
12. $9600.
13. $5276.

Exercise XCVIII.—Page 139.

1. $14 40.
2. $15.8).
3. $10.
4. $30.
5. $247.
6. $112.
7. $2500.
8. 7500 bu.
9. $788.75.
10. $1.75.
11. 5$\frac{1}{2}$ per cent.

Exercise XCIX.—Page 140.

1. $'8.
2. $34.50.
3. £236·412.
4. $568.05.
5. $151.50.
6. £236 61.
7. $503 36.
8. $95.
9. $842 19$\frac{3}{8}$.
10. $198.66.
11. $311 64.
12. $154.78.
13. $192·225.
14. $ 82.
15. $3 76 36.
16. $934.92.
17. $14 958...
18. $287 67...
19. $1306·849...
20. 8 per cent.
21. 7$\frac{1}{2}$ per cent

22. 6 per cent. 23. $3500. 24. $32500.
25. $9000. 26. 3 yr. 27. 3 yr.
28. Oct. 4, 1877. 29. $14\frac{2}{7}$ yr.

Exercise C.—Page 144.

1. $1168.70. 2. $457.50. 3. $900. 4. The latter.
5. $2100. 6. Gain $50. 7. $155.82. 8. $242.32.
9. $1.25. 10. $3.60. 11. $10.35...

Exercise CI.—Page 147.

1. 17. 2. 19. 3. 24. 4. 25.
5. 36. 6. 75. 7. 95. 8. 49.
9. 64. 10. 37. 11. 47. 12. 56.
13. 625. 14. 512. 15. 343. 16. 2401.
17. 6325. 18. 5008. 19. ·47. 20 ·27.
21. ·32. 22. ·8449... 23. ·946... 24. ·9486...
25. 8·4261... 26. 2·5298... 27. 3·794...

Exercise CII.—Page 148.

1. 96 sq. ft. 2. 91 sq. ft. 3. 525 sq. ft.
4. $18\frac{3}{4}$ sq. yd. 5. $351\frac{1}{3}$ sq. yd. 6. 470 sq. ft.

Exercise CIII.—Page 148.

1. 84 yd. 2. $26\frac{2}{3}$ yd. 3. 64 yd. 4. $138\frac{3}{8}$ yd.
5. $48. 6. $57.60. 7. $29.16\frac{2}{3}$. 8. $2\frac{1}{4}$ ft.

Exercise CIV.—Page 149.

1. 126 yd. 2. 578 sq. ft. 3. $115\frac{1}{3}$ yd. 4. $57\frac{3}{5}$ yd. 5. $6.80.

Exercise CV.—Page 150.

1. 240 cu. ft. 2. 95 cu. ft. 3. $187\frac{1}{2}$ cu. ft.
4. $45\frac{1}{2}$ cu. ft. 5. 16000. 6. $14.51\frac{3}{4}$.

Miscellaneous Problems.—Page 151

2. 16 days. 3. 5 days. 4. 9 months.
5. 5 days. 7. $6\frac{3}{4}$ hours. 8. 8 days.
9. $3\frac{7}{11}$ days. 10. $13\frac{1}{2}$ days. 11. $14\frac{2}{3}$ days.
12. $13\frac{1}{3}$ days. 14. $5\frac{1}{14}$ days. 15. 15 days.
16. 2 days. 17. $8\frac{8}{11}$ hours. 19. $3\frac{4}{7}$ days.
20. $13\frac{1}{13}$ days. 22. Man $27\frac{6}{13}$ da.; boy 120 da.
23. Man 90 da.; boy 180 da. 24. Man 28 da.; woman 40 da.
25. Woman 30 da.; boy 40 da. 27. $24\frac{1}{13}$ min.
28. 24 hours. 29. $23\frac{1}{13}$ min. 30 $17\frac{1}{7}$ min.
32. 180 min. 33. 72 min. 35. $23030.
36. 35 ft. 37. $1582. 38. $2843.75.
40. $5000. 42. 8 days. 43. $1827.
44. 36 days. 45. $94.50. 47. $32.
48. $192. 49. 300 men. 50. 12 oz.
51. $18. 52. $10.31\frac{1}{2}$. 53. $350.
55. 25 per cent. 56. 20 per cent. 57. $33\frac{1}{3}$ per cent.

39. A labourer in one week dug 5 rods more than $\frac{1}{2}$ the length of a ditch, and the next week he dug the remaining 20 rods; how long was the ditch?

Length of ditch dug first week $= \frac{1}{2}$ ditch + 5 rods;
$\therefore$ length remaining . . . $= \frac{1}{2}$ ditch less 5 rods;
$\therefore$ $\frac{1}{2}$ length of ditch less 5 rods $= 20$ rods;
$\therefore$ $\frac{1}{2}$ length of ditch . . . . $= 20$ rods + 5 rods
$= 25$ rods;
$\therefore$ length of ditch . . . . $= 50$ rods.

40. A man invested \$300 more than $\frac{3}{5}$ of his money in a house, and \$600 more than $\frac{4}{5}$ of the remainder in a lot, and had now \$900 left; how much was he worth?

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41. If 10 men can chop 90 cords of wood in 8 days, how many cords can be chopped by 20 men in 4 days?

Cords chopped by 10 men in 8 days $= 90$ cords;
" 1 *man* in 8 *days* $= \frac{90}{10} = 9$ *cords;*
" 1 man in 1 day $= \frac{9}{8}$ cords;
" 20 *men* in 1 *day* $= \frac{20 \times 9}{8} = \frac{45}{2}$ *cords;*
" 20 men in 4 days $= \frac{4 \times 45}{2} = 90$ cords.

NOTE.—When the pupil has become familiar with the unitary system, and thoroughly understands the reason of each step, the process may be abridged by leaving out the steps in italics.

42. If 8 men build 33 ft. of wall in 11 days, in how many days will 12 men build 36 feet?

43. If 36 men earn \$324 in 18 days, how much will 42 men earn in 87 days?

44. How many days will it take 15 men to cut 810 cords of wood, working 9 hours a day, if 13 men can cut 364 cords in 14 days, working 12 hours a day?

45. It costs a family of 5 persons \$135 for 6 weeks' board; how much will it cost a family of 7 persons at the same rate for 3 weeks?

46. If 12 men can dig a ditch 16 rods long in 8 days, in how many days can 24 men dig a ditch of the same depth and width, 32 rods in length?

Time in which 12 men will dig 16 rods $= 8$ days;
" 1 man " 1 rod $= \frac{12 \times 8}{16}$ days;
" 24 men " 32 rods $= \frac{32 \times 12 \times 8}{24 \times 16}$ days
$= 8$ days.

47. If 20 cwt. are carried the distance of 50 miles for $20, how much will 40 cwt. cost if carried 40 miles?

48. If $500 gain $60 in 2 yr. at 6%, how much will $800 gain in 3 yr. at 8%?

49. If 20 men can perform a piece of work in 12 days, required the number of men who could perform another piece of work 3 times as great in $\frac{1}{5}$ of the time?

50. If a 10-cent loaf weighs 15 oz. when flour is $8 a barrel, how much will a 6-cent loaf weigh when flour is worth $6 a barrel?

51. If it costs $36 to carpet a room 18 ft. long and 15 ft. wide, how much will it cost to carpet a room 15 ft. long and 9 feet wide?

52. If it costs $150 to dig a cellar 40 ft. long 30 ft. wide and 6 ft. deep, how much will it cost to dig a cellar 30 ft. long, 3 ft. wide, and $5\frac{1}{2}$ ft. deep?

53. If the rent of a house worth $3200 is $240 for 9 months, for what sum per year must a man rent a house worth $3500?

54. I bought a horse for $130 and sold him for $162.50; what was my gain per cent.?

On an outlay of $130 my gain is $32.50;

" $1 " $\frac{32.50}{130}$;

" $100 " $\frac{100 \times 32.50}{130}$ or $25;

∴ I gain 25 %.

55. If I buy a pair of boots for $6 and afterward sell them for $7.50 what per cent. do I gain?

56. A grocer sells a barrel of oranges for $7.50 which cost him $6.25; what is his gain per cent.?

57. A merchant buys sugar at 6 cents per pound and sells it at 8 cents; what per cent. does he gain?

58. I bought calico at 12 cents a yard; for what must I sell it to gain 25 per cent.?

That for which I gave $100 I must sell for $125;

" $1 " $\frac{125}{100}$;

$.12 " $\frac{12 \times 125}{100}$

= 15 cents.

59. A merchant bought silks at $1.25 per yard; for what must he sell them to gain 20 per cent?

60. *A* bought a house for $8500 and afterwards sold it at a loss of 15 %; what did he get for the house?

59. $1.50. 60. $7225. 61. $135.70
63. $45.80. 64. $1.45. 66. 576; 684.
67. $125; $225; $150. 68. $150; $200; $250.
70. 112 a.; 98 a. 71. 1520 bu.; 2280 bu.
73. $60; $144; $96. 74. $3900; $4800; $3300.
75. $14.40; $9.60. 76. $6.30; $5.25; $4.20.
78. $21.60; $23.04; $25.92. 79. $525; $500; $480.
80. $1440; $2385. 82. $124.16.... 83. $1215.50....
84. $6.8992. 85. 466.56. 86. $161.02....
87. $562.75 $\frac{1}{2}$.

Examination Papers.

ADMISSION TO HIGH SCHOOLS.

July, 1877.—Page 160.

1. 4547. 2. $\frac{14}{15}$. 3. £41 1s. 5$\frac{1}{4}$d.
4. 222$\frac{1}{2}$ cwt. 5. $93.75. 6. $2.77 $\frac{34}{75}$.
7. 37$\frac{1}{2}$ yds. 8. $35. 9. $4\frac{599}{900}$.
10. $210.65.

July, 1878.—Page 161.

1. 2, 2, 3, 3, 5, 7. 2. 31116. 3. 4 cents.
4. ·01. 5. 8 hrs. 6. $\frac{1}{108}$.
7. 120 boxes. 8. $101.85 $\frac{1}{3}$.

December, 1879.—Page 162.

1. 15. 2. $44. 3. 8$\frac{3}{4}$ miles.
4. $8\frac{2888}{11979}$. 5. $121.37 $\frac{1}{2}$. 6. 11$\frac{1}{4}$ cents.
7. $3.60.

UNIVERSITY OF HALIFAX.—Page 163.

1. 7s. 9$\frac{3}{4}$l; £457 0s. 7$\frac{1}{2}$. 2. $\frac{11}{180000}$. 3. $4\frac{41276}{83325}$, $\frac{7}{90}$, $\frac{164}{333}$.
4. $121·662; $20.50. 5. See Art. 141, 30·467532.
6. £378 8s. 2·64d.; £409 9s. 9$\frac{3}{4}$d. 7. $\frac{1194}{1295}$; ·9220077.
8. 1·414213; 8·774964. 9. 2 days.

CITY OF TORONTO.—Page 164.

1. 1 ton 8 cwt. 3 qr. 7 lb. 14 oz $8\frac{8}{75}$ dr,
2. 45926160 sec. 3. ·023. 4. $\frac{260}{421}$.
5. 2430 lb. 6 oz. 5 dr. 1 scr. 6. $1\frac{739}{9845}$.
7. 16 days. 8. $2\frac{4}{999999999}$. 9. $213.39 $\frac{1}{13}$.
10. $51.15. 11. $500. 12. 300.

COUNTY OF WATERLOO.—Page 165.

1. 300, and 180. 2. 7 and 6. 3. £177 1s. 7$\frac{3}{4}$d.
4. 9 tons. 5. 1 6. $175.02.
7. $10000.50. 8. $2166\frac{314}{821}$ bbl. 9. $5.
10. $15000; $[illegible]00. 11. 3600. 12. 18%.

COUNTY OF DURHAM.—Page 166.

1. 48 min. 2. 18 ft. 3. $2.10.
4. $1680; 50 cents. 5. $4.95. 6. 39 lb. tea; 21 lb. coffee.
7. $26. 8. $1\frac{1}{24}$; $\frac{19}{20}$.