ANSWERS TO THE QUESTIONS

IN

ADDITION OF VULGAR FRACTIONS.

(2) $1\frac{73}{84}$, *Ans.* (3) $4\frac{14}{15}$, *Ans.* (4) $8\frac{1}{15}$, *Ans.* (5) $\frac{11}{14}$, *Ans.* (6) $17\frac{1}{24}$, *Ans.* (7) $8\frac{11}{120}$, *Ans.* (8) $10\frac{53}{112}$, *Ans* (9) $7\frac{23}{24}$, *Ans.*

SUBTRACTION.

(2) $\frac{7}{30}$, *Ans.* (3) $5\frac{5}{548}$, *Ans.* (4) $\frac{29}{45}$, *Ans.* (5) $\frac{359}{420}$, *Ans.* (6) $63\frac{3}{4}$, *Ans.* (7) $2\frac{19}{20}$, *Ans.* (8) $\frac{33}{35}$, *Ans* (9) $\frac{53}{168}$, *Ans.*

MULTIPLICATION.

(2) $\frac{14}{27}$, *Ans.* (3) $672\frac{3}{10}$, *Ans.* (4) $7935\frac{12}{35}$, *Ans.* (5) $\frac{20}{49}$, *Ans.* (6) $\frac{15}{224}$, *Ans.* (7) $4\frac{37}{42}$, *Ans.* (8) 16, *Ans.* (9) $5\frac{29}{32}$, *Ans.* (10) £15 9*s.* 11*d.*$\frac{39}{411}$, *Ans.* (11) 8 *m.* 2 *r.* $188\frac{4}{7}$ *yds.*, *Ans.*

DIVISION.

(2) $\frac{28}{45}$, *Ans.* (3) $48\frac{3}{5}$, *Ans.* (4) $430\frac{3}{5}$, *Ans.* (5) $\frac{2}{3}$, *Ans* (6) $\frac{1}{8}$, *Ans.* (7) $\frac{196}{2805}$, *Ans.* (8) $2\frac{13}{24}$, *Ans.* (9) $\frac{9}{10}$, *Ans.* (10) $19\frac{41}{45}$, *Ans.*

ADDITION OF DECIMAL FRACTIONS.

(1) 480·8784, *Ans.* (2) 98·6091, *Ans.* (3) 981·2673, *Ans.* (4) 855·7195, *Ans.* (5) 4035·839, *Ans.* (6) 2803·3416, *Ans.*

SUBTRACTION.

(1) ·0383, *Ans.* (2) 0·61, *Ans.* (3) 55·3, *Ans.* (4) 194·7925, *Ans.* (5) 516·28, *Ans.* (6) 548·09, *Ans.* (7) 23·0408, *Ans.* (8) ·1063, *Ans.*

MULTIPLICATION.

(1) 4·70117, *Ans.* (2) 659·745, *Ans.* (3) ·05758775, *Ans.* (4) 1674833·05, *Ans.* (5) 5571·0985, *Ans.* (6) 559·3335, *Ans.* (7) 210·5980085, *Ans.* (8) ·429075, *Ans.* (9) 000049, *Ans.* (10) 5·450575, *Ans.* (11) ·0022675, *Ans.* (12) ·00055300185054, *Ans.*

DIVISION.

(1) 3·35, *Ans.* (2) 275·44, *Ans.* (3) 1196·17, *Ans.* (4) ·3′, *Ans.* (5) ·00069, *Ans.* (6) 23·25, *Ans.* (7) 1148·99, *Ans.* (8) ·0002013, *Ans.* (9) 20·163, *Ans.* (10) 535·68, *Ans.* (11) 13·549, *Ans.* (12) ·3183+, *Ans.* (13) 371·9, *Ans.* (14) ·0374, *Ans.* (15) ·1397 *Ans.* (16) ·003112, *Ans.*

ARITHMETICAL TABLES.

NUMERATION.

Units	1
Tens	12
Hundreds	123
Thousands	1,234
Tens of Thousands	12,345
C. of Thousands	123,456
Millions	1;234,567
X. of Millions	12;345,678
C. of Millions	123;456,789

CURRENT MONEY.

4 Farthings make .. 1 Penny
12 Pence .... 1 Shilling
20 Shillings, 1 Pound, or Sovereign

PENCE.

d.		s.	d.
20	are	1	8
24	..	2	0
30	..	2	6
36	..	3	0
40	..	3	4
48	..	4	0
50	..	4	2
60	..	5	0
70	..	5	10
72	..	6	0
80	..	6	8
84	..	7	0
90	..	7	6
96	..	8	0
100	..	8	4
108	..	9	0
120	..	10	0

SHILLINGS.

s.		£	s.
20	are	1	0
30	..	1	10
40	..	2	0
50	..	2	10
60	..	3	0
70	..	3	10
80	..	4	0
90	..	4	10
100	..	5	0
110	..	5	10
120	..	6	0
130	..	6	10
140	..	7	0
150	..	7	10
160	..	8	0
170	..	8	10
180	..	9	0

MULTIPLICATION.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Note. This table may be applied to division by reversing it; as the 2's in 4 are 2; the 2's in 6 are 3, &c.

LONG MEASURE.

3 barleycorns 1 inch
12 inches .... 1 foot
3 feet ...... 1 yard
6 feet ...... 1 fathom
5½ yds. ...... 1 pole
10 poles ..... 1 furlong
8 fur. ...... 1 mile
3 miles ..... 1 league
69½ miles .... 1 degree

LAND MEASURE.

9 feet make 1 yard
30¼ yds. ..... 1 pole
40 poles ..... 1 rood
4 roods .... 1 acre

CLOTH MEASURE.

2¼ inch make 1 nail
4 nails ..... 1 quarter
3 qrs. ...... 1 Flem. ell
4 qrs. ...... 1 yard
5 qrs. ...... 1 Eng. ell
6 qrs. ...... 1 Fren. ell

TIME.

60 seconds .... 1 minute
60 minutes .. 1 hour
24 hours .... 1 day
7 days ..... 1 week
4 weeks .... 1 month

DRY MEASURE.

2 gall. make 1 peck
4 pecks .... 1 bushel
4 bushels ... 1 sack
8 bushels ... 1 quarter
4 quarters ... 1 chaldron
10 quarters ... 1 last

SOLID MEASURE.

1728 inches .. 1 solid foot
27 feet .... 1 yard

COAL MEASURE.

3 bushels ... 1 sack
36 bushels ... 1 chaldron

Tables of Weights and Measures.

PRACTICE.

OF A POUND, OR SOVEREIGN.

s.	d.		£
10	0	are	1 half
6	8	..	1 third
5	0	..	1 fourth
4	0	..	1 fifth
3	4	..	1 sixth
2	6	..	1 eighth
2	0	..	1 tenth
1	8	..	1 twelfth
1	4	..	1 fifteenth
1	3	..	1 sixteenth
1	0	is	1 twentieth

OF A SHILLING.

d.		s.
6	are	1 half
4	....	1 third
3	....	1 fourth
2	....	1 sixth
1½	is	1 eighth
1	....	1 twelfth

OF A PENNY.

farth.		*d.*
2	are	1 half
1	is	1 fourth

OF A TON.

Cwt.		*T.*
10	are	1 half
5	..	1 fourth
4	..	1 fifth
2½	..	1 eighth
2	..	1 tenth
1	is	1 twentieth

OF A HUNDRED.

qr.	*lb.*		*Cwt.*
2	0	are	1 half
1	0	is	1 fourth
0	16	are	1 seventh
0	14	..	1 eighth

TROY.

24 gr. make 1 dwt.
20 dwt. .... 1 ounce
12 oz. .... 1 pound

AVOIRDUPOIS.

16 dr. make 1 oz.
16 oz. .... 1 lb.
14 lb. .... 1 stone
28 lb. ..... 1 quarter
4 qr. ..... 1 cwt.
20 cwt ..... 1 ton

APOTHECARIES'.

20 gr. make 1 scruple
3 scr. .... 1 dram
8 dr. .... 1 ounce
12 oz. ... ... 1 pound

WOOL.

7 lb. make 1 clove
2 cloves .... 1 stone
2 stones .... 1 tod
6½ tods ..... 1 wey
2 weys ..... 1 sack
12 sacks ..... 1 last

ALE AND BEER.

2 pints make 1 quart
4 quarts .... 1 gallon
9 gallons ... 1 firkin
2 firkins .... 1 kilder.
2 kilderkins . 1 barrel
1½ barrel ... 1 hhd.
2 barrels ... 1 punch.
3 barrels ... 1 butt

WINE.

2 pints make 1 quart
4 quarts .... 1 gallon
10 gallons ... 1 anker
42 gallons .. 1 tierce
63 gallons ... 1 hhd.
2 hhds. ..... 1 pipe
2 pipes ..... 1 tun

CUSTOMARY WEIGHT OF GOODS.

lb.
A firkin of butter is . 56
A firkin of soap 64
A barrel of pot ashes . 200
A barrel of anchovies 30
A barrel of soap 256
A barrel of butter ... 2[illegible]
A fother of lead, 19 cwt. 2 qrs. or . . . 21[illegible]
A barrel of candles .. 12[illegible]
A stone of iron or shot 14
A gallon of train oil . 7[illegible]
A fagot of steel 120
A stone of glass ...
A seam of glass, 24 stone, or .... 1[illegible]
A roll of parchment, 5 dozen skins
A barrel of figs, from nearly 96 to ...... 3[illegible]

THE

TUTOR'S ASSISTANT;

BEING A COMPENDIUM OF

PRACTICAL ARITHMETIC,

FOR THE

USE OF SCHOOLS, OR PRIVATE STUDENTS:

CONTAINING

I. *Arithmetic in Whole Numbers;* the Rules of which are expressed in a clear, concise, and intelligible manner; and the operations illustrated by examples worked at length, and by numerous explanatory Notes and Observations; with an ample variety of Examples for the exercise of Learners, calculated to initiate them in the *knowledge of real business.* Also the *New Commercial Tables,* adapted to the present *legislative regulations of Weights and Measures,* and the *modern practice of trade.*

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BY FRANCIS WALKINGAME,

WRITING-MASTER AND ACCOUNTANT.

THIRTEENTH EDITION,

REVISED, CORRECTED, AND ENLARGED BY THE ADDITION OF

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AND A COMPENDIUM OF

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MASTER OF AN ACADEMY IN DERBY.

St. John, N. B.

McMILLAN,

PHŒNIX BOOK AND STATIONERY WAREHOUSE,

PRINCE WILLIAM STREET.

1842.

THE EDITOR'S PREFACE.

THE immense circulation of WALKINGAME'S TUTOR'S ASSISTANT, even in its original form, is sufficiently evinced by the very extensive and uniformly increasing demand which the Proprietors of the present Edition have for many years experienced.

To advance the utility of a work held in such high estimation among Conductors of Schools; by simplifying the Rules, correcting and modernizing the antiquated phraseology, supplying deficiencies where there was a paucity of Examples, and incorporating with its original matter such emendations and additions as appear to be called for by the present improved state of Arithmetical Science; will, it is presumed, be rendering an acceptable service to the Public.

Amongst the various improvements introduced in this Edition, may be enumerated, a more intelligible elucidation of the system of Notation; of Direct, Inverse, and Compound Proportion, Practice, Interest, Progression, &c.; more perspicuous illustrations of the theory and practice of Vulgar and Decimal Fractions, Evolution, Duodecimals, &c.; the substitution of the *new Arithmetical* and *Commercial Tables*; the insertion of many additional Examples (particularly in the elementary Rules), adapted to exercise and improve the judgment of the Learner; also of Rules for the particular cases in Profit and Loss, of Involution, of Theorems for the solution of all the possible cases in Arithmetical and Geometrical Progression, Superficial Mensuration, a number of useful Supplemental Questions, and a Compendium of Book-keeping.

Such are the attempts that have been made to enhance the real worth of this popular *Treatise on Arithmetic*. How far the intention may have been judiciously or successfully executed, must be left to a candid Public to determine.

With a confident reliance, however, on the favourable consideration of those whose judgment and experience most essentially qualify them to discriminate between realities and specious pretensions to improvement, and duly to appreciate the difficulties of such an undertaking; the work is respectfully submitted to a trial before the tribunal of its legitimate judges, with an anxious and hopeful anticipation of obtaining their verdict of approval.

Derby, February 7, 1827.

Advertisement to the Sixth Edition.

WHEN the Editor first undertook the task of modernizing and improving this work, with an anxious desire of rendering it more commensurate with the progress of scientific information, and better adapted to the present improved systems of instruction, thereby to facilitate the arduous labours of the Teacher in the communication, and of the Pupil in the acquisition of a branch of knowledge so useful and indispensable; he was limited materially in the execution of his views by the necessity of conforming the whole of the arrangement, as well as a considerable part of the methods of operation that were susceptible of improvement, to those of FALCONAR'S KEY to the original work. The same necessity compelled him to retain every example contained in the old editions, however obsolete, useless, or absurd the subject might have become, from the innovations which time invariably introduces in the management of commercial transactions. Hence the student was subjected to the double inconvenience of referring, for some important parts of the additional matter, to the *Addenda* in the Arithmetic, and to an *Appendix* to the Key. This impediment to improvement is now removed; for, having constructed a new KEY with *correspondent* improvements, he has been enabled to complete his intentions by the regular incorporation of the newly introduced matter, by an arrangement more rational and more consistent with the modern practice of instruction, by the expunction of some useless subjects, and the substitution of others of more real utility.

Besides the emendations and additions enumerated in the preceding Preface, the worked examples under each Rule are *now* exhibited in a *more convenient form,* considerably amplified, and illustrated by *copious explanatory notes*;—the theory and practice of Circulating Decimals are comprehensively and clearly explained;—considerable improvements have been effected in that part which treats of the Doctrine of Compound Interest and Annuities, the *Tables of which have been greatly extended;* and, in order to correct the errors that abound in the Tables of this description contained in several scientific publications which have been collated, the *accuracy of these has been verified* by actual calculation. Some useful additions have also been made in the Mensuration and Book-keeping.

Three years have now elapsed since the IMPROVED TUTOR'S ASSISTANT was first issued from the press. That it has been greatly approved, has been manifested by the most unequivocal and satisfactory proof,—the sale of *several large impressions.* Under such auspicious symptoms of encouragement, therefore, the Editor feels an increased confidence in the prospect of obtaining the sanction of an enlightened public.

Derby, February 1, 1830.

THE AUTHOR'S PREFACE.

The public will, no doubt, be surprised to find there is another attempt made to publish a book of ARITHMETIC, when there are such numbers already extant on the same subject, and several of them that have so lately made their appearance in the world; but I flatter myself, that the following reasons which induced me to compile it, the method, and the conciseness of the Rules, which are laid down in so plain and familiar a manner, will have some weight towards its having a favourable reception.

Having some time ago drawn up a set of Rules and proper Questions, with their Answers annexed, for the use of my own school, and divided them into several books, as well for more ease to myself, as the readier improvement of my scholars, I found them, by experience, of infinite use; for when a master takes upon him that laborious (though unnecessary) method of writing out the Rules and Questions in the children's books, he must either be toiling and slaving himself after the fatigue of the school is over, to get ready the books for the next day, or else must lose that time which would be much better spent in instructing and opening the minds of his pupils. There was, however, still an inconvenience which prevented them from giving me the satisfaction I at first expected; *i.e.* where there are several boys in a class, some one or other must wait till the boy who first has the book finishes the writing of those rules or questions he wants, which detains the others from making that progress they otherwise might, had they a proper book of Rules and Examples for each boy; to remedy which, I was prompted to compile one, in order to have it printed, which might not only be of use in my own school, but in other schools, where the instructers wish their scholars to make a quick progress. It will also be of great use to such persons as have acquired some knowledge of numbers at school, to make them the more perfect; likewise to such as have completed themselves therein, it will prove, after an impartial perusal, on account of its great variety and brevity, a most agreeable and entertaining Exercise Book. I shall not presume to say any thing more in favour of this work, but beg leave to refer the unprejudiced reader to the remark of a certain Author,* concerning compositions of this nature. His words are as follow:

"And now, after all, it is possible that some who like best to tread the old beaten path, and to toil at their business, when they may do it with pleasure, may start an objection against the use of this well-intended Assistant, because the course of Arithmetic is always the same; and therefore say, that some boys, lazily inclined, when they see another at work upon the same question, will be apt to make his operation pass for their own. But these little forgeries are soon detected by the diligence of the Tutor: therefore, as different questions to different boys do not in the least promote their improvement, so neither do the same questions impede it. Neither is it in the power of any master (in the course of his business), how full of spirits

* Dilworth.

soever he may be, to frame new examples at pleasure in any Rule; but the same question will frequently occur in the same Rule, notwithstanding his greatest care and skill to the contrary.

"It may also be farther objected, that to teach by a printed book is an argument of ignorance and incapacity; which is no less trifling than the former. He, indeed (if such a one there be), who is afraid his scholars will improve too fast, will, undoubtedly, decry this method: but that master's ignorance can never be brought in question, who can begin and end it readily; and, most certainly, that scholar's non-improvement can be as little questioned, who makes a much greater progress by this, than by the common method."

To enter into a long detail of every Rule would tire the reader, and swell the Preface to an unusual length: I shall, therefore, only give a general idea of the method of proceeding, and leave the rest to speak for itself; which, I hope, the reader will find to answer the title, and the recommendation given it. As to the Rules, they follow in the same manner as the table of contents specifies, and in much the same order as they are generally taught in schools. I have gone through the four fundamental Rules in Integers first, before those of several denominations; in order that they being well understood, the latter will be performed with much more ease and despatch, according to the rules shown, than by the customary method of dotting. In Multiplication I have shown both the beauty and use of that excellent Rule, in resolving most Questions that occur in merchandising; and have prefixed to Reduction several Bills of Parcels, which are applicable to real business. In working Interest by Decimals, I have added tables to the Rules, for the more ready calculating of Annuities, &c. and have not only shown the use, but the method of making them: likewise a Table calculated for finding the Interest of Money for any number of days, at any rate per cent, by Multiplication and Addition only; which may also be applied to the calculation of Incomes, Salaries, or Wages, for any number of days; and I may venture to say, I have gone through the whole with so much plainness and perspicuity, that there is none better extant.

I have nothing farther to add, but a return of my sincere thanks to all those gentlemen, schoolmasters, and others, whose kind approbation and encouragement have now established the use of this book in almost every school of eminence throughout the kingdom: but I think my gratitude more especially due to those who have favoured me with their remarks; though I must still beg of every candid and judicious reader, that if he should, by chance, find a transposition of a letter or a false figure, to excuse it; for, notwithstanding there has been great care taken in correcting, yet errors of the press will inevitably creep in; and some may also have slipped my observation: in either of which cases, the admonition of a good-natured reader will be very acceptable to his

much obliged,
and most obedient humble servant,
F. WALKINGAME.

CONTENTS.

EXPLANATION OF
ARITHMETICAL SIGNS OR CHARACTERS.

$+$ is the sign of ADDITION, and is called *plus* or *more*.
$-$ is called *minus*, or *less*, and denotes SUBTRACTION.
$\times$ ——— *into*, and denotes MULTIPLICATION.
$\div$ ——— *by*, and denotes DIVISION.
$=$ is the sign of EQUALITY.
: : : are the signs of PROPORTION.
$\sqrt{}$ is the *Radical Sign*, or Sign of EVOLUTION.
$\backsim$ denotes the difference between two quantities, when it is uncertain which is the greater.

ILLUSTRATIONS.

$6+3=9$ signifies 6 *plus* 3 *equal* 9: that is, 6 added to 3 equal 9.
$7-4=3$ signifies 7 *minus* 4 *equal* 3: that is, 4 taken from 7 leaves 3.
$4\times 3=12$ is read 4 *into* 3 *equal* 12: that is, 4 multiplied by 3 equal 12.
$12\div 4=3$ is read 12 *by* 4 *equal* 3. But Division is more conveniently expressed in the form of a Fraction: thus, $\frac{12}{4}=3$; twelve *divided by* four *equal* three.
As $2:4::8:16$; As 2 *are to* 4, *so are* 8 *to* 16.
$\sqrt{16}=4$; the *Square Root* of 16 *equal* 4.
$\sqrt[3]{64}=4$; the *Cube Root* of 64 *equal* 4.

A *vinculum* connects two or more terms which are to be considered as forming *one* term or quantity. It is signified by a *line* drawn over them, or by *parentheses* including them.

A *point* is often used instead of the *cross* to denote *Multiplication*; and in Algebraical Theorems, &c. the multiplication of the quantities denoted by different letters is understood by placing the letters together without any sign between them. Thus, $2\,.\,\overline{5+7}$ denotes that 5 and 7 are to be added, and the sum (12) to be multiplied by 2. Also, $(ab+c).(n-1)$ denotes that a and b are to be multiplied, c added to the product, and the whole to be multiplied by *one less* than the number or quantity represented by n.

The common sizes of Books are			*marked*
Folio, ...	of which	2 leaves make a sheet,	fo.
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THE

TUTOR'S ASSISTANT;

BEING

A COMPENDIUM OF PRACTICAL ARITHMETIC.

INTEGERS, OR WHOLE NUMBERS.

ARITHMETIC is the science of numbers; or the art of numerical computation. A *whole number* is a *unit,* or a collection of *units.*

Numbers are expressed by ten written characters called figures, or digits: *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, which are *significant* figures, all declaring their own values by the names; and the *cipher,* or *nought* [0], an *insignificant* figure, indicating no value when it stands alone.

NUMERATION AND NOTATION.

A figure standing alone, or the *first on the right* of others, denotes only its simple value, as so many *units,* or *ones;* the second is so many *tens;* the third, so many *hundreds,* &c. increasing continually towards the left in a *tenfold* proportion.

Numeration is the art of *reading* numbers expressed in figures; and *Notation,* the art of *expressing numbers by figures.*

THE TABLE.

Billions		Hund. of Thous. of Millions	Tens of Thous. of Millions	Thousands of Millions		Hundreds of Millions	Tens	Millions		Hundreds of Thousands	Tens	Thousands		Hundreds	Tens	Units
3	;	5	7	8	,	6	9	5	;	2	7	3	,	8	4	1

Millions' period. Units' period.

This Table might be infinitely extended.

NOTE. *To read any Number.* Divide it into *periods* of six figures each, beginning at the right hand; and each period into *semi-periods* with a *different mark*, for the sake of distinction. The *first* on the right hand is the *Units'* period, the second the *Millions'* period, &c. Beginning at the left, observe that the three figures of every *complete semi-period* must be reckoned as so many *hundreds, tens,* and *units;* joining the word *thousands* when you come to the middle of the period, and the *proper name* of the period at the end of it.

2. *To express any given Number in Figures.* Begin at the left, and write the figures which denote (as so many *hundreds, tens,* and *units*), the number in that *semi-period;* and proceed thus with each successive semi-period, till the whole is completed; placing a separating comma in the middle of each period, or immediately after the thousands, and a semicolon between the periods. But observe, that though every semi-period but the first on the left must have its complete number of *three* figures, *that* may be incomplete, and consist of only one or two figures: also, where *significant figures are not required* in any part of a number, no semi-period must be omitted, but the places must be filled up with *ciphers.*

EXAMPLE. Write in figures, seventy thousand four hundred billions, two hundred and ten thousand millions, and ninety-six.

First, write 70 [seventy] with a comma, these being thousands; then 400 [four hundred] with a semicolon, denoting the end of the period; next, write 210 [two hundred and ten]; and, because they are thousands, put a comma after them, and then 000 [three ciphers, there being no more millions] followed by a semicolon, to denote the completion of the period; again, put 000 [three more ciphers, denoting the absence of thousands] with a comma after them, and then 096 [ninety-six], which will complete the number: thus, 70,400 ; 210,000 ; 000,096.

EXERCISES IN NUMERATION AND NOTATION.

Read, or write in words the following numbers.

* (1) 3	(13) 721	(25) 500050005
(2) 30	(14) 906	(26) 1010100
(3) 33	(15) 4294	(27) 11110101
(4) 300	(16) 94294	(28) 499994949
(5) 303	(17) 294294	(29) 3584600987
(6) 330	(18) 3703	(30) 584610070840
(7) 333	(19) 703703	(31) 5846100708400
(8) 127	(20) 311311	(32) 37613590200116
(9) 172	(21) 113113	(33) 5008000400090
(10) 217	(22) 131131131	(34) 601008000180070
(11) 271	(23) 708807780	(35) 3700000000075048
(12) 712	(24) 807078087	

* The figures in parentheses refer to the Editor's Key to this work. See Advertisement on the first page.

Express in figures the following numbers.

(1) Nine; ninety; ninety-nine; nine hundred; nine hundred and nine; nine hundred and ninety; nine hundred and ninety-nine.

(2) One hundred and eight; one hundred and eighty; eight hundred and one; eight hundred and ten; one hundred and sixteen; one hundred and sixty-one; six hundred and eleven.

(3) One hundred and twenty-three; one hundred and thirty-two; two hundred and thirteen; two hundred and thirty-one; three hundred and twelve; three hundred and twenty-one.

(4) Two thousand five hundred and seventy-two.

(5) Seventy-two thousand five hundred and seventy-two.

(6) Five hundred and seventy-two thousand five hundred and seventy-two.

(7) Ten thousand nine hundred and ten.

(8) Nine hundred and ten thousand nine hundred and ten.

(9) One hundred and nine thousand nine hundred and one.

(10) One hundred and ninety thousand and ninety-one.

(11) Nine hundred and one thousand and nineteen.

(12) One hundred and fourteen millions, one hundred and forty-one thousand four hundred and eleven.

(13) Four hundred and six millions, six hundred and four thousand four hundred and sixty.

(14) Six hundred and forty millions, forty-six thousand and sixty-four.

(15) Seven millions, seventy thousand seven hundred.

(16) Seven hundred millions, seven thousand and seventy.

(17) Ten millions, one thousand one hundred.

(18) One hundred and one millions, eleven thousand one hundred and ten.

(19) Twelve billions, seventeen thousand and nine millions, and eighty-nine.

(20) Seven thousand five hundred and four trillions, sixty thousand millions, eight hundred thousand.

Roman Numerals.

I	1	One.	VI	6	Six.	XI	11	Eleven.
II	2	Two.	VII	7	Seven.	XII	12	Twelve.
III	3	Three.	VIII	8	Eight.	XIII	13	Thirteen.
IV	4	Four.	IX	9	Nine.	XIV	14	Fourteen.
V	5	Five.	X	10	Ten.	XV	15	Fifteen.

XVI	16	Sixteen.
XVII	17	Seventeen.
XVIII	18	Eighteen.
XIX	19	Nineteen.
XX	20	Twenty.
XXX	30	Thirty.
XL	40	Forty.
L	50	Fifty.
LX	60	Sixty.
LXX	70	Seventy.
LXXX	80	Eighty.
XC	90	Ninety.
C	100	One hundred.
CC	200	Two hundred.
CCC	300	Three hundred.
CCCC	400	Four hundred.
D	500	Five hundred.
DC	600	Six hundred.
DCC	700	Seven hundred.
DCCC	800	Eight hundred.
DCCCC	900	Nine hundred.
M	1000	One thousand.

MDCCCXXXV .... 1835 .... One thousand eight hundred and thirty-five.

NOTE. A less numerical letter standing before a greater, must be taken from it, as I before V or X, and X before L or C, &c.; thus IV. Four; IX. Nine; XL. Forty; XC. Ninety, &c. And a less numerical letter standing after a greater, is to be added to it; thus, VI. Six; XI. Eleven; LX. Sixty; CX. One hundred and ten.

All operations in Arithmetic are comprised under four *elementary* or *fundamental Rules:* viz. *Addition, Subtraction, Multiplication,* and *Division.*

ADDITION

TEACHES to find the *sum* of several numbers.

RULE. Place the numbers one under another, so that units may stand under units, tens under tens, &c.; add the units, set down the units in their sum, and *carry* the *tens as so many ones* to the next row; proceed thus to the last row, under which set down the whole amount.

PROOF. Begin at the top and add the figures downwards: if the *sum* is found the same as before, it is presumed to be right.

* (1)	(2)	(3)	(4)
275	1234	75245	271048
110	7098	37502	325476
473	3314	91474	107584
354	6732	32145	625608
271	2546	47258	754087
352	6709	21476	279736

* Say 2 and 1 are 3, and 4 are 7, and 3 are ten, and 5 are 15, set down 5 and carry 1; 1 and 5 are 6, and 7 are 13, and 5 are 18, and 7 are 25, and 1 are 26, and 7 are 33, set down 3 and carry 3; 3 and 3 are 6, and 2 are 8, and 3 are 11, and 4 are 15, and 1 are 16, and 2 are 18, set down 18: so the *sum* is 1835.

After practising a few examples, it will be better for the learner

(5)	(6)	(7)
590046	370416	781943
73921	2890	56820
400080	60872	1693748
4987	998	300486
19874	47523	920437500
201486	9836	78632109
9883	26627	9408175

(8) What is the *sum* of 43, 401, 9747, 3464, 2263, 314, 974?

(9) Add 246034, 298765, 47321, 58653, 64218, 5376, 9821, and 640 together.

(10) If A has £56. B £104. C £274. D £1390. E £7003. F £1500. and G £998.; how much is the whole amount of their money?

(11) How many days are in the twelve calendar months?

(12) Add 87929, 135594, 7964, 3621, 27123, 8345, 35921, 2374, 64223, 42354, 3560, and 152165 together.

(13) Add 6228, 27305, 7856, 287, 7664, 100, 1423, 25258, 528, 3135, and 838.

(14) How many days are there in the first six months of the year; how many in the last six; and how many in the whole?

(15) In the year 1832, how many days from the Epiphany or Twelfth-day [Jan. 6th] to the last day of July?

(16) In the common year, how many days from each Quarter-day to the next?—that is, from Lady-day to Midsummer-day, from thence to Michaelmas-day, from thence to Christmas-day, and from Christmas-day to the ensuing Lady-day?

(17) When will the lease of a farm expire, which was granted in the year 1799, for ninety-nine years?

(18) A person deceased left his widow in possession of £2500. His eldest son inherited property of the value of £11340. To his two other sons he bequeathed a thousand pounds each more than to his daughter; whose portion exceeded the property left to her mother by £500. A nephew and a niece had legacies of £525. each; a public charity £105.; and his four servants the same sum to be divided

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to add the figures without naming them. Thus, in adding the first column of the above example, say 2, 3, 7, 10, 15; set down 5 and carry 1, &c.

This method will tend both to quickness and precision.

amongst them. What was the aggregate amount of his property?

(19) Tell the *name* and *signification* of the *sign* put between the following numbers: and find what they are equal to, as the sign requires?

$$1724 + 649 + 17 + 5400 + 12 + 999.$$

(20) Required the *sum* of forty-nine thousand and sixteen; four thousand eight hundred and forty; eight millions, seven hundred, and seven thousand one hundred; nine hundred and ninety-nine; and eleven thousand one hundred and ten.

(21) When will a person born in 1819, attain the age of 45?

(22) Henry came of age 13 years before the birth of his cousin James. How old will Henry be when James is of age?

(23) Homer, the celebrated Greek poet, is supposed to have flourished 907 years previous to the commencement of the Christian era. Admitting this to be fact, how many years was it from Homer's time to the close of the 18th century; and how long to A.D. 1827?

SUBTRACTION

Teaches to take a less number from a greater, to find the remainder or *Difference*.

The number to be subtracted is the *Subtrahend*, and the other is called the *Minuend*.

Rule. Having placed the Subtrahend under the Minuend (in the same order as in Addition), begin at the units, and subtract each figure from that above it, setting down the remainder underneath. But when the lower figure is the greater, *borrow ten*; which add to the upper, and then subtract: set down the remainder, and carry *one* to the next figure of the subtrahend for the *ten that was borrowed*.

Proof. Add the *Difference* to the *Subtrahend*, and their sum will be the *Minuend*.

(1)	From 2714754 Take 1542725	(4)	271508300 72841699	(7)	1000000000 987654321
(2)	42087296 34096187	(5)	375021599 278104609	(8)	2746981340 1095681539
(3)	45270509 32761684	(6)	400087635 9184267	(9)	666740825 109348172

(10) From 123456789 subtract 98765432.

(11) From 31147680975 subtract 767380799.

(12) Subtract 641870035 from 1630054154.

(13) Required the *difference* between 240914 and 24091.

(14) How much does twenty-five thousand and four exceed sixteen thousand three hundred and ninety?

(15) If eighty-four thousand and forty-eight be deducted from half a million, what will remain?

(16) The annual income of Mr. Lemmington, senior, is twelve thousand five hundred and sixty pounds. Mr. Lemmington, junior, has an income of seven thousand eight hundred and eighteen pounds per annum. How much is the son's income less than his father's?

(17) George the Fourth, at his accession to the throne, in 1820, was in the 58th year of his age. In what year was he born, and how long had he reigned on the 29th of January, 1829, the anniversary of his accession?

(18) The sum of two numbers is 36570, and one of them is twenty thousand and twelve: what is the other?

(19) Thomas has 115 marbles in two bags. In the green bag there are 68: how many are there in the other?

(20) Two brothers, who were sailors in Admiral Lord Nelson's fleet, were born, the elder in 1767, and the younger in 1775. What was the difference of their ages, and how old was each when they fought in the battle of Trafalgar, in 1805?

(21) Henry Jenkins died in 1670, at the age of 169. How long prior to his death was the discovery of the continent of America by Columbus, in 1498?—Also, how many years have elapsed from his birth to 1827?

EXAMPLE. From 32906547 subtract 8210468.

32906547	Minuend.
8210468	Subtrahend.
24696079	Difference.
32906547	Proof

Say 8 from 7 I cannot; borrow 10, and 7 are 17, 8 from 17, 9 remain; set down 9 and carry 1.—1 and 6 are 7, 7 from 4 I cannot; borrow 10, and 4 are 14, 7 from 14, 7; set down 7 and carry 1—1 and 4 are 5, 5 from 5, nothing; set down [0] nought.—0 from 6, 6; set down 6.—1 from 0 I cannot; but 1 from 10, 9; set down 9 and carry 1. Proceed in like manner to the end.

When the pupil is initiated in the practice by working an example or two, he may simplify the work by omitting to express some of the particulars. Thus, in the preceding example, it will be sufficient merely to say, 8 from 17, 9; set down 9 and carry 1: 1 and 6 are 7, 7 from 14, 7; set down 7 and carry 1, &c.

(22) Borrowed at various times, £644, £957, £90, £1378, and £1293; and paid again the different sums of £763, £591, £1161, £1000, and £847.—What remains unpaid?

(23) Explain the *name* and *signification* of the *sign* used; and work the two following examples.

10874 — 9999 | 51170 — 50049

(24) John is 17 years younger than Thomas: how old will Thomas be when John is of age; and how old will John be when Thomas is 50?

MULTIPLICATION

Teaches to repeat a given number as many times as there are units in another given number.

The number to be multiplied is called the *Multiplicand;* that by which we multiply is the *Multiplier;* and the number produced by multiplying is the *Product.*

Rule. When the multiplier is not more than 12, multiply the units' figure of the multiplicand, *set down the units* of the product, *reserving the tens;* multiply the next figure, to the product of which *carry the tens reserved:* proceed thus till the whole is multiplied, and set down the last product in full. *

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

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* **Example.** Multiply 713097 by 4.

```
713097
     4
-------
2852388
```

Say 4 times 7 are 28, set down 8 and carry 2; 4 times 9 are 36, and 2 are 38, set down 8 and carry 3; 4 times 0 [nought] and 3 are 3, set down 3; 4 times 3 are 12, set down 2 and carry 1; 4 times 1 are 4, and 1 are 5, set down 5; 4 times 7 are 28, set down 28.

(1) Multiply 25104736 by 2.
(2) Multiply 52471021 by 3.
(3) Multiply 7925437521 by 4.
(4) Multiply 27104107 by 5.
(5) Multiply 23104759 by 6.
(6) Multiply 7092516 by 7.
(7) Multiply 3725104 by 8.
(8) Multiply 4215466 by 9.
(9) Multiply 2701057 by 10.*
(10) Multiply 31040171 by 11.
(11) Multiply 73998063 by 12.

(12) Multiply 780149326 by 3, 4, 5, 6, 7, 8, 9, and 10.
(13) Multiply 123456789 by 4, 5, 6, 7, 8, and 9.
(14) Multiply 987654321 by 9, 10, 11, and 12.

When the multiplier is between 12 and 20, multiply by the units' figure in the multiplier, adding to each product the last figure multiplied.†

(15) 5710592 × 13.
(16) 5107252 × 14.
(17) 7653210 × 15.
(18) 2057165 × 16.
(19) 6251721 × 17.
(20) 9215324 × 18.
(21) 2571341 × 19.

When the multiplier consists of several figures, multiply by each of them separately, observing to put the first figure of every product under that figure you multiply by. Add the several products together, and their sum will be the total product.‡

PROOF. Make the former multiplicand the multiplier, and the multiplier the multiplicand; and if the work is right, the products of both operations will correspond. *Otherwise.* A presumptive or probable proof (not a positive one) may be obtained thus: Add together the figures in *each factor*, casting out or rejecting the *nines* in the sums as you proceed; set down the remainders on each side of a *cross*, multiply them together, and set down the *excess* above the nines

---

* To multiply by 10, annex a cipher to the multiplicand, for the product. To multiply by 100, annex two ciphers, &c.

EXAMPLES.

† Multiply 96048 by 15.

```
 96048
    15
-------
1440720
```

Say 5 times 8 are 40, set down 0 and carry 4; 5 times 4 are 20, and 4 are 24, and 8 are 32, set down 2 and carry 3; 5 times 0 and 3 are 3, and 4 are 7, set down 7; 5 times 6 are 30, set down 0 and carry 3; 5 times 9 are 45 and 3 are 48, and 6 are 54, set down 4 and carry 5; 5 and 9 are 14, set down 14.

‡ Multiply 76047 by 219.

```
  76047
    219                           Proof.
  -----
 684423 Product by   9.             0
304188     do.   by  40.           6×6
152094     do.   by 200.            0
-------
8935703 Total product.
```

in their product at the top of the cross. Then cast out the nines from the *product*, and place the *excess* below the cross. If these two correspond, the work is *probably* right; if not, it is *certainly* wrong.

(22) 271041071 × 5147.
(23) 62310047 × 1668.
(24) 170925164 × 7419.
(25) 9500985742 × 61879.
(26) 1701495868567 × 4768756.

When ciphers are intermixed with the significant figures in the multiplier, they may be omitted; but great care must be taken to place the first figure of the next product under the figure you multiply by.*

Ciphers on the right of the multiplier or multiplicand (if omitted in the work) must be placed in the total product.†

(27) 571204 × 27009.
(28) 7561240325 × 57002.
(29) 562710934 × 590030.
(30) 1379500 × 3400.
(31) 7271000 × 52600.
(32) 74837000 × 975000.

A number produced from multiplying two numbers together, is called a *compósite number;* and the two numbers producing it are called the *factors*, or *compōnent parts.* When the multiplier is a *compósite number*, you may multiply by one of the *factors;* and that product multiplied by the *other* will give the total product.‡

(33) 771039 × 35.
(34) 921563 × 32.
(35) 715241 × 56.
(36) 679998 × 132.
(37) 7984956 × 144.
(38) 8760472 × 999.§
(39) 7039654 × 99999.

(40) A boy can point 16000 pins in an hour. How many can five boys do in six days, supposing them to work 10 clear hours in a day.

EXAMPLES.

* Multiply 31864 by 7008.

```
  31864
   7008        Proof.
  ------          6
 254912         4 × 6
223048            6
---------
223302912
```

† Multiply 63850 by 5200.

```
 63850
  5200         Proof.
 -----            1
 12770          4 × 7
31925             1
---------
332020000
```

‡ Multiply 63175 by 45.

```
  63175
      5 × 9 = 45
 ------
 315875
      9
 -------
2842875
```

§ For an abridged method of multiplying by a series of *nines*, see the Key.

(41) If a person walks upon an average 7 miles a day, how many miles will he travel in 42 years, reckoning 365 days to a year?

(42) Multiply the *sum* of 365, 9081, and 22048, by the *difference* between 9081 and 22048.

(43) Required the *continued product* of 112, 45, 17, and 99.

NOTE. Multiply all the numbers one into another.

DIVISION

TEACHES to find how often one number is contained in another: or to divide a number into any equal parts required.

The number to be divided is called the *Dividend;* that by which we divide is the *Divisor;* and the number obtained by dividing is the *Quotient;* which shows how many times the divisor is contained in the dividend. When it is not contained an exact number of times, there is a part of the dividend left, which is called the *Remainder.*

RULE. When the divisor is not more than 12, find how often it is contained in the first figure (or two figures) of the dividend; set down the quotient underneath, and carry the overplus (if any) to the next in the dividend, *as so many tens;* find how often the divisor is contained therein, set it down, and continue in the same manner to the end.

When the divisor exceeds 12, find the number of times it is contained in a sufficient part of the dividend, which may be called a *dividual;* place the quotient figure on the right, multiply the divisor by it, subtract the product from the dividual, and to the remainder bring down the next figure of the dividend, which will form a new dividual: proceed with this as before, and so on, till all the figures are brought down.

PROOF. Multiply the divisor and quotient together, adding the remainder (if any), and the product will be the same as the dividend.

(1) Divide 725107 by 2.* | (2) Divide 7210472 by 3.

---

* EXAMPLE. Divide 7328105 by 4.

```
Divisor 4)7328105 Dividend.
Quotient  1832026 — 1 Rem.
                4
          7328105 Proof.
```

Say the fours in 7, once and 3 over; the fours in 33, 8 times 4 are 32 and 1 over; the fours in 12, 3 times; the fours in 8, twice; the fours in 1, 0 and 1 over; the fours in 10, twice 4 are 8, and 2 over; the fours in 25, six fours are 24, and 1 over.

(3) Divide 7210416 by 4.
(4) Divide 7203287 by 5.
(5) Divide 5231037 by 6.
(6) Divide 2532701 by 7.
(7) Divide 2547325 by 8.
(8) Divide 25047306 by 9.
(9) Divide 70312645 by 10.
(10) Divide 12804763 by 11.
(11) Divide 79043260 by 12.
(12) Divide 37000421 by 3, 5, 7, and 9.
(13) Divide 11111111 by 6, 9, 11, and 12.

(14) 7210473 ÷ 37.*
(15) 42749467 ÷ 347.
(16) 734097143 ÷ 5743.†
(17) 1610478407 ÷ 54716.
(18) 4973401891 ÷ 510834.
(19) 51704567874 ÷ 4765043.
(20) 17453798946123741 ÷ 31479461.
(21) 25473221 ÷ 27100.‡
(22) 725347216 ÷ 572100.
(23) 752473729 ÷ 373000.
(24) 6325104997 ÷ 215000.

When the divisor is a *compósite number*, you may divide the dividend by one of the *compōnent parts*, and that quotient by the *other;* which will give the quotient required. But the *true remainder* must be found by the following.

Rule. Multiply the second remainder by the first divisor: to that product add the first remainder, which will give the *true one.*

(25) 3210473 ÷ 27.§
(26) 7210473 ÷ 35.
(27) 6251043 ÷ 42.
(28) 5761034 ÷ 54.

---

* Example. Divide 40855 by 29.

```
                 Dividend.
       Divisor 29) 40855 (1408 Quotient.
                   29      29
                   ---    -----
                   118    12672
                   116    2816
                   ---    -----
                    255     23 Remainder.
                    232   -----
                    ---   40855 Proof.
                     23
```

† When the divisor is large, the quotient figures are most easily found by *trials of the first figure* (or *two*) in the *leading figures* of the dividend.

‡ Ciphers at the right of the diviser may be cut off, and as many figures from the right of the dividend; but these must be annexed to the remainder at last.

§ Example. Divide 314659 by 21.

```
21 = 7 × 3) 314659
          7) 104886—1 }
              14983—5 } = 5 × 3 + 1 = 16 rem.
```

A number may be divided by 10, 100, 1000, &c. by merely cutting off one, two, three, &c. figures on the right: the other figures are the quotient, those cut off are the remainder.

Thus $76390 \div 10 = 7639$; $238457 \div 10 = 23845$ and 7 rem. And $4598653 \div 1000 = 4598$ and 653 rem.

(29) $65941089 \div 10$.	(31) $18043329 \div 10000$.
(30) $7208465 \div 100$.	(32) $7406572 \div 1200$

(33) What is the difference between the twelfth part of 107724, and the 23d part of 346610?

(34) If a ship bound to Jamaica set sail from Liverpool on the 25th of January, 1828, and arrived at that island on the 8th of March, what was the velocity of her sailing per day and per hour; the distance being 4558 miles?

NOTE. This is the *direct* distance. The circuitous course of the ship would be considerably more.

(35) The period of Jupiter's revolution in his orbit round the sun, which is the year of that planet, is 4330 of our days. How many of our years, reckoning 365 days to the year, are equal to five years of Jupiter?

(36) I would plant 2072 elms in 14 rows, the trees in each row 17 feet asunder: what length will the grove be?

(37) If a chest of oranges, 1292 in number, be distributed, one moiety among 19 boys, the other among 17 girls; how many will fall to the share of each?

(38) The circumference of the earth's orbit, or annual path round the sun, is about 596440000 miles. Supposing the year to be exactly 365¼ days, or 8766 hours, how many miles in an hour, and how many in a minute, are we carried by this motion?

(39) Required the sum, the difference, the product, and the quotient, of 3679 and 283; and also the quotient of the product divided by the sum.

(40) The sum of two numbers is 4290; the less number is 143: what is their difference, product, and quotient; and the quotient of the product divided by the difference?

(41) The product of a certain number multiplied by 694, when 320 are added, is equal to 500000: what is that number?

(42) Allowing the earth to revolve on its axis in exactly 24 hours, and the circumference at the equator to be 24864 miles; at what rate per hour and per minute are the inhabitants of that part carried round by the revolution? Also, at what rate are the inhabitants of London carried round, the circumference in that latitude being 15480 miles?

ARITHMETICAL AND COMMERCIAL TABLES.

STERLING MONEY.

4 farthings (*qrs.*) make 1 penny, *d.*
12 pence .... 1 shilling, *s.*
5 shillings .... 1 crown, *cr.*
20 shillings .... 1 pound or sovereign, £.

¼*d.* denotes a farthing, ½*d.* a halfpenny, and ¾*d.* three farthings.

Qrs. 4 = 1 penny.
48 = 12 = 1 shilling.
240 = 60 = 5 = 1 crown.
960 = 240 = 20 = 4 = 1 pound.

OBSOLETE COINS.

A guinea (weight 5 *dwts.* 9½ *grs.*) value 21*s.* A moidore, 27*s.* A pistole, 17*s.* A mark, 13*s.* 4*d.* An angel, 10*s.* A noble, 6*s.* 8*d.* A tester, 6*d.* A groat, 4*d.*

Notes. Gold is considered the standard metal; and there is no alteration in the *new coin*, either in fineness or weight, from that of former coinages; 21 sovereigns being equal in weight to 20 guineas. 1869 sovereigns weigh exactly 40 lbs. troy. A sovereign is therefore a little more than 5 *dwts.* 3¼ *grs.* (5 *dwts.* 3·274 *grs.*); and a half sovereign rather exceeds 2 *dwts.* 13½ *grs.* (2 *dwts.* 13·637 *grs.*) The *new silver coin* is of the same fineness as that of former coinages; but 1lb. of silver is now coined into 66*s.* instead of 62*s.* as it was formerly, so that one shilling now weighs 3 *dwts.* 15$\frac{3}{11}$ *grs.* and other silver pieces in proportion.

The mint value of gold is £3..17..10½. per ounce, and of silver 5*s.* 6*d.*

The standard for gold coin is 22 parts (commonly called *carats*) of fine gold, and 2 parts (or *carats*) of copper, melted together. For silver coin, 11 *oz.* 2 *dwts.* of fine silver, alloyed with 18 *dwts.* of copper.

MONEY TABLE.

Farthings.		Farthings.		Pence.		Pence.			Pence.			Shillings.		
qrs.	*d.*	*qrs.*	*d.*	*d.*	*s.*	*d.*	*s.*	*d.*	*d.*	*s.*	*d.*	*s.*	£	*s.*
4 are	1	32 are	8	36 are	3	20 are	1	8	160 are	13	4	80 are	4	0
6 ..	1½	34 ..	8½	48 ..	4	30 ..	2	6	170 ..	14	2	90 ..	4	10
8 ..	2	36 ..	9	60 ..	5	40 ..	3	4	180 ..	15	0	100 ..	5	0
10 ..	2½	38 ..	9½	72 ..	6	50 ..	4	2	190 ..	15	10	110 ..	5	10
12 ..	3	40 ..	10	84 ..	7	60 ..	5	0	200 ..	16	8	120 ..	6	0
14 ..	3½	42 ..	10½	96 ..	8	70 ..	5	10				130 ..	6	10
16 ..	4	44 ..	11	108 ..	9	80 ..	6	8	Shillings.			140 ..	7	0
18 ..	4½	46 ..	11½	120 ..	10	90 ..	7	6	*s.*	£	*s.*	150 ..	7	10
20 ..	5	48 ..	1*s.*	132 ..	11	100 ..	8	4	20 are	1	0	160 ..	8	0
22 ..	5½			144 ..	12	110 ..	9	2	30 ..	1	10	170 ..	8	10
24 ..	6	Pence.		156 ..	13	120 ..	10	0	40 ..	2	0	180 ..	9	0
26 ..	6½	*d.*	*s.*	168 ..	14	130 ..	10	10	50 ..	2	10	190 ..	9	10
28 ..	7	12 are	1	180 ..	15	140 ..	11	8	60 ..	3	0	200 ..	10	0
30 ..	7½	24 ..	2	192 ..	16	150 ..	12	6	70 ..	3	10	210 ..	10	10

NOTE. When the units' figure is cut off from any number of shillings, half the remaining figures will be the pounds. Thus, 256s.= £12. 16s. because half of 25 = 12; and the *one* over prefixed to the 6, gives 16s.

WEIGHTS AND MEASURES.

TROY WEIGHT.

24 grains (*gr.*) make	1 pennyweight,	*dwt.*
20 pennyweights	1 ounce.........	*oz.*
12 ounces	1 pound	lb.

Grains. 24 =	1 pennyweight.		
480 =	20 =	1 ounce.	
5760 =	240 =	12 =	1 pound.

Gold, silver, and gems, are weighed by this weight.

APOTHECARIES' WEIGHT.

20 grains (*gr.*) make	1 scruple	℈
3 scruples...........	1 dram	ʒ
8 drams	1 ounce.........	℥
12 ounces	1 pound.........	lb.

Grains. 20 =	1 scruple.			
60 =	3 =	1 dram.		
480 =	24 =	8 =	1 ounce.	
5760 =	288 =	96 =	12 =	1 pound.

This is used only in the mixing of medicines.
These are the same grain, ounce, and pound, as those in Troy Weight.

AVOIRDUPOIS WEIGHT.

16 drams (*dr.*) make	1 ounce	*oz.*
16 ounces	1 pound......	*lb.*
14 pounds	1 stone	*st.*
28 pounds, or 2 stones	1 quarter ...	*qr.*
4 quarters, or 8 stones, or 112 lb..	1 hundred ...	*cwt.*
20 hundreds	1 ton	*t.*

Drams. 16 =	1 ounce.					
256 =	16 =	1 pound.				
3584 =	224 =	14 =	1 stone.			
7168 =	448 =	28 =	2 =	1 quarter.		
28672 =	1792 =	112 =	8 =	4 =	1 cwt.	
573440 =	35840 =	2240 =	160 =	80 =	20 =	1 ton.

By this weight nearly all the common necessaries of life are weighed. A truss of hay = 56 lb. and one of straw = 36 lb. A load is 36 trusses. A peck loaf weighs 17 lb. 6 oz. 1 dr. In the metropolis, 8 lb. are a stone of meat. A fother of lead is $19\frac{1}{2}$ cwt. In some districts, goods of various descriptions (as cheese, coal, &c.) are sold by the *long cwt* or 120 lb.

WOOL.

When wool is purchased from the grower, the legal stone of 14 ℔. and the tod of 28 ℔. are used. But in the dealings between woolstaplers and manufacturers,

15 pounds are	1 stone.
2 stones, or 30 ℔.	1 tod.
8 tods, or 240 ℔.	1 pack or sack.

COMPARISON OF WEIGHTS.

A grain is the elementary or standard weight.

1 ounce avoirdupois is	$437\frac{1}{2}$ grains.
1 ounce troy	480
1 pound troy	5760
1 pound avoirdupois	7000

175 pounds troy = 144 pounds avoirdupois.

175 ounces troy = 192 ounces avoirdupois.

We may, therefore, reduce ℔s. Troy into Avoirdupois, by multiplying them by 144, and dividing by 175, &c.

LINEAL, OR LONG MEASURE.

12 inches (*in.*) make	1 foot	*ft.*
3 feet, or 36 inches............	1 yard	*yd.*
2 yards, or 6 feet	1 fathom	*fa.*
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	1 pole, rod, or perch,	*p.*
4 poles, or 22 yards	1 land-chain*	*ch.*
40 poles, or 10 ch. or 220 yds.	1 furlong	*fur.*
8 furlongs, or 1760 yards ...	1 mile	*m.*
3 miles	1 league	*l.*

Barley-corns.						
3 =	1 inch					
36 =	12 =	1 foot.				
108 =	36 =	3 =	1 yard.			
594 =	198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 pole.		
23760 =	7920 =	660 =	220 =	40 =	1 furlong.	
190080 =	63360 =	5280 =	1760 =	320 =	8 =	1 mile.

NOTE. It is commonly supposed that the English inch was originally taken from three grains of barley, selected from the middle of the ear, and well dried.

A twelfth part of an inch is called a *line*.

4 inches are a hand, used in measuring the height of horses. 5 feet are a pace. A cubit = $1\frac{1}{2}$ feet nearly.

This measure determines the length of lines. A line has the dimension of length only, without breadth or thickness.

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* The chain consists of 100 links, each link being = 7·92 inches.

CLOTH MEASURE.

$2\frac{1}{4}$ inches (*in.*) make	1 nail,.........	*n.*
4 nails, or 9 inches	1 quarter, ...	*qr.*
4 quarters	1 yard, ...	*yd.*
5 quarters	1 English ell,	*E. e.*

A Flemish ell is 3 qrs. A French ell 6 qrs.

Used for all drapery goods.

SUPERFICIAL, OR SQUARE MEASURE.

144 square inches (*sq. in.*) make ..	1 square foot,.	*sq. ft.*
9 square feet	1 square yard,	*sq. yd.*
$30\frac{1}{4}$ sq. yards, or $272\frac{1}{4}$ sq. feet...	1 sq. rod, pole, or perch.	

Also, in the measure of land,

40 perches make	1 rood, ...	*r.*
4 roods, or 4840 yards	1 acre, ...	*a.*
10,000 square links	1 square chain,	*sq. c.*
10 sq. chains, or 100,000 links	1 acre, ...	*a.*
640 acres	1 square mile,	*sq. m.*

Inches. 144 =	1 foot.				
1296 =	9 =	1 yard.			
39204 =	$272\frac{1}{4}$ =	$30\frac{1}{4}$ =	1 pole.		
1568160 =	10890 =	1210 =	40 =	1 rood.	
6272640 =	43560 =	4840 =	160 =	4 =	1 acre.

Roofing, flooring, &c. are commonly charged by the *Square*, containing 100 square feet.

By this measure is expressed the area of any superficies, or surface. A superficies has measurable length and breadth.

CUBIC, OR SOLID MEASURE.

1728 cubic inches (*in.*) make.........	1 cubic foot.
27 cubic feet	1 cubic yard.*
40 feet of round timber, or / 50 feet of hewn timber	1 ton, or load.
42 feet	1 ton of shipping.

A cord of wood is 4 feet broad, 4 feet deep, and 8 feet long, being 128 cubic feet.

A stack of wood is 3 feet broad, 3 feet deep, and 12 feet long, being 108 cubic feet.

This determines the solid contents of bodies. A solid has three dimensions, length, breadth, and thickness.

* A solid yard of earth is called a load.

IMPERIAL MEASURE.

This is the standard now established by Act of Parliament, as a *general measure of capacity* for liquid and dry articles.

2 pints (*pt.*) make	1 quart, ...	*qt.*
4 quarts	1 gallon.	*gal.*

The imperial or standard gallon must contain 10 lbs. Avoirdupois weight of pure water, at the temperature of 62° of Fahrenheit's thermometer. This quantity measures $277\frac{1}{4}$* cubic inches; being about *one-fifth greater* than the old wine measure, *one thirty-second greater* than the old dry measure, and *one-sixtieth less* than the old ale measure.

In DRY MEASURE,

2 gallons (*gal.*) make	1 peck, ...	*pk.*
4 pecks	1 bushel, ...	*b.*
8 bushels	1 quarter, ...	*qr.*

Corn to be stricken off the measure with a round stick or roller.

Obsolete. A coom = 4 bushels; a chaldron = 4 quarters; a wey = 5 quarters; a last = 2 weys.

Solid inches.	$277\frac{1}{4}$ =	1 gallon.			
	$554\frac{1}{2}$ =	2 =	1 peck.		
	2218 =	8 =	4 =	1 bushel.	
	17744 =	64 =	32 =	8 =	1 quarter.

OF COALS,

3 bushels make	1 sack.
12 sacks, or 36 bushels ...	1 chaldron.
21 chaldrons	1 score.

All the measures used for heaped goods are to be of cylindrical form; the diameter being at least double the depth. The height of the raised cone to be equal to three-fourths of the depth of the measure.

The old dry gallon contained $268\frac{4}{5}$ cubic inches.

NOTE. The bushel, for measuring heaped goods, must be 17·8 inches in diameter, and 8·904 inches deep; or if made 18 inches in diameter, the depth will be 8·717 inches. The cone to be raised 6·6 inches in height.

In WINE AND SPIRIT MEASURE, the old gallon contained 231 cubic inches.

63 gallons were	a hogshead, *hhd.*
2 hogsheads, or 126 gallons,	a pipe or butt.
4 hogsheads, or 252 gallons,	a ton.

* More accurately, 277·274 cubic inches.

Some other denominations have been long obsolete; as, an anker (10 gallons); a runlet (18 gallons); a tierce (42 gallons); a puncheon (84 gallons). But casks of most descriptions are generally charged according to the number of gallons contained.

Solid inches. $34\frac{21}{32}$ =	1 pint.						
$69\frac{5}{16}$ =	2 =	1 quart.					
$277\frac{1}{4}$ =	8 =	4 =	1 gallon.				
$17466\frac{3}{4}$ =	504 =	252 =	63 =	1 hogshead.			
$34933\frac{1}{2}$ =	1008 =	504 =	126 =	2 =	1 pipe.		
69867 =	2016 =	1008 =	252 =	4 =	2 =	1 tun.	

In ALE, BEER, or PORTER MEASURE, the old gallon contained 282 cubic inches; and measures of the following denominations have been in use:

A firkin, containing	9 gallons.
A kilderkin	18 gallons.
A barrel	36 gallons.
A hogshead	54 gallons.
A butt	108 gallons.

Cubic inches. $34\frac{21}{32}$ =	1 pint.							
$69\frac{5}{16}$ =	2 =	1 quart.						
$277\frac{1}{4}$ =	8 =	4 =	1 gallon.					
$2495\frac{1}{4}$ =	72 =	36 =	9 =	1 firkin.				
$4990\frac{1}{2}$ =	144 =	72 =	18 =	2 =	1 kilderkin.			
9981 =	288 =	144 =	36 =	4 =	2 =	1 barrel.		
$14971\frac{1}{2}$ =	432 =	216 =	54 =	6 =	3 =	$1\frac{1}{2}$ =	1 hogshead.	
29943 =	864 =	432 =	108 =	12 =	6 =	3 =	2 =	1 butt.

*RULES FOR CHANGING OLD MEASURES TO IMPERIAL.

ALE. Multiply by 60, and divide by 59, or add $\frac{1}{59}$ part. (True, within $\frac{1}{10000}$ part of the whole.)

Or, multiply by 179, and divide by 176. (True, within $\frac{1}{1000000}$ part.)

DRY. Multiply by 32, and divide by 33, or deduct $\frac{1}{33}$ part. (Error, less than $\frac{1}{3700}$ part.

WINE. Multiply by 5, and divide by 6, or deduct $\frac{1}{6}$ part. (Error, less than $\frac{1}{4000}$ part.)

Or, multiply by 624, and divide by 749. (Error, less than $\frac{1}{8000000}$ part.)

*RULES FOR CHANGING IMPERIAL TO OLD MEASURES.

ALE. Multiply by 59, and divide by 60, or deduct $\frac{1}{60}$ part.

Or, multiply by 176, and divide by 179.

DRY. Multiply by 33, and divide by 32, or add $\frac{1}{32}$ part.—That is, add one peck in every quarter, one quart in every bushel, or half a pint in every peck.

WINE. Multiply by 6, and divide by 5, or add $\frac{1}{5}$ part.

Otherwise, multiply by 749, and divide by 624.

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* Examples applying to these Rules will be found in the Miscellaneous Questions in the latter part of the book.

TIME.

60 seconds (*sec.*) make	1 minute, ...	*min.*
60 minutes	1 hour, ...	*hr.*
24 hours	1 day,*	*d.*
7 days	1 week, ...	*wk.*
52 weeks, 1 day, 6 hours, or 365 days, 6 hours...............	1 Julian year,	*yr.*
365 days, 5 hours, 48 min. 51½ seconds ..	The Solar year.†	
100 years	1 century.	

Seconds. 60 = 1 minute.
3600 = 60 = 1 hour.
86400 = 1440 = 24 = 1 day.
604800 = 10080 = 168 = 7 = 1 week.
31557600 = 525960 = 8766 = 365 *d.* 6 *h.* = 52 *w.* 1 *d.* 6 *h.* = 1 Julian year.
31556931 = 525948 = 8765 = 365 *d.* 5 *h.* 48 *m.* 51½″ = 1 Solar year.

The year is divided into 12 Calendar months; January, February, March, April, May, June, July, August, September, October, November, December.

The days are thirty in September,
In April, June, and in November;
Twenty-eight in February alone,
And in each other thirty-one:
But every leap-year we assign
To February twenty-nine.

The *leap-years* are those which can be *exactly* divided by 4; as, 1824, 1828, &c. Hence it appears that the year is accounted 365 days, for *three years together;* and 366 days in the *fourth:* the average being 365¼ days. (*The Julian year.*)

Four weeks are frequently called a *month;* but in this sense it is better to avoid the term.

NOTE. In all questions in this book, where the proposed or required time consists of years, months, weeks, &c. allow 4 weeks to a month, and 13 months to a year.

GEOMETRY.

60 seconds (″), make......	1 minute, ′
60 minutes	1 degree, °
360 degrees	1 circle.

Many highly important calculations in the mathematical sciences are founded on this division of the circle.

In Astronomy, the great circle of the *ecliptic* (or of the *zodiac*) is divided into 12 *signs*, each 30°.

* A day is the time in which the earth revolves once upon its axis: by law and custom it is reckoned from midnight to midnight; but the astronomical day begins at noon.

† The Solar, or true year, is that portion of time in which the earth makes one entire revolution round the sun.

In *Geography*, a degree of latitude, or of longitude on the equator, measures nearly $69\frac{1}{10}$ British miles. But a minute of a degree is called a geographical mile.

ARTICLES SOLD BY TALE.

12 articles of any kind, are	1 dozen.	24 sheets of paper	1 quire.
12 dozen	1 gross.	20 quires	1 ream.
12 gross	1 great gross.	2 reams	1 bundle.
20 articles	1 score.		

DEFINITIONS.

1. A NUMBER is called *ábstract*, when it is considered *simply*, or without reference to any subject; as seven, a thousand, &c.

2. When a number is applied to denote so many of a particular subject, it is a *concréte* number; as seven pounds, a thousand yards, &c.

3. A *denomination* is a name of any particular distinctive part of money, weight, or measure; as penny, pound, yard, &c.

4. The association of a concrete number with its subject, forms a *quantity*.

5. A *simple quantity* has only *one denomination*; as seven pounds.

6. A *compound quantity* consists of *more denominations* than one; as seven pounds five shillings.

REDUCTION

Is the method of changing quantities of one denomination into another denomination, retaining the same value.

RULE. Consider how many of the *less name* make *one* of the *greater*; and *multiply* by that number to reduce the *greater name to the less*, or *divide* by it to reduce the *less name to the greater*.

EXAMPLES.

Reduce £8..8..6½ into farthings.

```
£  s. d.
8  8  6½
20
168 s.
12
2022 d.
4
8090 qrs. Ans.
```

The £8 being multiplied by 20, and the 8s. added, make 168s.; these being multipled by 12, and the 6d. added, make 2022d.; which being multiplied by 4 and the 2 farthings added, make in the whole 8090 *farthings*.

(1) In £12 how many shillings, pence, and farthings? *Ans.* 240*s.* 2880*d.* 11520 *qrs.*

(2) In 311520 farthings, how many pounds? *Ans.* £324..10.

(3) Change 21 guineas into farthings. *Ans.* 21168 *qrs.*

(4) In £17..5..3¼, how many farthings? *Ans.* 16573 *qrs.*

(5) In £25..14..1, how many pence? *Ans.* 6169*d.*

(6) Reduce 17940 pence to crowns. *Ans.* 299 *crowns.*

(7) In 15 crowns, how many shillings and sixpences? *Ans.* 75*s.* 150 *sixpences.*

(8) Change 57 half-crowns into threepences, pence, and farthings. *Ans.* 570 *threepences,* 1710*d.* 6840 *farthings.*

(9) How many half-crowns, and how many sixpences, are equivalent to £25..17..6? *Ans.* 207 *half-cr.* 1035 *sixpences.*

(10) Convert £17..11..9 into threepences. *Ans.* 1407 *threep.*

(11) Change £10..13..10½ into halfpence. *Ans.* 5133.

(12) In 52 crowns, as many half-crowns, shillings, and pence, how many farthings? *Ans.* 21424 *far.*

(13) Convert 17380 farthings into pounds. *Ans.* £18..2..1.

(14) In 21424 farthings, how many crowns, half-crowns, shillings, and pence, of each an equal number? *Ans.* 52.

(15) Reduce 60 guineas to shillings, crowns, and pounds. *Ans.* 1260*s.* 252 *crowns,* £63.

(16) Reduce 76 moidores † into pounds. *Ans.* £102..12.

(17) How many shillings, half-crowns, and crowns, an equal number of each, are there in £556? *Ans.* 1308 *of each, and* 2*s. over.*

(18) In 1308 crowns, as many half-crowns, and as many shillings, how many pounds? *Ans.* £555..18.

(19) Seven men brought £15..10 each into the mint, to be exchanged for guineas: how many would they have? *Ans.* 103 *guineas and* 7*s. over.*

(20) In 525 American dollars, at 4*s.* 6*d.* each, how many pounds sterling? *Ans.* £118..2..6.

Converse to the preceding EXAMPLE.

In 8090 farthings, how many pounds?

4) 8090 *qrs.*	Dividing the farthings by 4, we obtain 2022*d.* and 2 over, which are *farthings*, because the remainder is a part of the dividend. Divide 2022 by 12, and we obtain 168*s.* and 6*d.* over: these shillings divided by 20, give £8..8*s.* so that the answer is £8..8..6½.
12) 2022½ *d.*	
2|0) 16|8*s.* 6½*d.*	
Ans. £8..8..6½.	

† 27 shillings. The moidore is current in Portugal, but not in England.

WEIGHTS AND MEASURES.

TROY WEIGHT.

(21) In 27 ounces of gold, how many grains? *Ans.* 12960.

(22) Reduce 3 *lb.* 10 *oz.* 7 *dwts.* 5 *gr.* to grains? *Ans.* 22253.

(23) In 8 ingots of silver, each ingot weighing 7 *lb.* 4 *oz.* 17 *dwts.* 15 *gr.* how many grains? *Ans.* 341304 *gr.*

(24) How many ingots weighing 7 *lb.* 4 *oz.* 17 *dwts.* 15 *gr.* each are there in 341304 grains? *Ans.* 8 *ingots.*

APOTHECARIES' WEIGHT.

(25) In 27 ℔. 7 ℥. 2 ʒ. 1 ℈. 2 *gr.* how many grains?
Ans. 159022 *grains.*

(26) In a compound of 9 ℥. 4 ʒ. 1 ℈. how many pills of 5 *grains* each? *Ans.* 916 *pills.*

AVOIRDUPOIS WEIGHT.

(27) In 14769 ounces, how many cwt.?
Ans. 8 *cwt.* 0 *qr.* 27 *lb.* 1 *oz.*

(28) In 34 *tons.* 17 *cwt.* 1 *qr.* 19 *lb.* how many pounds?
Ans. 78111 *lb.*

(29) In 9 *cwt.* 2 *qrs.* 14 *lb.* of indigo, how many half stones, and how many pounds? *Ans.* 154 *half stones*, 1078 *lb.*

(30) How many stones and pounds are there in 27 hogsheads of tobacco, each weighing neat 8¾ *cwt.*?
Ans. 1890 *stones*, 26460 *lb.*

(31) Bought 32 bags of hops, each bag 2 *cwt.* 1 *qr.* 14 *lb.* and another of 150 *lb.* how many cwt. are there in the whole?
Ans. 77 *cwt.* 1 *qr.* 10 *lb.*

(32) In 27 *cwt.* of raisins, how many parcels of 18 *lb.* each?
Ans. 168.

CLOTH MEASURE.

(33) In 27 yards, how many nails? *Ans.* 432.

(34) In 75 English ells, how many yards? *Ans.* 93 *yds.* 3 *qrs.*

(35) In 24 pieces, each containing 32 Flemish ells, how many English ells? *Ans.* 460 *English ells*, 4 *qrs.*

(36) In 17 pieces of cloth, each 27 Flemish ells, how many yards? *Ans.* 344 *yards*, 1 *qr.*

(37) In 911¼ yards, how many English ells? *Ans.* 729

(38) In 12 bales of cloth, each containing 25 pieces, of 15 English ells, how many yards? *Ans.* 5625.

LONG MEASURE.

(39) In 57½ miles, how many furlongs and poles?
Ans. 460 *furlongs*, 18400 *poles.*

(40) In 7 miles, how many feet and inches?

Ans. 36960 *feet,* 443520 *inches.*

(41) In 72 leagues, how many yards? *Ans.* 380160 *yards.*

(42) If the distance from London to Bawtry be accounted 150 miles, what is the number of leagues, and also the number of yards, feet, and inches?

Ans. 50 *leagues,* 264000 *yards,* 792000 *feet,* 9504000 *inches.*

(43) How often will the wheel of a coach, that is 17 feet in circumference, turn in 100 miles? *Ans.* 31058$\frac{14}{17}$ *times round.*

(44) How many barleycorns will reach round the globe, the circumference being 360 degrees, supposing that each degree were 69 miles and a half? *Ans.* 4755801600.

See table of Geometry, page 30.

LAND MEASURE.

(45) In 27 *a.* 3 *r.* 19 *p.* how many perches? *Ans.* 4459.

(46) A person having a piece of ground, containing 37 acres, 1 perch, intends to dispose of 15 acres. How many perches will he have left? *Ans.* 3521 *perches.*

(47) There are 4 fields to be divided into shares of 75 perches each; the first field contains 5 acres; the second 4 acres, two perches; the third 7 acres, 3 roods; and the fourth 2 acres, 1 rood. How many shares will there be?

Ans. 40 *shares,* 42 *perches rem.*

(48) In a field of 9 acres and a half, how many gardens may be made, each containing 500 square yards?

Ans. 91, *and* 480 *yards rem.*

IMPERIAL MEASURE.

(49) In 10080 pints of port wine, how many tuns?

Ans. 5 *tuns.*

(50) In 35 pipes of Madeira, how many gallons and pints?

Ans. 4410 *gals.* 35280 *pints.*

(51) A gentleman ordered his butler to bottle off $\frac{2}{3}$ of a pipe of French wine into quarts, and the rest into pints. How many dozen of each had he? *Ans.* 28 *dozen of each.*

(52) In 46 barrels of beer, how many pints? *Ans.* 13248.

(53) In 10 barrels of ale, how many gallons and quarts?

Ans. 360 *gals.* 1440 *qts.*

(54) In 12480 pints of porter, how many kilderkins?

Ans. 86 *kil.* 1 *fir.* 3 *gals.*

(55) In 108 barrels of ale, how many hogsheads? *Ans.* 72.

(56) In 120 quarters of corn, how many bushels, pecks, gallons, and quarts? *Ans.* 960 *bu.* 3840 *pks.* 7680 *gal.* 30720 *qts.*

(57) How many bushels are there in 970 pints?
Ans. 15 *bu.* 1 *gal.* 2 *pts.*

(58) In 1 score, 16 chaldrons of coals, how many sacks and bushels? *Ans.* 444 *sacks,* 1332 *bushels.*

TIME.

(59) In 72015 hours, how many weeks?
Ans. 428 *weeks,* 4 *days,* 15 *hours.*

(60) How many days were there from the birth of Christ, to Christmas, 1794, estimating 365¼ days to the year?
Ans. 655258½ *days.*

(61) Stowe writes, that London was built 1108 years before our Saviour's birth. Find the number of hours to Christmas, 1794. *Ans.* 25438932 *hours.*

(62) From July 18th, 1799, to April 18th, 1826, how many days? *Ans.* 9770½ *days, reckoning* 365¼ *days to a year.*

(63) In a lunar month, containing 29 days, 12 hours, 44 minutes, 2 seconds, and eight-tenths, how many tenth parts of seconds? *Ans.* 25514428.

(64) How many seconds are there in 18 centuries, estimating the solar year at 365 days, 5 hours, 48 minutes, 51½ seconds? *Ans.* 56802476700 *seconds.*

COMPOUND ADDITION

TEACHES to find the *sum* of *Compound Quantities.*

RULE. Add the numbers of the *least denomination;* divide the sum by as many as make *one* of the *next greater;* set down the remainder (if any), and carry the quotient to those of the next greater: proceed thus to the *greatest denomination,* which add as in Simple Addition.

PROOF. As in Simple Addition.

EXAMPLE.

£	s.	d.
15..	7..	4½
7..	18..	10¾
11..	19..	5
6..	10..	11¼
4..	0..	9¼
45..	17..	4¾

Say 1, 2, 5, 7 farthings are 1 penny 3 farthings; set down ¾ and carry 1*d.*—1, 10, 11, 16, 20, 30, 40*d.* are 3*s.* 4*d.*; set down 4*d.* and carry 3*s.*—3, 12, 20, 27, 37, 47, 57*s.* are £2..17*s.*; set down 17*s.* and carry £2. The rest as in Simple Addition.

In Addition of Money, the reduction of one denomination to the *next greater* is generally done without the trouble of *dividing,* by the knowledge previously acquired of the Money Tables.

MONEY.

(1)

£	s.	d.
2	13	5½
7	9	4¼
5	15	4½
9	17	6¼
7	16	3
5	14	7¾

(2)

£	s.	d.
27	7	0
34	14	10¼
57	19	2¼
91	16	0
75	18	7¾
97	13	5

(3)

£	s.	d.
35	17	0
59	14	10½
97	13	10¼
37	16	8¼
97	15	7
59	16	0½

(4)

£	s.	d.
75	3	0
54	17	1
91	15	11¼
35	16	1¾
29	19	11¼
91	17	3¼

(5)

£	s.	d.
257	1	5¼
734	3	7¾
595	5	3
159	14	7½
207	5	4
798	16	7¼

(6)

£	s.	d.
525	2	4¼
179	3	5
250	4	7¼
975	3	5¼
254	5	7
379	4	5¾

(7)

£	s.	d.
21	14	7¼
75	16	0
79	2	4¼
57	16	5½
26	13	8¾
54	2	7

(8)

£	s.	d.
73	2	1½
25	12	7
96	13	5½
76	17	3¼
97	14	1½
54	11	7¼

(9)

£	s.	d.
127	4	7½
525	3	10
271	0	0
524	9	1
379	4	0½
215	5	11¾

(10)

£	s.	d.
261	17	1¼
379	13	6
257	16	7¾
184	13	5
725	2	3¼
359	6	5

(11)

£	s.	d.
31	1	1½
75	13	1
39	19	7¼
97	17	3¼
36	13	5
24	16	3¼

(12)

£	s.	d.
27	13	5½
16	12	10¼
9	13	0½
15	2	10½
37	19	0
56	19	1[illegible]

WEIGHTS AND MEASURES.

TROY WEIGHT.

(13)

oz.	dwt.	gr.
5	11	4
7	19	21
3	15	14
7	19	22
9	18	15
8	13	12

(14)

lb.	oz.	dwt.	gr.
5	2	15	22
3	11	17	14
3	7	15	19
9	1	13	21
3	9	7	23
5	2	15	17

APOTHECARIES' WEIGHT.

(15)

℔	℥	ʒ	℈
17	10	7	1
9	5	2	2
27	11	1	2
9	5	6	1
37	10	5	2
49	0	7	0

(16)

℥	ʒ	℈	gr.
2	1	0	12
1	7	1	17
10	2	0	14
5	7	1	15
9	5	2	13
1	4	1	18

AVOIRDUPOIS WEIGHT.

(17)

lb.	*oz.*	*dr.*
152	15	15
272	14	10
303	15	11
255	10	4
173	6	2
635	13	13

(18)

cwt.	*qrs.*	*lb.*
25	1	17
72	3	26
54	1	16
24	1	16
17	0	19
55	2	16

(19)

t.	*cwt.*	*qrs.*	*lb.*
7	17	2	12
5	5	3	14
2	4	1	17
3	18	2	19
7	9	3	20
8	5	1	24

LONG MEASURE.

(20)

yds.	*ft.*	*in.*
225	1	9
171	0	3
52	2	6
397	0	10
154	2	7
137	1	4

(21)

lea.	*m.*	*fur.*	*po.*
72	2	1	19
27	1	7	22
35	2	5	31
79	0	6	12
51	1	6	17
72	0	5	21

(22)

m.	*fur.*	*yds.*
39	6	36
14	7	214
3	4	160
45	3	202
17	1	19
32	4	176

CLOTH MEASURE.

(23)

yds.	*qrs.*	*n.*
135	3	3
70	2	2
95	3	0
176	1	3
26	0	1
279	2	1

(24)

E.e.	*qrs.*	*n.*
272	2	1
152	1	2
79	0	1
156	2	0
79	3	1
154	2	1

LAND MEASURE.

(25)

a.	*r.*	*p.*
726	1	31
219	2	17
1455	3	14
879	1	21
438	2	14
757	0	0

(26)

a.	*r.*	*p.*
1232	1	14
327	0	19
131	2	15
1219	1	18
223	2	8
256	0	9

IMPERIAL MEASURE.

WINE.

(27)

hhd.	*gal.*	*qts.*
31	57	1
97	18	2
76	13	1
55	46	2
87	38	3
55	17	1

(28)

t.	*hhd.*	*gal.*	*qts.*
14	3	27	2
19	2	56	3
17	0	39	2
75	2	16	1
54	1	19	2
97	3	54	3

ALE AND BEER.

(29)

bar.	*fir.*	*gal.*
25	2	7
17	3	5
96	2	6
75	1	8
96	3	7
75	0	5

(30)

hhd.	*gal.*	*qts.*
76	51	2
57	3	3
97	27	3
22	17	2
32	19	3
55	38	3

DRY.							TIME.							
(31)			(32)				(33)			(34)				
qrs.	*b.*	*p.*	*b.*	*p.*	*gal.*	*qts.*	*w.*	*d.*	*h.*	*w.*	*d.*	*h.*	*m.*	*s.*
300	2	1	16	2	1	2	71	3	11	57	2	15	42	41
167	0	1	21	0	1	3	51	2	9	95	3	21	27	51
369	7	0	7	3	0	0	76	0	21	76	0	15	37	28
50	3	2	15	1	1	2	95	3	21	53	2	21	42	27
74	6	3	3	2	0	1	79	1	15	98	2	18	47	38

(35) A, B, C, and D, were partners in the purchase of a quantity of goods: A laid out £7, half-a-guinea, and a crown; B, 49*s.* C, 54*s.* 6*d.* D, 87*d.* What was the purchase?
Ans. £13..6..3.

(36) A man lent his friend, at different times, these several sums, *viz* £63—£25..15—£32..7—£15..14..10, and four score and nineteen pounds, half-a-guinea, and a shilling. How much was the whole loan? *Ans.* £236..8..4.

(37) Bought goods, for which I paid £54..17; for packing, 13*s.* 8*d.*; carriage, £1..5..4; and expenses over making the bargain, 14*s.* 3*d.* What was the whole cost?
Ans. £57..10..3.

(38) A nobleman, previous to quitting town, wished to discharge his tradesmen's bills. On inquiry he found that he owed 82 guineas for rent;—to his wine-merchant, £72..5;—to his confectioner, £12..13..4;—to his draper, £47..13..2;—to his tailor, £110..15..6;—to his coachmaker, £157..8;—to his tallow-chandler, £8..17..9;—to his corn-factor, £170..6..8;—to his brewer, £52..17..0;—to his butcher, £122..11..5;—to his baker, £37..9..5;—and to his servants for wages, £53..18. What money must he draw from his banker, including £100 that he wished to take with him?
Ans. £1032..17..3.

(39) A father was 24 years of age (allowing 13 months to a year, and 28 days to a month) at the birth of his first child; between the eldest and next born was 1 year, 11 months, and 14 days; between the second and third were 2 years, 1 month, and 15 days; between the third and fourth, 2 years, 10 months, and 25 days. When the fourth was 27 years, 9 months, and 12 days old, what age was the father?
Ans. 58 *years,* 7 *months,* 10 *days.*

(40) A clerk, having been out collecting debts, presented an account that A paid him £7..5..2;—B, £15..18..6½;—

C, £150..13..2¼;—D, £17..6..8;—E, 5 guineas, 2 crown pieces, 4 half-crowns and 4*s.* 2*d.*—F paid him only twenty groats;—G, £76..15..9½;—and H, £121..12..4. How much was the whole amount? *Ans.* £396..7..6¾.

(41) A nobleman had a service of plate, which consisted of twenty dishes, weighing 203 *oz.* 8 *dwts.*; 36 plates, 408 *oz.* 9 *dwts.*; 5 dozen spoons, 112 *oz.* 8 *dwts.*; 6 salts, and 6 pepper-boxes, 71 *oz.* 7 *dwts.*; knives and forks, 73 *oz.* 5 *dwts.*; two large cups, a tankard, and a mug, 121 *oz.* 4 *dwts.*; a tea-urn and lamp, 131 *oz.* 7 *dwts.*; with sundry other small articles, weighing 105 *oz.* 5 *dwts.* The weight of the whole is required. *Ans.* 102 *lb.* 2 *oz.* 13 *dwts.*

(42) A hop-merchant buys 5 bags of hops, of which the first weighed 2 *cwt.* 3 *qrs.* 13 *lb.*; the second, 2 *cwt.* 2 *qrs.* 11 *lb.*; the third, 2 *cwt.* 3 *qrs.* 5 *lb.*; the fourth, 2 *cwt.* 3 *qrs.* 12 *lb.*; the fifth, 2 *cwt.* 3 *qrs.* 15 *lb.* He purchased also two pockets, each pocket weighing 84 *lb.* I desire to know the weight of the whole. *Ans.* 15 *cwt.* 2 *qrs.*

COMPOUND SUBTRACTION

Teaches to find the *difference* of *Compound Quantities.*

Rule. Subtract as in integers: but borrow (when there is occasion) *as many* as are *equal* to *one* of the *next greater* denomination: observing to *carry one* to the next for that which was borrowed.*

Proof. As in Simple Subtraction.

MONEY.

	(1) £	*s.*	*d.*	(2) £	*s.*	*d.*	(3) £	*s.*	*d.*
From	715	2	7¼	316	3	5½	87	2	10
Take	476	3	8½	218	2	1¼	79	3	7½

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* Example. Subtract £54..17..9¾ from £89..12..7½.

£ *s.* *d.*
89..12..7½
54..17..9¾
34..14..9¾

Because 3 farthings cannot be taken from 2, say 3 from 4, 1, and 2 are 3; set down 3 and carry 1.—1 and 9 are 10, 10 from 12, 2, and 7 are 9; set down 9 and carry 1.—1 and 17 are 18, 18 from 20, 2, and 12 are 14; set down 14 and carry 1 to the pounds.

(4)

£	s.	d.
3	15	1½
1	14	7

(7)

£	s.	d.
321	17	1½
257	14	7

(10)

£	s.	d.
527	3	5¼
139	5	7½

(13)

£	s.	d.
10	7	6
9	19	7

(5)

£	s.	d.
25	2	5¼
17	9	8½

(8)

£	s.	d.
59	15	3¼
36	17	2

(11)

£	s.	d.
300	15	0
296	15	10

(14)

£	s.	d.
500	0	0
499	19	11¾

(6)

£	s.	d.
37	3	4¼
25	5	2¼

(9)

£	s.	d.
71	2	4
19	13	7¾

(12)

£	s.	d.
68	13	9
44	19	10½

(15)

£	s.	d.
779	12	0
689	13	6

(16)

	£	s.	d.
Borrowed	350	0	0
Paid at different times	26	5	0
	73	10	6
	41	9	8½
	66	14	9
Paid in all			
Remains to pay			

(17)

	£	s.	d.
Lent	577	10	0
Received at several times	95	10	0
	80	0	0
	74	15	9
	23	17	4½

WEIGHTS AND MEASURES.

TROY WEIGHT.

(18)

lb.	oz.	dwt.	gr.
52	1	7	2
39	0	15	7

(19)

lb.	oz.	dwt.	gr.
7	2	2	7
5	7	1	5

APOTHECARIES' WEIGHT.

(20)

℔	℥	ʒ	℈
5	2	1	0
2	5	2	1

21)

℔	℥	ʒ	℈	gr.
9	7	2	1	13
5	7	3	1	18

AVOIRDUPOIS WEIGHT.

(22)

lb.	oz.	dr.
35	10	5
29	12	7

(23)

cwt.	qr.	lb.
35	1	21
25	1	27

(24)

t.	cwt.	qrs.	lb.
21	1	2	7
9	11	3	15

LONG MEASURE.

(25)			(26)			
yds.	*ft.*	*in.*	*lea.*	*mi.*	*fur.*	*po.*
107	2	10	147	2	6	29
78	2	11	58	2	7	33

IMPERIAL MEASURE—WINE.

(31)				(32)			
hhd.	*gal.*	*qts*	*pts.*	*tun*	*hhd.*	*gal.*	*qts.*
47	47	2	1	42	2	37	2
28	59	3	0	17	3	49	3

CLOTH MEASURE.

(27)			(28)		
yds.	*qrs.*	*n.*	*E.e.*	*qrs.*	*n.*
71	1	2	35	2	1
3	2	1	14	3	2

ALE AND BEER.

(33)			(34)		
ar.	*fir.*	*gal.*	*hhd.*	*gal.*	*qts.*
37	2	1	27	27	1
25	1	7	12	50	2

LAND MEASURE.

(29)			(30)		
a.	*r.*	*p.*	*a.*	*r.*	*p.*
175	1	27	325	2	1
59	0	37	279	3	5

CORN AND COAL.

(35)			(36)			
qr.	*b.*	*p.*	*sc.*	*ch.*	*sa.*	*b.*
65	2	1	3	16	1	0
57	2	3	2	12	2	1

TIME.

* (37)				(38)			† (39)		
yrs.	*mo.*	*w.*	*d.*	*h.*	*m.*	*sec.*	*yrs.*	*m.*	*d.*
79	8	2	4	24	42	45	10	7	20
23	9	3	5	19	53	47	5	8	29

(40) When an estate of £300 per annum is reduced, by the payment of taxes, to 12 score and £14..6, what are the taxes? *Ans.* £45..14.

(41) A horse with his furniture is worth £37..5; without it, 14 guineas: how much does the price of the furniture exceed that of the horse? *Ans.* £7..17.

(42) A merchant commencing trade, owed £750; he had in cash, commodities, the stocks, and good debts, £12510..7; he cleared the first year, by commerce, £452..3..6. What was he then worth? *Ans.* £12212..10..6.

(43) A gentleman left £45247 to his two daughters, of

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* In this example allow 4 weeks to a month, and 13 months to the year.

† In this, reckon 30 days to a month, and 12 months to the year.

which the younger was to have 15 thousand, 15 hundred, and twice £15. What was the elder sister's fortune?

Ans. £28717.

(44) A tradesman, being insolvent, called all his creditors together, and found he owed to A, £53..7..6;—to B, £105..10;—to C, £34..5..2;—to D, £28..16..5;—to E, £14..15..8;—to F, £112..9;—and to G, £143..12..9. The value of his stock was £212..6; and the amount of good book-debts was £112..8..3; besides £21..10..5, money in hand. How much would his creditors lose by taking the whole of his effects?

Ans. The creditors lost £146..11..10.

(45) My agent at Seville, in Spain, renders me the following account of money received for the sale of goods sent him on commission, *viz.* for bees' wax, £37..15..4; stockings, £37..6..7; tobacco, £125..11..6; linen cloth, £112..14..8, tin, £115..10..5. He informs me, at the same time, that he has shipped, agreeably to my order, wines, value £250..15; fruit, £51..12..6; figs, £19..17..6; oil, £19..12..4; and Spanish wool, value £115..15..6. How stands the balance of the account between us? *Ans. Due to the agent,* £28..14..4.

(46) The great bell at Oxford, the heaviest in England, is stated to weigh 7 *tons*, 11 *cwt.* 3 *qrs.* 4 *lbs.*; that of St. Paul's, in London, 5 *tons*, 2 *cwt.* 1 *qr.* 22 *lbs.*; and that of Lincoln called the *Great Tom*, 4 *tons*, 16 *cwt.* 3 *qrs.* 16 *lbs.* How much is the aggregate weight of these three bells inferior to that of the great bell at Moscow, which is 198 *tons?*

Ans. 180 *tons*, 8 *cwt.* 3 *qrs.* 14 *lbs.*

COMPOUND MULTIPLICATION

Is the method of multiplying *Compound Quantities.*

RULE. Multiply the *least denomination;* reduce the product, and carry to the next as directed in Compound Addition, and the same with the rest.

When the multiplier is a *compósite* number above 12, multiply (as before directed) by its *compōnent parts.* For other numbers, multiply by the *factors* of the *nearest compósite, adding* to the last product, *so many times* the *top line* as will supply the deficiency; or *subtracting so many times,* if there is an *excess.*

MONEY.

* (1)			(2)			(3)			(4)		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
35	12	$7\frac{3}{4}$	75	13	$1\frac{1}{2}$	62	5	$4\frac{1}{4}$	57	2	$4\frac{3}{4}$
		2			3			4			5
71	5	$3\frac{1}{2}$									

	£	s.	d.			£	s.	d.	
(5)	57	18	$7\frac{1}{4}$	× 6.	(9)	135	13	$6\frac{3}{4}$	× 10.
(6)	81	9	$11\frac{1}{2}$	× 7.	(10)	79	16	$7\frac{1}{2}$	× 11.
(7)	64	10	5	× 8.	(11)	247	14	$11\frac{1}{2}$	× 12.
(8)	118	6	$4\frac{1}{4}$	× 9.	(12)	119	7	$5\frac{3}{4}$	× 12.

	£	s.	d.			s.	d.			£	s.	d.	
(13)	0	9	6	× 18.†	(16)	15	$3\frac{1}{2}$	× 35.	(19)	1	5	3	× 97.
(14)	1	2	6	× 26.‡	(17)	7	$2\frac{3}{4}$	× 75.	(20)	0	6	4	× 43.
(15)	0	7	$8\frac{1}{2}$	× 21.	(18)	9	7	× 37.					

(21) What is the value of 127 lb of souchong tea, at 12*s.* 3*d.* per lb? *Ans.* £77..15..9.

(22) 135 stones of soap, at 7*s.* 5*d.* per stone? *Ans.* £50..1..3.

(23) 74 ells of diaper, at 1*s.* $4\frac{1}{2}$*d.* per ell? *Ans.* £5..1..9.

(24) 6 doz. pairs of gloves, at 1*s.* 10*d.* per pair? *Ans.* £6..12

NOTE. When the fraction $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ is connected with the multiplier, take *half* the given price (or the price of *one*) for $\frac{1}{2}$, *half of that* for $\frac{1}{4}$, and for $\frac{3}{4}$ add them both together.§

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* In this example, say twice 3 are 6, 6 farthings are $1\frac{1}{2}$*d.* set down $\frac{1}{2}$*d.* and carry 1; twice 7 are 14 and 1 are 15, 15*d.* are 1*s.* 3*d.* set down 3*d.* and carry 1; twice 12 are 24 and 1 are 25, 25*s.* are £1..5, set down 5*s.* and carry 1; twice 5 are 10 and 1 are 11, set down 1 and carry 1; twice 3 are 6 and 1 are 7, set down 7.

	s.	d.	
†	9..	6	
		2	× 9 = 18
	19..	0	
		9	
£8..	11..	0	*Ans.*

	£	s.	d.	
‡	1..	2..	6	
			8	× 3 + 2 = 26
	9..	0..	0	
			3	
	27..	0..	0	
Multiplicand × 2 =	2..	5..	0	
	29.	5..	0	*Ans.*

§ EXAMPLE.

What is the value of $11\frac{3}{4}$ lbs. of tea, at 10*s.* 9*d.* per lb?

	£	s.	d.	
$\frac{1}{2}$ ×		10..	9	
			11	
	£5..	18..	3	= the value of 11.
$\frac{1}{2}$ ×		5..	$4\frac{1}{2}$	= .... do. .... $\frac{1}{2}$.
		2..	$8\frac{1}{4}$	= .... do. .... $\frac{1}{4}$.
	£6..	6..	$3\frac{3}{4}$	*Ans.*

(25) What is the value of 25½ ells of Holland, at 3*s.* 4½*d.* per ell? *Ans.* £4..6..0¾.

(26) 75½ lb of hemp, at 1*s.* 3*d.* per lb? *Ans.* £4..14..4½.

(27) 19½ yds. of muslin, at 4*s.* 3*d.* per yd.? *Ans.* £4..2..10½.

(28) 35½ cwt. of raw sugar, at £4..15..6 per cwt? *Ans.* £169..10..3.

(29) 154½ cwt. of raisins, at £4..17..10 per cwt? *Ans.* £755..15..3.

(30) 117¼ gallons of gin, at 12*s.* 6*d.* per gallon? *Ans.* £73..5..7½.

(31) 85¾ cwt. of logwood, at £1..7..8 per cwt? *Ans.* £118..12..5.

(32) 17¾ yards of superfine scarlet cloth, at £1..3..6 per yard? *Ans.* £20..17..1½.

(33) 37½ lb of hyson tea, at 12*s.* 4*d.* per lb? *Ans.* £23..2..6.

(34) 56¾ cwt. molasses, at £2..18..7 per cwt? *Ans.* £166..4..7¼.

(35) 87¾ lb of Turkey coffee, at 4*s.* 3*d.* per lb? *Ans.* £18..12..11¼.

(36) 120¾ cwt. of hops, at £4..7..6 per cwt? *Ans.* £528..5..7½.

When the multiplier is large, multiply the given quantity (or price) by a series of *tens,* to find 10, 100, 1000 times, &c. as far as to the value of the *highest place* of the multiplier; multiply the last product by the figure in that place, and each preceding product by the figure of corresponding value; that is, the product for 100 by the *number of hundreds,* the product for 10 by the *number of tens,* and the *original quantity* by the *units' figure,* &c. The *sum* of the products thus obtained will be the *total product.**

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* EXAMPLE. Multiply £7..14..9½ by 3645.

	£ s. d.		£ s. d.	times.
	7..14.. 9½	× 5 =	38..13..11½ =	5
	10			
The product for 10	77.. 7..11	× 4 =	309..11.. 8 =	40
	10			
The product for 100	773..19.. 2	× 6 =	4643..15.. 0 =	600
	10			
The product for 1000	7739..11.. 8	× 3 =	23218..15.. 0 =	3000
		Ans.	£28210..15.. 7½ =	3645

(37) 407 lb of gall-nuts, at 3*s.* 9½*d.* per lb? *Ans.* £77..3..2½.

(38) 729 stones of beef, at 7*s.* 7¼*d.* per stone?
Ans. £277..3..5¼.

(39) 2068 yards of lace, at 9*s.* 5½*d.* per yard?
Ans. £977..19..10.

(40) What is the produce of a toll-gate in the course of the year, if the tolls amount, on an average, to 11*s.* 7½*d.* per day? *Ans.* £212..3..1½.

(41) How much money must be equally divided among 18 men, to give each £14..6..8½? *Ans.* £258..0..9.

(42) A privateer manned with 250 sailors captured a prize, of which each man shared £125..15..6. What was the value of the prize? *Ans.* £31443..15.

(43) What sum did a gentleman receive as a dower with his wife, whose fortune was a cabinet with two divisions, in each division 87 drawers, and each drawer containing 21 guineas? *Ans.* £3836..14.

(44) A merchant began trade with £19118; for 5 years together he cleared £1086 a year; and the next 4 years, £2715..10..6 a year; but, the last 3 years he was in trade, he had the misfortune to lose, upon an average, £475..4..6 a year. What was his real fortune at the end of the 12 years? *Ans.* £33984..8..6.

(45) In many parts of the kingdom, coals are weighed in the wagon or cart upon a machine, constructed for the purpose. If three of these draughts amounted together to 137 *cwt.* 2 *qrs.* 10 *lb.*; and the tare, or weight of the wagon, was 13 *cwt.* 1 *qr.*; how many coals had the customer in 12 such draughts? *Ans.* 391 *cwt.* 1 *qr.* 12 *lb.*

(46) A certain gentleman lays up every year £294..12..6, and spends daily £1..12..6. What is his annual income?
Ans. £887..15.

WEIGHTS AND MEASURES.

(47) Multiply 9 lb. 10 oz. 15 dwts. 19 gr. by 9, 11, and 12.

(48) Multiply 23 tons, 9 cwt. 3 qrs. 18 lb. by 7, 8, and 9.

(49) Multiply 107 yards, 3 qrs. 2 nails, by 10, 17, and 29.

(50) Multiply 33 bar. 2 fir. 3 gal. by 11 and 12.

(51) Multiply 110 miles, 6 fur. 26 poles, by 12, 13, and 39.

(52) A lunar month contains 29 days, 12 hours, 44 min. 3 seconds nearly. What time is contained in 13 lunar months?

COMPOUND DIVISION

TEACHES to find any required *part* of a *Compound quantity.*

RULE. Divide the *greatest denomination:* reduce the remainder to the *next less*, to which add the next; divide that, and proceed as before to the end.

When the divisor is above 12, the work must be done at length: unless it is a *compósite* number, for which observe the directions in Simple Division.—*Proof by Multiplication.*

MONEY.

* (1)	(2)	(3)	(4)
£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*	£ *s.* *d.*
2)25 2 4	3)37 7 7	4)57 5 7	5)52 7 0

	£	*s.*	*d.*			£	*s.*	*d.*
(5)	78	10	$9\frac{1}{2} \div 6$.		(9)	87	14	0 by 10.
(6)	25	19	$7\frac{3}{4} \div 7$.		(10)	68	0	0 by 11.
(7)	16	14	$1\frac{1}{2} \div 8$.		(11)	49	14	7 by 12.
(8)	124	15	$2\frac{1}{4} \div 9$.		(12)	496	8	6 by 12.
(13)	66	6	$6\frac{3}{4} \div 25$.		(16)	248	17	4 by 99.
(14)	596	12	$7\frac{1}{4} \div 36$.		(17)	928	12	8 by 110.
(15)	564	4	$6 \div 63$.		(18)	608	13	9 by 144.

(19) Divide £1407..17..7 by 243.
(20) Divide £700791..14..4 by 1794.
(21) Divide £490981..3..$7\frac{1}{2}$ by 31715.
(22) Divide £19743052..5..$7\frac{1}{2}$ by 214723.

(23) If a man spend £257..2..5 in 12 months, what is that per month? *Ans.* £21. 8..$6\frac{1}{4}$ $\frac{8}{12}$.

(24) The clothing of 35 charity boys came to £57..3..7: what was the expense of each boy? *Ans.* £1..12..$8\frac{12}{35}$.

(25) If I gave £37..6..$4\frac{3}{4}$ for nine pieces of cloth, what was that per piece? *Ans.* £4..2..$11\frac{7}{9}$.

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* EXAMPLE. Divide £27..14..$11\frac{1}{2}$ by 5.

£ *s.* *d.*
5)27..14..$11\frac{1}{2}$
5..10..$11\frac{3}{4}$ $\frac{3}{5}$

Say the fives in 27, 5 times 5 are 25 and 2 over; £2 are 40*s.* and 14 are 54, the fives in 54, 10 times 5 are 50 and 4 over; 4*s.* are 48*d.* and 11 are 59, the fives in 59, 11 fives are 55 and 4 over: 4*d.* are 16 *qrs.* and 2 are 18, the fives in 18, 3 times five are 15, and 3 over, or $\frac{3}{5}$.

(26) If 20 cwt. of tobacco cost £27..5..4½, at what rate did I buy it per cwt? *Ans.* £1..7..$3\frac{18}{20}$.

(27) What is the value of one hogshead of beer, when 120 hogsheads are sold for £154..17..10? *Ans.* £1..5..$9\frac{3}{4}$ $\frac{16}{120}$.

(28) Bought 72 yards of cloth for £85..6. What was the price per yard? *Ans.* £1..3..$8\frac{1}{4}$ $\frac{24}{72}$.

(29) Gave £275..3..4 for 18 bales of cloth. What is the price of one bale? *Ans.* £15..5..$8\frac{3}{4}$ $\frac{10}{18}$.

(30) A prize of £7257..3..6 is to be equally divided among 500 sailors. What is each man's share? *Ans.* £14..10..$3\frac{1}{4}$ $\frac{388}{500}$.

(31) A club of 25 persons joined to purchase a lottery ticket of £10 value, which was drawn a prize of £4000. What was each man's contribution, and his share of the prize-money? *Ans. Each contribution* 8s. *and share of prize* £160.

(32) A tradesman cleared £2805 in 7½ years: what was his yearly profit? *Ans.* £374.

(33) What was the weekly salary of a clerk who received £266..18..1½ for 90 weeks? *Ans.* £2..19..3¾.

(34) If 100000 quills cost me £187..17..1, what is the price per thousand? *Ans.* £1..17..$6\frac{3}{4}$ $\frac{40}{100}$.

WEIGHTS AND MEASURES.

(35) Divide 83 ℔. 5 oz. 10 dwts. 17 gr. by 8, 10, and 12

(36) Divide 29 tons, 17 cwt. 0 qrs. 18 ℔. by 9, 15, and 19

(37) Divide 114 yards, 3 qrs. 2 nails, by 10 and 16.

(38) Divide 1017 miles, 6 fur. 38 poles, by 11 and 49.

(39) Divide 2019 acres, 3 roods, 29 perches, by 26.

(40) Divide 117 years, 7 months, 26 days, 11 hours, 27 minutes, by 37.

PROMISCUOUS EXAMPLES.

(1) Of three numbers, the first is 215, the second 519, and the third is equal to the other two. What is the sum of them all? *Ans.* 1468.

(2) The less of two sums of money is £40, and their difference £14. What is the greater sum, and the amount of both? *Ans.* £54 *the greater,* £94 *the sum.*

(3) What number added to ten thousand and eighty-nine, will make the sum fifteen thousand and forty? *Ans.* 4951.

(4) What is the difference between six dozen dozen, and half a dozen dozen; and what is their sum and product?
Ans. Diff. 792, *sum* 936, *product* 62208.

(5) What difference is there between twice eight and fifty and twice fifty-eight, and what is their product?

Ans. 50 *difference,* 7656 *product.*

(6) The greater of two numbers is 37 times 45, and their difference is 19 times 4: required their sum and product?

Ans. 3254 *sum,* 2645685 *product.*

(7) A gentleman left his elder daughter £1500 more than the younger, whose fortune was 11 thousand, 11 hundred, and £11. Find the portion of the elder, and the amount of both.

Ans. Elder's portion £13611; *amount* £25722.

(8) The sum of two numbers is 360, the less is 144. What is their difference and their product?

Ans. 72 *difference,* 31104 *product.*

(9) There are 2545 bullocks to be divided among 509 men. Required the number and the value of each man's share, supposing every bullock worth £9..14..6?

Ans. Each man had 5 *bullocks, and* £48..12..6 *for his share.*

(10) How many cubic feet are contained in a room, the length of which is 24 feet, the breadth 14 feet, and the height 11 feet? * *Ans.* 3696.

(11) A gentleman's garden containing 9625 square yards, is 35 yards broad. What is the length? *Ans.* 275 *yards.*

(12) What sum added to the 43d part of £4429, will make the total amount = £240? *Ans.* £137.

(13) Divide 20*s.* among A, B, and C, so that A may have 2*s.* less than B, and C 2*s.* more than B.

Ans. A 4*s.* 8*d.* *B* 6*s.* 8*d.* *and C* 8*s.* 8*d.*

(14) In an army consisting of 187 squadrons of horse, each 157 men, and 207 battalions of foot, each 560 men, how many effective soldiers are there, supposing that in 7 hospitals there are 473 sick? *Ans.* 144806.

(15) A tradesman gave his daughter, as a marriage portion, a scrutoire, containing 12 drawers; in each drawer were six divisions, and in each division there were £50, four crown pieces, and eight half-crown pieces. How much had she to her fortune? *Ans.* £3744.

(16) There are 1000 men in a regiment, of whom 50 are officers: how many privates are there to one officer? *Ans.* 19.

(17) What number must 7847 be multiplied by, to produce 3013248? *Ans.* 384.

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* Multiply the three dimensions continually together.

(18) Suppose I pay eight guineas and half-a-crown for a quarter's rent, but am allowed 15*s* for repairs; what does my apartment cost me annually, and how much in seven years? *Ans. In one year,* £31..2. *In seven,* £217..14.

(19) The quotient is 1083; the divisor, 28604; and the remainder, 1788: what is the dividend? *Ans.* 30979920.

(20) An assessment was made on a certain hundred, for the sum of £386..15..6, the amount of the damage done by a riotous assemblage. Four parishes paid £37..14..2 each; four hamlets, £31..4..2 each; and four townships, £18..12..6 each: how much was deficient? *Ans.* £36..12..2.

(21) An army consisting of 20,000 men, got a booty of £12,000: what was each man's share, if the whole were equally divided among them? *Ans.* 12*s*.

(22) A gentleman left by will, to his wife, £4560;—to a public charity, £572..10;—to four nephews, £750..10 each;—to four nieces, £375..12..6 each;—to thirty poor housekeepers, 10 guineas each;—and to his executors, 150 guineas. What was the amount of his property? *Ans.* 10109..10.

(23) My purse and money, said Dick to Harry, are worth 12*s*. 8*d*. but the money is worth seven times the value of the purse: what did the purse contain? *Ans.* 11*s*. 1*d*.

(24) Supposing 20 to be the remainder of a division, 423 the quotient, and the divisor the *sum* of both, *plus* 19; what is the dividend? *Ans.* 195446.

(25) A merchant bought two lots of tobacco, which weighed 12 *cwt.* 3 *qrs.* 15 *lb.* for £114..15..6; their difference in weight was 1 *cwt.* 2 *qrs.* 13 *lb.* and in price £7..15..6. Required their respective weights and value?*

Ans. Greater weight, 7 *cwt.* 1 *qr. value* £61..5..6.
Less weight, 5 *cwt.* 2 *qrs.* 15 *lb. value* £53..10.

(26) Divide 1000 crowns in such a manner among A, B, and C, that A may receive 129 crowns more than B, and B 178 less than C. *Ans. A* 360 *crowns, B* 231, *C* 409.

(27) If 103 guineas and 7*s*. be divided among 7 men, how many pounds sterling is the share of each? *Ans.* £15..10.

(28) A certain person had 25 purses, each purse containing 12 guineas, a crown, and a moidore; how many pounds sterling had he in all? *Ans.* £355.

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* Add the difference to the sum, and divide by 2 for the *greater*; subtract the difference from the sum, and divide by 2 for the *less*.

(29) A gentleman, in his will, left £50 to the poor, and ordered that $\frac{1}{3}$ should be given to old men, each man to have 5*s.*—$\frac{1}{4}$ to old women, each woman to have 2*s.* 6*d.*—$\frac{1}{5}$ to poor boys, each boy to have 1*s.*—$\frac{1}{6}$ to poor girls, each girl to have 9*d.* and the remainder to the person who distributed it. How many of each sort were there, and what remained for the person who distributed the money?

Ans. 66 *men,* 100 *women,* 200 *boys,* 222 *girls;* £2..13..6 *for the distributor.*

(30) A gentleman sent a tankard to his goldsmith, that weighed 50 *oz.* 8 *dwts.* to be made into spoons, each weighing 2 *oz.* 16 *dwts.*: how many would he have? *Ans.* 18.

(31) A gentleman has sent to a silversmith 137 *oz.* 6 *dwts.* 9 *gr.* of silver, to be made into tankards of 17 *oz.* 15 *dwts.* 10 *gr.* each; spoons of 21 *oz.* 11 *dwts.* 13 *gr.* per dozen; salts, of 3 *oz.* 10 *dwts.* each; and forks, of 21 *oz.* 11 *dwts.* 13 *gr.* per dozen; and for every tankard to have one salt, a dozen spoons, and a dozen forks. What number of each will he have?

Ans. Two of each sort, 8 *oz.* 9 *dwts.* 9 *gr. over.*

(32) How many parcels of sugar of 16 *lb.* 2 *oz.* each, are there in 16 *cwt.* 1 *qr.* 15 *lb.*?

Ans. 113 *parcels, and* 12 *lb.* 14 *oz. over.*

(33) In an arc of 7 signs, 14° 3′ 53″, how many seconds?

Ans. 806633″.

(34) How many *lbs.* of *lead* would counterpoise a mass of *bullion* weighing 100 *lbs.* Troy?* *Ans.* 82 *lb.* 4 *oz.* $9\frac{25}{175}$ *dr.*

(35) If an apothecary mixes together 1 *lb.* avoirdupois of white wax, 4 *lbs.* of spermaceti, and 12 *lbs.* of olive oil; how many *ounces, apothecaries' weight,* will the mass of ointment weigh, and how many masses of 3 *drams* each will it contain?

Ans. The whole 247 *oz.* $7\frac{64}{193}$ *dr. and* 661 *of* 3 *dr. each.*

PROPORTION.

PROPORTION is either DIRECT OR INVERSE. It is commonly called the RULE OF THREE; there being always *three numbers* or *terms given,* two of which are terms of *supposition;* and the other is the term of *demand:* because it requires a *fourth*

* *Bullion* is the term denoting gold or silver in the mass. Lead is weighed by Avoirdupois weight. See the Table of COMPARISON OF WEIGHTS.

term to be found, in the same proportion to itself, as that which is between the other two.

GENERAL RULE FOR STATING THE QUESTION. Put the *term of demand* in the *third* place; that *term of supposition* which is of the *same kind* as the *demand*, the *first;* and the other, which is of the *same kind* as the *required term*, the *second.**

Also, the terms being thus arranged, reduce the first and third (if necessary) into one name, and the second into the lowest denomination mentioned.

THE RULE OF THREE DIRECT

REQUIRES the *fourth* term to be *greater* than the *second*, when the *third* is *greater* than the *first;* or the *fourth*, to be *less* than the *second*, when the *third* is less than the *first*.

RULE. Multiply the second and third together, and divide their product by the first: the quotient will be the answer, in the same denomination as the second.†

The following methods of contracting the operations in the RULE OF THREE are highly important, and should never be lost sight of.

1. Let the first and third terms be reduced *no lower* than is *necessary*, to make them of the *same denomination*.

2. Let the *dividing term* and *either* (but not *both*) of *the other* terms be divided by any number that will divide them *exactly;* and use the quotients instead of the original numbers.

3. When it is conveniently practicable, work by Compound Multiplication and Division, instead of reducing the terms.

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* Some modern authors prefer placing the *term of demand* the *second*, and that *similar* to the *required term* the *third*. This arrangement will answer the purpose equally well, observing that those of *like kind* must be reduced (if necessary) to the *same name*.

† The following GENERAL RULE comprehends both the cases of DIRECT and INVERSE PROPORTION under one head; which is considered by many scientific men of the present day as a more systematic arrangement.

RULE. The question being stated, and the terms prepared, consider, from the nature of the case, whether the *required term* is to be *greater* or *less* than the *second*, or term of *similar kind:* if greater, multiply that *similar to the answer* by the greater of the other two, and divide the product by the less; if less, multiply it by the less, and divide the product by the greater. In either case the quotient will be the *term required*, in the same denomination as the *similar term*.

NOTE. It is evident that the above Rule will answer generally, whether the term of demand is put in the second or third place.

(1) If 1 *lb.* of sugar cost $4\frac{1}{2}$*d.* what will 54 *lb.* cost?*

(2) If a gallon of beer cost 10*d.* what is that per barrel? *Ans.* £1..10.

(3) If a pair of shoes cost 4*s.* 6*d.* what is the value of 12 dozen pairs?†

(4) If one yard of cloth cost 15*s.* 6*d.* what will 32 yards cost at the same rate? *Ans.* £24..16.

(5) If 32 yards of cloth cost £24..16, what is the value of one yard? *Ans.* 15*s.* 6*d.*

(6) If I gave £4..18 for 1 *cwt.* of sugar, at what rate did I buy it per *lb.*? *Ans.* $10\frac{1}{2}$*d.*

(7) Bought 20 pieces of cloth, each piece 20 ells, for 12*s.* 6*d.* per ell: what is the value of the whole? *Ans.* £250.

(8) What will 25 *cwt.* 3 *qrs.* 14 *lb.* of tobacco come to, at $15\frac{1}{2}$*d.* per *lb.*? *Ans.* £187..3..3.

(9) Bought $27\frac{1}{4}$ yards of muslin, at 6*s.* $9\frac{1}{2}$*d.* per yard: what is the amount of the whole? *Ans.* £9..5..$0\frac{3}{4}$ $\frac{1}{2}$.

(10) Bought 17 *cwt.* 1 *qr.* 14 *lb.* of iron, at $3\frac{1}{4}$*d.* per *lb.*: what was the price of the whole? *Ans.* £26..7..$0\frac{1}{2}$

(11) If coffee is sold for $5\frac{1}{2}$*d.* per ounce, what will be the price of 2 *cwt.*? *Ans.* £82..2..8.

(12) How many yards of cloth may be bought for £21..11..$1\frac{1}{2}$, when $3\frac{1}{2}$ yards cost £2..14..3? *Ans.* 27 *yards*, 3 *qrs.* $1\frac{1}{31}$ *nail.*

(13) If 1 *cwt.* of Cheshire cheese cost £1..14..8, what must I give for $3\frac{1}{2}$ *lb.*? *Ans.* 1*s.* 1*d.*

(14) Bought 1 *cwt.* 24 *lb.* 8 *oz.* of old lead, at 9*s.* per *cwt.*: what did the lead cost? *Ans.* 10*s.* $11\frac{1}{2}$ $\frac{112}{224}$*d.*

(15) If a gentleman's income be £500 a year, and he spend 19*s.* 4*d.* per day, what is his annual saving? *Ans.* £147..3..4.

(16) If 14 yards of cloth cost 10 guineas, how many Flemish ells can I buy for £283..17..6? *Ans.* 504 *Fl. ells*, 2 *qrs.*

(17) If 504 Flemish ells, 2 quarters, cost £283..17..6, what is the cost of 14 yards? *Ans.* £10..10.

```
      lb.   d.     lb.
* As  1 : 4½ : : 54
           4       18
          ---     ----
          18   4)972 qrs.
          ---     ----
              12)243 d.
                 ------
                 20s. 3d. = £1..0..3. Ans.
```

```
      pr.  s. d.    prs.
† As  1 : 4..6 : : 144
                12
             -------
             2..14..0
                12
             -------
            £32..8..0. Ans.
```

(18) At the rate of £1..1..8 for 3 *lb.* of gum acacia, what must be given for 29 *lb.* 4 *oz.*? *Ans.* £10..11..3.

(19) If 1 English ell, 2 quarters, cost 4*s.* 7*d.* what will $39\frac{1}{2}$ yards cost at the same rate? *Ans.* £5..3..$5\frac{1}{4}$ $\frac{1}{7}$.

(20) If 27 yards of Holland cost £5..12..6, how many English ells can I buy for £100? *Ans.* 384 *ells.*

(21) If 7 yards of cloth cost 17*s.* 8*d.* what is the value of 5 pieces, each containing $27\frac{1}{2}$ yards? *Ans.* £17..7..$0\frac{1}{4}$ $\frac{1}{7}$.

(22) A draper bought 420 yards of broad cloth, at the rate of 14*s.* $10\frac{3}{4}$*d.* per ell English: what was the amount of the purchase money? *Ans.* £250..5.

(23) A grocer bought 4 hogsheads of sugar, each hogshead weighing neat 6 *cwt.* 2 *qrs.* 14 *lb.* at £2..8..6 per cwt. what is the value? *Ans.* £64..5..3.

(24) A draper bought 8 packs of cloth, each pack containing 4 parcels, each parcel 10 pieces, and each piece 26 yards; at the rate of £4..16 for 6 yards: what was the purchase money? *Ans.* £6656.

(25) If 24 *lb.* of raisins cost 6*s.* 6*d.* what will 18 frails cost, each frail weighing neat 3 *qrs.* 18 *lb.*? *Ans.* £24..17..3.

(26) When the price of silver is 5*s.* per ounce, what is the value of 14 ingots, each ingot weighing 7 *lb.* 5 *oz.* 10 *dwts.*? £313..5.

(27) What is the value of a pack of wool, weighing 2 *cwt.* 1 *qr.* 19 *lb.* at 17*s.* per tod of 28 *lb*? *Ans.* £8..4..$6\frac{1}{4}$ $\frac{20}{28}$.

(28) Bought 171 tons of lead, at £14 per ton; paid carriage and other incidental charges, £4..10. Required the whole cost, and the cost per *lb*? *Ans.* £2398..10, *the whole cost; and the cost per lb.* $1\frac{1}{2}$*d.* $\frac{432}{38304}$.

(29) If a pair of stockings cost 10 groats, how many dozen pairs can I buy for £43..5? *Ans.* 21 *doz.* $7\frac{1}{2}$ *pairs.*

(30) Bought 27 *doz.* 5 *lb.* of candles, at the rate of 5*s.* 9*d.* a dozen: what did they cost? *Ans.* £7..17..$7\frac{3}{4}$.

(31) A factor bought 86 pieces of stuff, which cost him £517..17..10, at 4*s.* 10*d.* per yard. How many yards were there in the whole, and how many English ells in a piece?
Ans. 2143 *yds.; and* 19 *ells*, 4 *qrs.* $2\frac{60}{86}$ *nails, in a piece.*

(32) A gentleman has an annuity of £896..17. What may he spend daily, that at the year's end he may lay up 200 guineas, after giving to the poor quarterly 10 moidores?
Ans. £1..14..8 $\frac{44}{365}$.

RULE OF THREE INVERSE

REQUIRES the *fourth* term to be *less* than the *second*, when the *third* is *greater* than the *first*; or the *fourth* to be *greater* than the *second*, when the *third* is *less* than the *first*.

RULE. Multiply the first and second together, and divide their product by the third: the quotient will be the answer, as before.

(1) If 8 men can do a piece of work in 12 days, in how many days can 16 men do the same?*

(2) If 54 men can build a house in 90 days, how many men can do the same in 50 days? *Ans.* $97\frac{1}{5}$ *men.*

(3) If, when a peck of wheat is sold for 2*s.* the penny loaf weighs 8 *oz.* how much must it weigh when the peck is worth but 1*s.* 6*d.*? *Ans.* $10\frac{2}{3}$ *oz.*

(4) How many sovereigns, of 20*s.* each, are equivalent to 240 pieces, value 12*s.* each? *Ans.* 144.

(5) How many yards of stuff three quarters wide, are equal in measure to 30 yards of 5 quarters wide? *Ans.* 50 *yds.*

(6) If I lend a friend £200 for 12 months, how long ought he to lend me £150? *Ans.* 16 *months.*

(7) If for 24*s.* I have 1200 *lb.* carried 36 miles, what weight can I have carried 24 miles for the same money? *Ans.* 1800 *lb.*

(8) If I have a right to keep 45 sheep on a common 20 weeks, how long may I keep 50 upon it? *Ans.* 18 *weeks.*

(9) A besieged town has a garrison of 1000 soldiers, with provisions for only 3 months. How many must be sent away, that the provisions may last 5 months? *Ans.* 400.

(10) If £20 worth of wine be sufficient to serve an ordinary of 100 men, when the price is £30 per tun; how many will £20 worth suffice, when the price is only £24 per tun? *Ans.* 125 *men.*

(11) A courier makes a journey in 24 days, by travelling 12 hours a day: how many days will he be in going the same journey, travelling 16 hours a day? *Ans.* 18 *days.*

(12) How much will line a cloak, which is made of 4 yards of plush, 7 quarters wide, the stuff for the lining being but 3 quarters wide? *Ans.* $9\frac{1}{3}$ *yards.*

$$\text{* As } \overset{m.}{8} : \overset{d.}{12} :: \overset{m}{16} : \frac{8 \times 12}{16} = 6 \textit{ days. Ans.}$$

DIRECT AND INVERSE PROPORTION PROMISCUOUSLY ARRANGED.

(1) If 14 yards of broad cloth cost £9..12, what is the purchase of 75 yards? *Ans.* £51..8..6¾ $\frac{6}{14}$.

(2) If 14 pioneers make a trench in 18 days, in how many days would 34 men make a similar trench, working in both cases 12 hours a day? *Ans. 7 days, 4 hours,* 56$\frac{8}{17}$ *minutes.*

(3) How much must I lend to a friend for 12 months, to requite his kindness in having lent me £64 for 8 months? *Ans.* £42..13..4.

(4) Bought 59 *cwt.* 2 *qrs.* 21 *lb.* of tobacco, at £2..17..4 per *cwt.* what does it come to? *Ans.* £171..2..1.

(5) A woollen-draper purchased 147 yards of broad cloth, at 14*s.* 6*d.* per yard. Suppose that he sold it in pieces for coats, each 1¾ yard, how much must he charge for each, so as to gain £16..10..9 by the whole? *Ans.* £1..9..3¾.

(6) If £100 gain £4..10 interest at 12 months, what sum will gain the same in 18 months? *Ans.* £66..13..4.

(7) A draper having sold 147 yards of cloth, at the rate of £1..9..3¾ for 1¾ yard, found that he had gained £16..10..9. What did the whole cost him, and how much per yard?

Ans. The whole £106..11..6; *and* 14*s.* 6*d. per yard.*

(8) If £100 in 12 months gain £4..10 interest, in what time will £66..13..4 gain the same interest? *Ans.* 18 *months.*

(9) If a draper bought 147 yards of cloth, at 14*s.* 6*d.* per yard, and sold it in pieces for coats, each 1¾ yard, for £1..9..3¾; how much would he gain per yard, and by the whole? *Ans.* 2*s.* 3*d. per yard;* £16..10..9 *by the whole.*

(10) If 1 *cwt.* cost £12..12..6, what must be given for 14 *cwt.* 1 *qr.* 19 *lb.*? *Ans.* £182..0..11½ $\frac{3}{112}$.

(11) If £100 gain £4..10 in 12 months, what interest will £375 gain in the same time? *Ans.* £16..17..6.

(12) A regiment of soldiers, consisting of 1000 men, are to have new coats, each to be made of 2½ yards of cloth, 5 quarters wide, and to be lined with shalloon of 3 quarters wide. How many yards of shalloon will line them?

Ans. 4166 *yards,* 2 *qrs.* 2⅔ *nails.*

THE DOUBLE RULE OF THREE

Has *five terms given,* three of supposition and two of demand, to find a *sixth,* in the same proportion with the terms of demand, as that of the terms of supposition. It comprises *two* operations of the SINGLE RULE.—But it may comprise *three*, *four*, or *more* operations of the Single Rule; as there may be *seven* terms given to find an *eighth*, or *nine* to find a *tenth*, &c. In this respect it is unlimited; and is therefore more properly called COMPOUND PROPORTION.

RULE 1. Put the *terms of demand* one under another in the *third place;* the *terms of supposition* in the same order in the *first place;* except that which is of the *same nature* as the *required term*, which must be in the *second place.*

Examine the statings separately, using the middle term in each, to know if the proportion is *direct* or *inverse.* When direct, mark the *first* term with an asterisk: when inverse, mark the *third* term.

Find the product of the *marked* terms for a *Divisor,* and the product of *all the rest* for a *Dividend:* divide, and the quotient will be the answer.*

RULE 2. (1) Of the conditional terms, put the principal cause of action, gain or loss, &c. in the *first* place. (2) Put that which denotes time or distance, &c. in the *second,* and the other in the *third.* (3) Put the terms of demand under the *like* terms of supposition. (4) If the blank falls in the *third* place, multiply the first and second terms for a divisor, and the other three for a dividend. (5) But if the blank is in the *first* or *second* place, divide the product of the rest by the product of the third and fourth terms, for the answer.

NOTE. It will save much labour to write the terms of the Dividend *over*, and those of the Divisor *under a line*, like those of a *compound fraction*, and to cancel them accordingly. See Reduction of Vulgar Fractions, Case 6.

PROOF. By two operations of the Single Rule of Three.

(1) If 14 horses eat 56 bushels of oats in 16 days, how many bushels will serve 20 horses 24 days?†

* See also Supplemental Questions, Nos. 6 and 7.

† By Rule 1.

$$\text{As}\left.\begin{array}{r} h. \\ {}^*14 \\ d. \\ {}^*16\end{array}\right\} : \begin{array}{c} b. \\ 56 \end{array} :: \left\{\begin{array}{c} h. \\ 20 \\ d. \\ 24\end{array}\right\} : \begin{array}{c} \begin{array}{ccc} 1 & & \\ 8 & 10 & 12 \end{array} \\ \dfrac{\cancel{56}\times\cancel{20}\times\cancel{24}}{\cancel{14}\times\cancel{16}} \\ \begin{array}{cc} 2 & 2 \\ 1 & 1 \end{array} \end{array} = 120 \text{ bushels.}$$

By Rule 2.

$$\begin{array}{ccccc} h. & & d. & & b. \\ 14 & : & 16 & : & 56 \\ 20 & : & 24 & : & — \end{array} \quad \begin{array}{c} \begin{array}{ccc} 1 & & \\ 8 & 10 & 12 \end{array} \\ \dfrac{\cancel{56}\times\cancel{20}\times\cancel{24}}{\cancel{14}\times\cancel{16}} \\ \begin{array}{cc} 2 & 2 \\ 1 & 1 \end{array} \end{array} = 120$$

By two single statings.

$$\begin{array}{l} \qquad\qquad\quad h. \quad b. \quad\;\; h. \quad b. \\ b\;\;(1)\ \text{As } 14 : 56 :: 20 : 80 \\ \qquad\qquad\quad d. \quad b. \quad\;\; d. \quad b. \\ \quad\;\;(2)\ \text{As } 16 : 80 :: 24 : 120 \end{array}$$

(2) If 8 men in 14 days can mow 112 acres of grass, how many men can mow 2000 acres in 10 days? *Ans.* 200 *men.*

(3) If £100 in 12 months gain £6 interest, how much will £75 gain in 9 months? *Ans.* £3..7..6.

(4) If £100 in 12 months gain £6 interest, what principal will gain £3..7..6 in 9 months? *Ans.* £75.

(5) If £100 gain £6 interest in 12 months, in what time will £75 gain £3..7..6 interest? *Ans.* 9 *months.*

(6) If a carrier charges £2..2 for the carriage of 3 *cwt.* 150 miles, how much ought he to charge for the carriage of 7 *cwt.* 3 *qrs.* 14 *lb.* 50 miles? *Ans.* £1..16..9.

(7) If 40 acres of grass be mown by 8 men in 7 days, how many acres can be mown by 24 men in 28 days? *Ans.* 480.

(8) If £2 will pay 8 men for 5 days' work, how much will pay 32 men for 24 days' work? *Ans.* £38..8.

(9) If a regiment of soldiers, consisting of 1360 men, consume 351 quarters of wheat in 108 days, how much will 11232 soldiers consume in 56 days? *Ans.* $1503\frac{9}{85}$ *qrs.*

(10) If 939 horses consume 351 quarters of oats in 168 days, how many horses will consume 1404 quarters in 56 days? *Ans.* 11268.

(11) If I pay £14..10 for the carriage of 60 *cwt.* 20 miles, what weight can I have carried 30 miles for £5..8..9, at the same rate? *Ans.* 15 *cwt.*

(12) If 144 *threepenny* loaves serve 18 men for 6 days, how many *fourpenny* loaves will serve 21 men for 9 days? *Ans.* 189.

PRACTICE

Is so called from its general use among merchants and tradesmen.

It is a concise method of computing the value of articles, &c. by taking *aliquot parts.*

The General Rule is to *suppose* the *price* one pound, one shilling, or one penny each. Then will the given number of articles, considered accordingly as *pounds*, or *shillings*, or *pence*, be the *supposed* value of the whole; out of which the *aliquot part* or *parts* are to be taken for the *real* price.

Note. An *aliquot part* of a number is such a part as being taken a certain number of times will produce the *number exactly:* thus, 4 is an aliquot part of 12; because 3 fours are 12.

ALIQUOT PARTS.

Of a pound.

s.	d.		£
10	0	are	$\frac{1}{2}$
6	8	...	$\frac{1}{3}$
5	0	...	$\frac{1}{4}$
4	0	...	$\frac{1}{5}$
3	4	...	$\frac{1}{6}$
2	6	...	$\frac{1}{8}$
2	0	...	$\frac{1}{10}$
1	8	...	$\frac{1}{12}$
1	4	...	$\frac{1}{15}$
1	3	...	$\frac{1}{16}$
1	0	...	$\frac{1}{20}$
0	8	...	$\frac{1}{30}$
0	6	...	$\frac{1}{40}$

Of a shilling.

d.		s.
6	are	$\frac{1}{2}$
4	...	$\frac{1}{3}$
3	...	$\frac{1}{4}$
2	...	$\frac{1}{6}$
$1\frac{1}{2}$	...	$\frac{1}{8}$
1	is	$\frac{1}{12}$

Of a penny.

2 *qrs.* are $\frac{1}{2}$*d.*
1 *qr.* is $\frac{1}{4}$*d.*

Of a ton.

cwt.		*ton.*
10	are	$\frac{1}{2}$
5	...	$\frac{1}{4}$
4	...	$\frac{1}{5}$
2 3 *qr.* 12 *lb.*		$\frac{1}{7}$
$2\frac{1}{2}$	...	$\frac{1}{8}$
2	...	$\frac{1}{10}$
1	is	$\frac{1}{20}$

Of a cwt.

qr.	*lb.*		*cwt.*
2 or	56	are	$\frac{1}{2}$
1 or	28	...	$\frac{1}{4}$
	16	...	$\frac{1}{7}$
	14	...	$\frac{1}{8}$
	8	...	$\frac{1}{14}$
	7	...	$\frac{1}{16}$

Of a quarter.

lb.		*qr.*
14	are	$\frac{1}{2}$
7	...	$\frac{1}{4}$
4	...	$\frac{1}{7}$
$3\frac{1}{2}$	...	$\frac{1}{8}$
2	...	$\frac{1}{14}$
$1\frac{3}{4}$	...	$\frac{1}{16}$
1	is	$\frac{1}{28}$

Of a lb.

oz.		*lb.*
8	are	$\frac{1}{2}$
4	...	$\frac{1}{4}$
2	...	$\frac{1}{8}$
1	is	$\frac{1}{16}$

Of a lb. Troy.

oz.		*lb.*
6	are	$\frac{1}{2}$

4, &c. as in the parts of a shilling.

Of an oz. Troy.

The same as the parts of a £, changing the names from *shillings* to *dwts.*

Of a dwt.

gr.		*dwt.*
12	are	$\frac{1}{2}$
8	...	$\frac{1}{3}$
6	...	$\frac{1}{4}$
4	...	$\frac{1}{6}$
3	...	$\frac{1}{8}$
2	...	$\frac{1}{12}$
$1\frac{1}{2}$	...	$\frac{1}{16}$
1	is	$\frac{1}{24}$

RULE 1. When the price is less than a penny, call the given number *pence*, and take the *aliquot parts* that are in a penny; then divide by 12 and 20, to reduce the answer to pounds.

(1) $\frac{1}{4}$ is $\frac{1}{4}$ 5704 *lb.* at $\frac{1}{4}$
12) 1426
2|0) 11|8..10
Ans. £ 5..18..10.

(2) 7695 at $\frac{1}{2}$*d.*
Ans. £ 16..0..$7\frac{1}{2}$.

(3) 5470 at $\frac{1}{2}$*d.*
Ans. £ 11..7..11.

(4) 6547 at $\frac{3}{4}$*d.*
Ans. £ 20..9..$2\frac{1}{4}$.

(5) 4573 at $\frac{3}{4}$*d.*
Ans. £ 14..5..$9\frac{3}{4}$.

RULE 2. When the price is less than a shilling, call the given number *shillings*, take the *aliquot part* or *parts* that are in a shilling, add the quotients together, and divide by 20, as in the preceding rule.

*(1) 7547 at 1*d.*
Ans. £ 31..8..11.

†(2) 3751 at $1\frac{1}{4}$*d.*
Ans. £ 19..10..$8\frac{3}{4}$.

(3) 54325 at $1\frac{1}{2}$*d.*
Ans. £ 339..10..$7\frac{1}{2}$.

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* 1*d.* = $\frac{1}{12}$ 7547*s.*
2|0) 62|8..11
Ans. £ 31..8..11

† 1*d.* = $\frac{1}{12}$ 3751*s.*
$\frac{1}{4}$ = $\frac{1}{4}$ 312..7
78..$1\frac{3}{4}$
2|0) 39|0..$8\frac{3}{4}$
£ 19..10..$8\frac{3}{4}$. *Ans.*

(4) 6254 at $1\frac{3}{4}$*d.* *Ans.* £45..12..0$\frac{1}{2}$.	(18) 2715 at $5\frac{1}{4}$*d.* *Ans.* £59..7..0$\frac{3}{4}$.	(32) 9872 at $8\frac{3}{4}$*d.* *Ans.* £359..18..4.
(5) 2351 at 2*d.* *Ans.* £19..11..10.	(19) 3120 at $5\frac{1}{2}$*d.* *Ans.* £71..10.	(33) 5272 at 9*d.* *Ans.* £197..14.
(6) 7210 at $2\frac{1}{4}$*d.* *Ans.* £67..11..10$\frac{1}{2}$.	(20) 7521 at $5\frac{3}{4}$*d.* *Ans.* £180..3..9$\frac{3}{4}$.	(34) 6325 at $9\frac{1}{4}$*d.* *Ans.* £243..15..6$\frac{1}{4}$.
(7) 2710 at $2\frac{1}{2}$*d.* *Ans.* £28..4..7.	(21) 3271 at 6*d.* *Ans.* £81..15..6.	(35) 7924 at $9\frac{1}{2}$*d.* *Ans.* £313..13..2.
(8) 3250 at $2\frac{3}{4}$*d.* *Ans.* £37..4..9$\frac{1}{2}$.	(22) 7914 at $6\frac{1}{4}$*d.* *Ans.* £206..1..10$\frac{1}{2}$.	(36) 2150 at $9\frac{3}{4}$*d.* *Ans.* £87..6..10$\frac{1}{2}$.
(9) 2715 at 3*d.* *Ans.* £33..18..9.	(23) 3250 at $6\frac{1}{2}$*d.* *Ans.* £88..0..5.	(37) 6325 at 10*d.* *Ans.* £263..10..10.
(10) 7062 at $3\frac{1}{4}$*d.* *Ans.* £95..12..7$\frac{1}{2}$.	(24) 2708 at $6\frac{3}{4}$*d.* *Ans.* £76..3..3.	(38) 5724 at $10\frac{1}{4}$*d.* *Ans.* £244..9..3.
(11) 2147 at $3\frac{1}{2}$*d.* *Ans.* £31..6..2$\frac{1}{2}$.	(25) 3271 at 7*d.* *Ans.* £95..8..1.	(39) 6327 at $10\frac{1}{4}$*d.* *Ans.* £270..4..3$\frac{3}{4}$.
(12) 7000 at $3\frac{3}{4}$*d.* *Ans.* £109..7..6.	(26) 3254 at $7\frac{1}{4}$*d.* *Ans.* £98..5..11$\frac{1}{2}$.	(40) 3254 at $10\frac{1}{2}$*d.* *Ans.* £142..7..3.
(13) 3257 at 4*d.* *Ans.* £54..5..8.	(27) 2701 at $7\frac{1}{2}$*d.* *Ans.* £84..8..1$\frac{1}{2}$.	(41) 7291 at $10\frac{3}{4}$*d.* *Ans.* £326..11..6$\frac{1}{4}$.
(14) 2056 at $4\frac{1}{4}$*d.* *Ans.* £36..8..2.	(28) 3714 at $7\frac{3}{4}$*d.* *Ans.* £119..18..7$\frac{1}{2}$.	(42) 3256 at 11*d.* *Ans.* £149..4..8.
(15) 3752 at $4\frac{1}{2}$*d.* *Ans.* £70..7..0.	(29) 2710 at 8*d.* *Ans.* £90..6..8.	(43) 7254 at $11\frac{1}{4}$*d.* *Ans.* £340..0..7$\frac{1}{2}$.
(16) 2107 at $4\frac{3}{4}$*d.* *Ans.* £41..14..0$\frac{1}{4}$.	(30) 3514 at $8\frac{1}{4}$*d.* *Ans.* £120..15..10$\frac{1}{2}$.	(44) 3754 at $11\frac{1}{2}$*d.* *Ans.* £179..17..7.
(17) 3210 at 5*d.* *Ans.* £66..17..6.	(31) 2759 at $8\frac{1}{2}$*d.* *Ans.* £97..14..3$\frac{1}{2}$.	(45) 7972 at $11\frac{3}{4}$*d.* *Ans.* £390..5..11.

RULE 3. When the price is more than one shilling, and less than two, take the *part* or *parts* for the *excess* above a shilling, add the quotients to the given quantity, and reduce the whole to pounds as before. Or, when convenient, take the *aliquot part* of a pound.

* (1) 2106 at $12\frac{1}{4}$*d.* *Ans.* £107..9..10$\frac{1}{2}$.	(2) 3715 at $12\frac{1}{2}$*d.* *Ans.* £193..9..9$\frac{1}{2}$.	(3) 2712 at $12\frac{3}{4}$*d.* *Ans.* £144..1..6.

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* $d.$

$$\frac{1}{4} = \left\{\begin{array}{l} \frac{1}{12} \\ \text{of} \\ \frac{1}{4} \end{array}\right. \begin{array}{r} 2106s. \\ \hline (175..6) \\ \hline 43..10\frac{1}{2} \\ \hline 2|0)\,214|9..10\frac{1}{2} \\ \hline \end{array}$$

Ans. £107..9..10$\frac{1}{2}$

This example is worked by taking $\frac{1}{12}$, and then $\frac{1}{4}$ of that; because a farthing is $\frac{1}{48}$ of a shilling; which is $= \frac{1}{12}$ of $\frac{1}{4}$, or $\frac{1}{4}$ of $\frac{1}{12}$, because 4 *twelves* are 48.

(4) 2107 at 1*s.* 1*d.* *Ans.* £114..2..7.	(19) 2750 at 1*s.* 4¾*d.* *Ans.* £191..18..6½.	(34) 7104 at 1*s.* 8½*d.* *Ans.* £606..16.
(5) 3215 at 1*s.* 1¼*d.* *Ans.* £177..9..10¾.	(20) 3725 at 1*s.* 5*d.* *Ans.* £263..17..1.	(35) 1004 at 1*s.* 8¾*d.* *Ans.* £86..16..1.
(6) 2790 at 1*s.* 1½*d.* *Ans.* £156..18..9.	(21) 7250 at 1*s.* 5¼*d.* *Ans.* £521..1..10½.	(36) 2104 at 1*s.* 9*d.* *Ans.* £184..2.
(7) 7904 at 1*s.* 1¾*d.* *Ans.* £452..16..8.	(22) 2597 at 1*s.* 5½*d.* *Ans.* £189..7..3½.	(37) 2571 at 1*s.* 9¼*d.* *Ans.* £227..12..9¾.
(8) 3750 at 1*s.* 2*d.* *Ans.* £218..15.	(23) 7210 at 1*s.* 5¾*d.* *Ans.* £533..4..9½.	(38) 2104 at 1*s.* 9½*d.* *Ans.* £188..9..8.
(9) 3291 at 1*s.* 2¼*d.* *Ans.* £195..8..0¾.	(24) 7524 at 1*s.* 6*d.* *Ans.* £564..6.	(39) 7506 at 1*s.* 9¾*d.* *Ans.* £680..4..7½.
(10) 9264 at 1*s.* 2½*d.* *Ans.* £559..1..11.	(25) 7103 at 1*s.* 6¼*d.* *Ans.* £540..2..5¾.	(40) 1071 at 1*s.* 10*d.* *Ans.* £98..3..6.
(11) 7250 at 1*s.* 2¾*d.* *Ans.* £445..11..5½.	(26) 3254 at 1*s.* 6½*d.* *Ans.* £250..16..7.	(41) 5200 at 1*s.* 10¼*d.* *Ans.* £482..1..8.
(12) 7591 at 1*s.* 3*d.* *Ans.* £474..8..9.	(27) 7925 at 1*s.* 6¾*d.* *Ans.* £619..2..9¾.	(42) 2117 at 1*s.* 10½*d.* *Ans.* £198..9..4½.
(13) 6325 at 1*s.* 3¼*d.* *Ans.* £401..18..0¼.	(28) 9271 at 1*s.* 7*d.* *Ans.* £733..19..1.	(43) 1007 at 1*s.* 10¾*d.* *Ans.* £95..9..1¼.
(14) 5271 at 1*s.* 3½*d.* *Ans.* £340..8..4½.	(29) 7210 at 1*s.* 7¼*d.* *Ans.* £578..6..0½.	(44) 5000 at 1*s.* 11*d.* *Ans.* £479..3..4.
(15) 3254 at 1*s.* 3¾*d.* *Ans.* £213..10..10½.	(30) 2310 at 1*s.* 7½*d.* *Ans.* £187..13..9.	(45) 2105 at 1*s.* 11¼*d.* *Ans.* £203..18..5¼.
(16) 2915 at 1*s.* 4*d.* *Ans.* £194..6..8.	(31) 2504 at 1*s.* 7¾*d.* *Ans.* £206..1..2.	(46) 1006 at 1*s.* 11½*d.* *Ans.* £98..10..1.
(17) 3270 at 1*s.* 4¼*d.* *Ans.* £221..8..1½.	(32) 7152 at 1*s.* 8*d.* *Ans.* £596.	(47) 2705 at 1*s.* 11¾*d.* *Ans.* £267..13..7¾.
(18) 7059 at 1*s.* 4½*d.* *Ans.* £485..6..1½.	(33) 2905 at 1*s.* 8¼*d.* *Ans.* £245..2..2¼.	(48) 5000 at 1*s.* 11½*d.* *Ans.* £489..11..8.

RULE 4. When the price is an *even* number of shillings, the given quantity may be multiplied by *half* that number, doubling the units' figure of the product for shillings, and the rest of the product will be pounds. Or take the aliquot part of a pound.

(1) 2750 at 2*s.* *Ans.* £275.	(4) 1572 at 8*s.* *Ans.* £628..16.	(7) 5271 at 14*s.* *Ans.* £3689..14.
(2) 3254 at 4*s.* *Ans.* £650..16.	(5) 2102 at 10*s.* *Ans.* £1051.	(8) 3123 at 16*s.* *Ans.* £2498..8.
(3) 2710 at 6*s.* *Ans.* £813.	(6) 2101 at 12*s.* *Ans.* £1260..12.	(9) 1075 at 16*s.* *Ans.* £860.

(10) 1621 at 18*s*.
Ans. £1458..18.

NOTE. At 2*s*. take the *tenth*, and at 10*s*. take the *half* of so many £.

RULE 5. When the price is an *odd* number of shillings, work by Rule 4th for the *greatest even* number, and add $\frac{1}{20}$ of the given quantity for the odd shilling.—Or, take such parts of a pound, as will make the given price.

* (1) 3270 at 3*s*. *Ans.* £490..10.	(4) 3214 at 9*s*. *Ans.* £1446..6.	(7) 2150 at 15*s*. *Ans.* £1612..10.
(2) 3271 at 5*s*. *Ans.* £817..15.	(5) 2710 at 11*s*. *Ans.* £1490..10.	(8) 3142 at 17*s*. *Ans.* £2670..14.
(3) 2715 at 7*s*. *Ans.* £950..5.	(6) 3179 at 13*s*. *Ans.* £2066..7.	(9) 2150 at 19*s*. *Ans.* £2042..10.

RULE 6. When the price consists of shillings and pence, suppose the given number to be *pounds*, and take such *aliquot parts*, or the *sum* of such *aliquot parts*, as will make the given price.—Or, work for the shillings as in the preceding Rules, and take parts for the residue.

† (1) 2710 at 6*s*. 8*d*. *Ans.* £903..6..8.	‡ (7) 2710 at 3*s*. 2*d*. *Ans.* £429..1..8.	(13) 7152 at 17*s*. 6¾*d*. *Ans.* £6280..7.
(2) 3150 at 3*s*. 4*d*. *Ans.* £525.	(8) 7514 at 4*s*. 7*d*. *Ans.* £1721..19..2.	(14) 2510 at 14*s*. 7¼*d*. *Ans.* £1832..16..5½.
(3) 2715 at 2*s*. 6*d*. *Ans.* £339. 7..6.	(9) 2517 at 5*s*. 3*d*. *Ans.* £660..14..3.	(15) 3715 at 9*s*. 4½*d*. *Ans.* £1741..8..1½.
(4) 7150 at 1*s*. 8*d*. *Ans.* £595..16..8.	(10) 2547 at 7*s*. 3½*d*. *Ans.* £928..11. 10½.	(16) 2572 at 13*s*. 7½*d*. *Ans.* £1752..3..6.
(5) 3215 at 1*s*. 4*d*. *Ans.* £214..6..8.	(11) 3271 at 5*s*. 9¼*d*. *Ans.* £943..16..4¾.	(17) 7251 at 14*s*. 8¼*d*. *Ans.* £5324..19..0¾.
(6) 7211 at 1*s*. 3*d*. *Ans.* £450..13..9.	(12) 2103 at 15*s*. 4½*d*. *Ans.* £1616..13..7½.	(18) 3210 at 15*s*. 7¾*d*. *Ans.* £2511..3..1½.

RULE 7. When the price consists of pounds, shillings and pence, multiply the given quantity by the *number* of pounds, and take *aliquot parts* for the residue.—Or, work for the shillings as in the preceding Rules, &c.—Or, when the given number of articles is not large, work by Compound Multiplication.

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s		*s*.
* 2	= $\frac{1}{10}$	3270
1	= $\frac{1}{2}$	327
		163..10
	Ans.	£490..10

s. *d*.		£
† 6..8	= $\frac{1}{3}$	2710
	Ans.	£903..6..8

s. *d*.		£
‡ 2..6	= $\frac{1}{8}$	2710
8	= $\frac{1}{30}$	338..15
		90.. 6..8
	Ans.	£429.. 1..8

* (1) 7215 at £7..4. *Ans.* £51948.	(7) 2107 at £1..13. *Ans.* £3476..11.	(13) 3210 at £1..18..6¾. *Ans.* £6189..5..7½.
(2) 2104 at £5..3. *Ans.* £10835..12.	(8) 3215 at £4..6..8. *Ans.* £13931..13..4.	(14) 2157 at £2..7..4½. *Ans.* £5109..7..10½.
(3) 2107 at £2..8. *Ans.* £5056..16.	(9) 2154 at £7..1..3. *Ans.* £15212..12..6.	(15) 142 at £1..15..2¼. *Ans.* £250..2..6½.
(4) 7156 at £5..6. *Ans.* £37926..16.	(10) 2701 at £2..3..4. *Ans.* £5852..3..4.	(16) 95 at £15..14..7¼. *Ans.* £1494..7..4¾.
(5) 2710 at £2..3..7½. *Ans.* £5911..3..9.	(11) 2715 at £1..17..2½. *Ans.* £5051..0..7½.	(17) 37 at £1..19..5¼. *Ans.* £73..0..8¾
(6) 3215 at £1..17. *Ans.* £5947..15.	(12) 2157 at £3..15..2¼. *Ans.* £8108..19..5¼.	(18) 2175 at £2..15..4½. *Ans.* £6022..0..7½.

RULE 8. When the given quantity consists of several denominations, multiply the price by the number of the highest, and take aliquot parts for the inferior denominations.

(1) At £3..17..6 per *cwt.* what is the value of 25 *cwt.* 2 *qrs.* 14 *lb.* of soap?†

(2) At £1..4..9 per *cwt.* what is the value of 17 *cwt.* 1 *qr.* 17 *lb.*? *Ans.* £21..10..8.

(3) Sold 85 *cwt.* 1 *qr.* 10 *lb.* of iron, at £1..7..8 per *cwt.* what is the value of the whole? *Ans.* £118..1..0¾.

(4) If hops are sold at £4..5..8 per *cwt.* what must be given for 72 *cwt.* 1 *qr.* 18 *lb.*? *Ans.* £310..3..2.

(5) What is the value of 27 *cwt.* 2 *qrs.* 15 *lb.* of logwood, at £1..1..4 per *cwt.*? *Ans.* £29..9..6¼.

(6) Bought 78 *cwt.* 3 *qrs.* 12 *lb.* of molasses, at £2..17..9 per *cwt.* what must I give for the whole? *Ans.* £227..14.

(7) Sold 56 *cwt.* 1 *qr.* 17 *lb.* of sugar, at £2..15..9 per *cwt.* how much is the whole charge? *Ans.* £157..4..4¼.

(8) What is the value of 97 *cwt.* 15 *lb.* of currants, at £3..17..10 per *cwt.*? *Ans.* £378..0..3.

(9) At £4..14..6 the *cwt.* what is the value of 37 *cwt.* 2 *qrs.* 13 *lb.* of raw sugar? *Ans.* £177..14..8½.

---

s.	
* 4 = ⅕	7215
	7
	50505
	1443
	£51948 *Ans.*

† 2 *qrs.* = ½	£3..17..6
	5 × 5 = 25
	19..7..6
	5
lb.	96..17..6
14 = ¼	1..18..9
	9..8¼
	£99..5..11¼ *Ans.*

(10) Bought sugar at £3..14..6 the *cwt.*: what did I give for 15 *cwt.* 1 *qr.* 10 *lb.*? *Ans.* £57..2..9.

(11) Required the value of 17 *oz.* 8 *dwts.* 18 *grs.* of gold, at £3..17..10½ per *ounce.* *Ans.* £67..17..11.

(12) At £37..6..8 per *cwt.* the value of 1 *cwt.* 2 *qrs.* 10½ *lb.* of cochineal is required. *Ans.* £59..10.

(13) Required the value of 13 *hhds.* 42 *gals.* of Champagne wine, at £25..13..6 per *hhd.* *Ans.* £350..17..10.

(14) A gentleman purchased, at an auction, an estate of 149*a.* 3*r.* 20*p.* at £54..10 per acre. What was the whole purchase money, including the auction duty of 7*d.* in the £; the attorney's bill for the deeds of conveyance, £33..6..8; and his surveyor's charge for measuring it, at 1*s.* per acre?

Ans. £8447..5..0½.

RULE 9. To find the price of 1 *lb.* at a given *number of shillings per cwt.*

Multiply the shillings by 3, and divide the product by 7; the quotient will be the price of 1 *lb.* in farthings.*

(1) What is the price of 1 *lb.* at 44*s.* 4*d.* per *cwt.*?†

(2) What are the respective prices per *lb.* at 86*s.* 4*d.*; 91*s.*; and 116*s.* 8*d.* per *cwt.*? *Ans.* 9¼*d.*; 9¾*d.*; *and* 1*s.* 0½*d.*

RULE 10. It is sometimes expedient to change the price and the quantity for each other. Thus, 48 yards at 2*s.* 9*d.* will be equivalent to 33 yards at 4*s.*; because 2*s.* 9*d.* = 33*d.* and 4*s.* = 48*d.*

(1) What is the value of 72 *yds.* at 3*s.* 5*d.* and at 14*s.* 7*d.* per yard? *Ans.* £12..6, *and* £52..10.

(2) 80 *yds.* at 15*s.* 3*d.* and at 16*s.* 8*d.* per yard?

Ans. £61, *and* £66..13..4.

(3) 42 *lbs.* at 11½*d.* and at 1*s.* 3¼*d.* per *lb.*?

Ans. £2..0..3, *and* £2..13..4½.

TARE AND TRET.

Gross weight is the weight of any goods, together with that of the package which contains them.

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* Multiplying by 3 reduces the shillings to *fourpences*, and 7 fourpences (or 2*s.* 4*d.*) are the value of 1 *cwt.* at 1 *farthing* per *lb.*

† 44*s.* 4*d.*
3
7) 133
19 farthings = 4¾*d.* per *lb.* *Ans.*

Neat weight is that of the *articles alone,* or what remains after the deduction of all allowances.

Tare is an allowance for the weight of the package. It is either so much in the whole, or at so much per bag, box, barrel, &c. or at so much in the cwt.

Tret is an allowance of 4 *lb.* in 104 *lb.* (or $\frac{1}{26}$ part) for waste.

Cloff is an allowance of 2 *lb.* in 3 *cwt.* on some goods: but both these are nearly obsolete.

Suttle is the remainder when any particular allowance has been deducted.

RULE. When the *Tare* is at so much for each bag, &c. the *whole Tare* may be found by multiplying by the number of them. When it is at so much per *cwt.* take the *aliquot parts* of the *Gross* for the *Tare*. Subtract the *Tare* from the *Gross;* the remainder is the *Neat:* unless there is *Tret* allowed.

If *Tret* is allowed, it is $\frac{1}{26}$ of the *Tare suttle*, which being subtracted from it, the remainder is the *Neat*. But if *Cloff* also is to be allowed, the *cwts. Tret-suttle*, multiplied by 2, and divided by 3, will be the lbs. *Cloff*, which subtract to find the *Neat.*

(1) In 7 frails of raisins, each weighing 5 *cwt.* 2 *qrs.* 5 *lb.* gross, tare at 23 *lb.* per frail, how much neat weight?*

(2) What is the neat weight of 25 hogsheads of tobacco, weighing gross 163 *cwt.* 2 *qrs.* 15 *lb.* tare 100 *lb.* per hogshead? *Ans.* 141 *cwt.* 1 *qr.* 7 *lb.*

(3) In 16 bags of pepper, each weighing 85 *lb.* 4 *oz.* gross, tare per bag, 3 *lb.* 5 *oz.* how many pounds neat? *Ans.* 1311 *lb.*

(4) What is the neat weight of 5 hogsheads of tobacco, weighing gross 75 *cwt.* 1 *qr.* 14 *lb.* tare in the whole 752 *lb?* *Ans.* 68 *cwt.* 2 *qrs.* 18 *lb.*

(5) In 75 barrels of figs, each 2 *qrs.* 27 *lb.* gross, tare in the whole 597 *lb.* how much neat weight? *Ans.* 50 *cwt.* 1 *qr.*

(6) What is the neat weight of 18 butts of currants, each 8 *cwt.* 2 *qrs.* 5 *lb.* gross, tare at 14 *lb.* per *cwt.*?†

---

	cwt. qr. lb.	
*	5..2.. 5	gross.
	23	tare.
	5..1..10	neat of 1 frail.
	7	
Ans.	37..1..14	neat of the whole.

	cwt. qr. lb.	
†	8..2.. 5	
	9	$9 \times 2 = 18$
	76..3..17	
lb.	2	
$14 = \frac{1}{8}$	153..3.. 6	whole gross.
	19. 0 .25¼	tare.
Ans.	134 .2.. 8¾	neat.

(7) In 25 barrels of figs, each 2 *cwt.* 1 *qr.* gross, tare per *cwt.* 16 *lb.* how much neat weight? *Ans.* 48 *cwt.* 0 *qr.* 24 *lb.*

(8) What is the neat weight of 9 hogsheads of sugar, each weighing gross 8 *cwt.* 3 *qrs.* 14 *lb.* tare 16 *lb.* per *cwt?*
Ans. 68 *cwt.* 1 *qr.* 24 *lb.*

(9) In 1 butt of currants, weighing 12 *cwt.* 2 *qrs.* 24 *lb.* gross, tare 14 *lb.* per *cwt.* tret 4 *lb.* per 104 *lb.* what is the neat weight?*

(10) In 7 *cwt.* 3 *qrs.* 27 *lb.* gross, tare 36 *lb.* tret according to custom, how many pounds neat? *Ans.* 826 *lb.*

(11) In 152 *cwt.* 1 *qr.* 3 *lb.* gross, tare 10 *lb.* per *cwt.* tret as u ual, how much neat weight? *Ans.* 133 *cwt.* 1 *qr.* 12 *lb.*

(12) What is the neat weight of 3 hogsheads of tobacco, weighing 15 *cwt.* 3 *qrs.* 20 *lb.* gross, tare 7 *lb.* per *cwt.* tret and cloff as usual?†

(13) In 7 hogsheads of tobacco, each weighing gross 5 *cwt.* 2 *qrs.* 7 *lb.*; tare 8 *lb.* per *cwt.* tret and cloff as usual, how much neat weight? *Ans.* 34 *cwt.* 2 *qrs.* 8 *lb.*

INVOICES, OR BILLS OF PARCELS.

(1) Mrs. Bland. *London, Sept.* 1, 1830.

Bought of Jane Harris.

		s.	d.		£	s.	d.
15 pairs worsted stockings	at	4	6	per pr. .			
1 doz. thread ditto . .	at	3	2	....			
½ doz. black silk ditto .	at	8	3	....			
1½ doz. milled hose . .	at	4	2	....			
2 doz. cotton ditto . .	at	7	6	....			
17 pairs kid gloves . .	at	1	8	....			
					£21..18..4		

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lb.	*cwt. qrs. lb.*	
* 14 = $\frac{1}{8}$	12.. 2..24	gross.
lb.	1.. 2..10	tare.
4 = $\frac{1}{26}$	11.. 0..14	suttle.
	1..19	tret.
Ans.	10.. 2..23	neat.

lb.	*cwt. qr. lb.*	
† 7 = $\frac{1}{16}$	15.. 3..20	gross.
	3..27½	tare.
26)	14.. 3..20½	suttle.
	2.. 8	tret.
	14.. 1..12½	suttle.
$\overline{14 \times 2} \div 3 =$	9½	cloff.
Ans.	14.. 1.. 3	neat.

(2) Mr. Isaac Pearson, *Derby, June* 3, 1830.

Bought of John Sims and Son.

		s.	d.		£	s.	d.
15 yds. satin	at	9	6	per yard .			
$18\frac{1}{4}$ yds. flowered silk. .	at	17	4	....			
12 yds. rich brocade. .	at	19	8	....			
$16\frac{1}{2}$ yds. sarcenet . . .	at	3	2	....			
$13\frac{1}{8}$ yds. Genoa velvet .	at	27	6	....			
23 yds. lustring . . .	at	6	3	....			
					£62..11..$9\frac{1}{4}$		

(3) Miss Enfield, *Nottingham, June* 4, 1830.

Bought of Joseph Thompson.

		s.	d.		£	s.	d.
$4\frac{1}{2}$ yds. cambric . . .	at	12	6	per yard .			
$12\frac{1}{2}$ yds. muslin . . .	at	8	3	....			
15 yds. printed calico .	at	5	4	....			
2 doz. napkins . . .	at	2	3	each			
14 ells diaper	at	1	7	per ell . .			
35 ells dowlas	at	1	$1\frac{1}{2}$	....			
					£17..14..11		

Received the above,

JOSEPH THOMPSON.

(4) Mrs. Mary Bright sold to the Right Honourable Lady Anna Maria Lamb, 18 yards of French lace, at 12*s.* 3*d.* per yd.; 5 pairs of fine kid gloves, at 2*s.* 2*d.* per pair; 1 dozen French fans, at 3*s.* 6*d.* each; two superb silk shawls, at three guineas each; 4 dozen Irish lamb, at 1*s.* 3*d.* per pair; and 6 sets of knots, at 2*s.* 6*d.* per set.—Please to make out the Invoice for her. *Total amount,* £23..14..4.

(5) Mr. Thomas Ward sold to James Russell Vernon, Esq. $17\frac{1}{4}$ yards of fine serge, at 3*s.* 9*d.* per yd.; 18 yds. of drugget, at 9*s.* per yd.; $15\frac{1}{2}$ yds. of superfine scarlet, at 22*s.* per yd.; $16\frac{1}{4}$ yds. of Yorkshire black, at 18*s.* per yd.; 25 yds. of shalloon, at 1*s.* 9*d.* per yd.; and 17 yards of drab, at 17*s.* 6*d.* per yd.—Make an Invoice of these articles. *Total amount,* £60..10..$5\frac{1}{4}$.

(6) Mr. Samuel Green, of Wolverhampton, sent to Messrs. Wright and Johnson, agreeably to order, 27 calf skins, at 3*s.* 6*d.* each; 75 sheep skins, at 1*s.* 7*d.*; 39 coloured ditto, at

1s. 8d.; 15 buck skins, at 11s. 6d.; 17 Russia hides, at 10s. 7d.; and 125 lamb skins, at 1s. 2½d.—Draw up the Invoice.

Total amount, £39..1..8½.

(7) Mr. Richard Groves sent the following articles to the Rev. Samuel Walsingham; *viz.* 2 stones of raw sugar, at 6½d. per ℔; 2 loaves of sugar, 15½ ℔, at 11½d. per ℔; a stone of East India rice, at 3½d. per ℔; 2 stone Carolina rice, at 5d. per ℔; 15 oz. nutmegs, at 5½d. per oz.; and half a stone of Dutch coffee, at 1s. 10d. per ℔.—Make a copy of the Invoice.

Total amount, £3..5..5¾.

BILLS OF BOOK-DEBTS.

(8) Mr. Charles Cross, *Chester.*

To Samuel Grant and Co. Dr.

1830.		s.	d.		£	s.	d.
Apr. 14.	Belfast butter, 1 cwt.	0	6½	per ℔.			
	Cheese, 17cwt. 3qrs. 12 ℔ at	56	0	long cwt.			
May 8.	Butter ½ firkin, 28 ℔ at	0	5½	per ℔.			
July 17.	5 Cheshire cheeses, 127 ℔ at	0	6¼	....			
Sept. 4.	2 Stilton ditto, 15 ℔ at	0	10½	....			
	Cream cheese, 13 ℔ at	0	8½	....			
					£30..1..6¾		

Dec. 28. Received the contents,

SAMUEL GRANT.

(9) Mr Charles Septimus Twigg, *Newark.*

To Isaac Jones, Dr.

1829.		s.	d.		£	s.	d.
Oct. 22.	Tares, 39 bushels, at	1	10	per bush.			
1830.	Pease, 18 bushels, at	30	4	per qr....			
Feb. 18.	Malt, 7 qrs. . at	63	6	per qr....			
	Hops, 2cwt. 1 qr. at	1	5	per ℔....			
Feb. 20.	Oats, 6qrs. . at	2	4½	per bush.			
	Beans, 17qrs. . at	37	4	per qr....			
					£84..9..11		

1830, *July* 1. *Received the above for Isaac Jones,*

THOMAS WEST.

SIMPLE INTEREST

Is the premium allowed for the loan of any sum of money during a given space of time.

The *Principal* is the money lent, for which *Interest* is to be received.

The *Rate per cent per annum* is the quantity of *Interest* (agreed on between the Borrower and the Lender) to be paid for the use of every £100 of the *Principal*, for one year.

The *Amount* is the Principal and Interest added together.

I. *To find the Interest of any Sum of Money for a Year.*

RULE. Multiply the *Principal* by the *Rate per cent*, and that Product divided by 100, will give the *Interest* required.

NOTE. When the *Rate* is an aliquot part of 100, the *Interest* may be calculated more expeditiously by taking such part of the *Principal*. Thus, for 5 per cent, take $\frac{1}{20}$; for 4 per cent, $\frac{1}{25}$, or $\frac{1}{5}$ of $\frac{1}{5}$; for 2 per cent, $\frac{1}{50}$; for $2\frac{1}{2}$ per cent, $\frac{1}{40}$; for 3 per cent, $\frac{1}{50}$, *plus* $\frac{1}{2}$ of that; &c.

This rule is applied to the calculation of Commission, Brokerage, Purchasing Stocks, Insurance, Discounting of Bills, &c.*

II. *For several Years.* Multiply the *Interest* of *one* year by the *number* of years, and the product will be the answer.

For parts of a year, as months and days, &c. the *Interest* may be found by taking the aliquot parts of a year; or by the Rule of Three: and it is customary to allow 12 months to the year, and 30 days to a month.†

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* To discount a Bill of Exchange is to advance the cash for it before it becomes due; deducting the *Interest* for the time it has to run. Bankers always charge *Discount* as the *Interest* of the sum.

† At the rate of 5 per cent, the interest of £1 for a year is 1*s.*; or *one penny* for a *month*. Therefore, the *principal* × the *number of months*, gives the *interest* in *pence*.

Or, take the parts of a year for the *months*, out of *as many shillings* as there are *pounds* in the *principal*.

Thus, to find the interest of £40..10 for 2 months, say $40\frac{1}{2}d. \times 2 = 81d. = 6s. 9d.$; or, 2 months being $\frac{1}{6}$ of a year, $40s. 6d. \div 6 = 6s. 9d.$ *Ans.*

For days, take the aliquot parts of a month. The interest for days, at 5 per cent, may also be found by multiplying the principal by the number of days, and the product divided by 365 will give the answer in *shillings*; or divided by 7300 (= 365 × 20) will give the answer in pounds.

(1) What is the interest of £375 for a year, at £5 per cent per annum?*

(2) What is the interest of £945..10 for a year, at £4 per cent per annum? *Ans.* £37..16..4¾.

(3) What is the interest of £547..15, at £5 per cent per annum, for 3 years? *Ans.* £82..3..3.

(4) What is the interest of £254..17..6 for 5 years, at £4 per cent per annum? *Ans.* £50..19..6.

(5) What is the amount of £556..13..4, at £5 per cent per annum, in 5 years? *Ans.* £695..16..8.

NOTE. *Commission* and *Brokerage* (commonly called *Brokage*) are allowances of so much per cent to an agent or broker, for buying or selling goods, or transacting business for another.

(6) My correspondent informs me that he has bought goods to the amount of £754..16, on my account: what is his commission at £2½ per cent? *Ans.* £18..17..4¾.

(7) If I allow my factor £3¾ per cent for commission, what will he require on £876..5..10? *Ans.* £32..17..2½.

NOTE. *Stock* is a general term to designate the capitals of our trading companies; or to denote *property* in the *public Funds;* which means the money paid by Government for the interest of the *National Debt.* The quantity of Stock is a *nominal* sum, for which the owner receives a certain rate of interest while he holds the same.

(8) At £110¼ per cent, what is the purchase of £2054..16, South Sea stock? *Ans.* £2265..8..4.

(9) At £104⅜ per cent, South Sea annuities, what is the purchase of £1797..14? *Ans.* £1876..6..11¾.

(10) At £96¾ per cent, what is the purchase of £577..19, Bank annuities? *Ans.* £559..3..3¾.

(11) At £124⅝ per cent, what is the purchase of £758..17..10, India stock? *Ans.* £945..15..4¼.

(12) What sum will purchase £1284, of the 3 per cent consols, at £59⅞ per cent; including the broker's charge of ⅛, or 2*s.* 6*d.* per cent, on the amount of stock? *Ans.* £770..7..11¼.

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* £375	
5	Better thus:
£ 18|75	£ £
20	5 = $\frac{1}{20}$ 375
s. 15,00	*Ans.* 18..15
Ans. 18..15	

Cutting the two figures in the above divides the number by 100; see Division, p. 23.

(13) If I employ a broker to buy goods for me, to the amount of £2575..17..6, what is the brokerage at 4*s.* per cent?*

(14) What is the broker's charge on a sale amounting to £7105..5..10, at 5*s.* 6*d.* per cent? *Ans.* £19..10..9¼.

(15) What is the brokage on goods sold for £975..6..4, at 6*s.* 6*d.* per cent? *Ans.* £3..3..4½.

(16) What is the interest of £257..5..1, at £4 per cent per annum, for a year and three quarters? *Ans.* £18..0..1½.

(17) What is the interest of £479..5, for 5¼ years, at £5 per cent per annum? *Ans.* £125..16..0¾.

(18) What is the amount of £576..2..7, in 7¼ years, at £4½ per cent per annum? *Ans.* £764..1..8½.

(19) What is the interest of £259..13..5, for 20 weeks, at £5 per cent per annum? *Ans.* £4..19..10¼.

(20) What is the interest of £2726..1..4, at £4½ per cent per annum, for 3 years, 154 days? *Ans.* £419..15..6¼.

(21) Compute the interest of £155, for 49 days, and for 146 days, at £5 per cent per annum?

Ans. £1..0..9½, *and* £3..2..0.

(22) What will a banker charge for the discount of a bill of £76..10, and another of £54, negotiated on the 18th of May; the former becoming due June 30, and the latter July 13; discounting at £5 per cent? *Ans.* 8*s.* 11*d.* *and* 8*s.* 3*d.*

When the Amount, Time, and Rate per cent are given, to find the Principal.

RULE. As the amount of £100, at the rate and for the time given, is to £100; so is the amount given, to the principal required.

(23) What principal being put to interest will amount to £402..10 in 5 years, at £3 per cent per annum?†

(24) What principal being put to interest for 9 years, will amount to £734..8, at £4 per cent per annum? *Ans.* £540.

---

s.	£ *s.* *d.*	
* 4 = ⅕	2575..17..6	03
	£ 5|15.. 3..6	12
	20	42
	s. 3|03	4
	Ans. £5..3..0¼.	1|68

† £3 × 5 + 100 = £115. As £115 : £100 : : £402..10*s.* : £350. *Ans.*

(25) What principal being put to interest for 7 years, at £5 per cent per annum, will amount to £334..16?

Ans. £248.

When the Principal, Rate per cent, and Amount are given, to find the Time.

RULE. As the interest for one year, is to 1 year; so is the whole interest, to the number of years.

(26) In what time will £350 amount to £402..10, at £3 per cent per annum?*

(27) In what time will £540 amount to £734. 8, at £4 per cent per annum? *Ans.* 9 *years.*

(28) In what time will £248 amount to £334..16, at £5 per cent per annum? *Ans.* 7 *years.*

When the Principal, Amount, and Time are given, to find the Rate per cent.

RULE. As the principal, is to the whole interest; so is £100, to its interest for the given time. Divide that interest by the number of years, and the quotient will be the rate per cent.

(29) At what rate per cent will £350 amount to £402..10, in 5 years?†

(30) At what rate per cent will £248 amount to £334..16, in 7 years? *Ans.* £5 *per cent.*

(31) At what rate per cent will £540 amount to £734..8, in 9 years? *Ans.* £4 *per cent.*

DISCOUNT

Is the abatement of so much money, on any sum received before it is due, as the money received, if put to interest, would gain at the rate, and in the time given. Thus £100 *present money* would discharge a debt of £105 to be paid a year hence, *Discount* being made at £5 per cent.

---

* $\frac{\overset{\pounds}{350} \times 3}{100} = $ £10..10, the interest for 1 year.
£402..10—£350 = £52..10, the whole interest.
As £10..10 : 1 year : : £52..10 : 5 years. *Ans.*

† As £350 : £52..10 : : £100 : £15 = the interest of £100 for 5 years. Then 15 ÷ 5 = £3, the rate per cent.

RULE. As £100 with its interest for the time given, is to that interest; so is the sum given, to the *Discount* required.

Also, as that *Amount* of £100, is to £100; so is the given sum, to the *Present worth.*

But, if either the *Discount* or the *Present worth* be found by the proportion, the other may be found by subtracting that from the given sum.

(1) What are the discount and present worth of £386..5 for 6 months, at £6 per cent per annum?*

(2) How much shall I receive in present payment for a debt of £357..10, due 9 months hence; allowing discount at £5 per cent per annum? *Ans.* £344..11..6$\frac{3}{4}$ $\frac{15}{83}$.

(3) What is the discount of £275..10 for 7 months, at £5 per cent per annum? *Ans.* £7..16..1$\frac{3}{4}$ $\frac{95}{247}$.

(4) What is the present worth of £527..9..1, payable in 7 months, at £4$\frac{1}{4}$ per cent per annum?

Ans. £514..13..10$\frac{1}{2}$ $\frac{6930}{24595}$.

(5) Required the present worth of £875..5..6, due in 5 months, at £4$\frac{1}{2}$ per cent per annum? *Ans.* £859..3..3$\frac{1}{4}$ $\frac{73}{4075}$.

(6) What is the present worth of £500, payable in 10 months, at £5 per cent per annum? *Ans.* £480.

(7) How much ready money ought I to receive for a note of £75, due in 15 months, at £5 per cent per annum?

Ans. £70..11..9$\frac{3}{17}$.

(8) What will be the present worth of £150, payable at 3 instalments of four months; *i.e.* one third at 4 months, one third at 8 months, and one third at 12 months, discounting at £5 per cent per annum? *Ans.* £145..3..8$\frac{1}{2}$.

(9) Of a debt of £575..10, one moiety is to be paid in 3 months, and the other in 6 months. What discount must be allowed for present payment, at £5 per cent per annum?

Ans. £10..11..4$\frac{1}{4}$.

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* 6 m. = $\frac{1}{2}$ £6 £
100 + 3 = 103 = amount of £100 in 6 months.

```
    £    £        s.
As 103 : 3 : : 386.. 5   £   s.
                     3  386..5
          103)1158..15( 11..5 discount.
              1133     375..0 present worth.
                25
                20
           103)515 = 5s.
```

(10) What is the present worth of £500, at £4 per cent per annum; £100 being to be paid down, and the rest at two 6 months? *Ans.* £488..7..8½.

(11) Bought goods amounting to £109..10, at 6 months' credit, or £3½ per cent discount for prompt payment. How much ready money will discharge the account?*

Ans. £105..13..4½.

NOTE. The Rule to find the present worth of any sum of money is precisely identical with that case in Simple Interest in which the Amount, Time, and Rate per cent are given to find the Principal. See page 70.

COMPOUND INTEREST

Is that which arises from both the Principal and Interest: that is, when the Interest of money, having become due, and not being paid, is added to the Principal, and the subsequent Interest is computed on the *Amount.*

RULE. Compute the first year's interest, which add to the principal: then find the interest of that amount, which add as before, and so on for the number of years. Subtract the given sum from the *last Amount,* and the remainder will be the *Compound Interest.*

(1) What is the compound interest of £500, forborne 3 years, at £5 per cent per annum?†

(2) What is the amount of £400 in 3½ years, at £5 per cent per annum, compound interest? *Ans.* £474..12..6¼.

(3) What will £650 amount to in 5 years, at £5 per cent per annum, compound interest? *Ans.* £829..11..7½.

(4) What is the amount of £550..10 for 3½ years, at £6 per cent per annum, compound interest? *Ans.* £675..6..5.

(5) What is the compound interest of £764 for 4 years and 9 months, at £6 per cent per annum? *Ans.* £243..18..8.

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* The discount in cases of this sort is so much per cent on the sum, without regard to time. It is, therefore, computed as a year's interest.

† $\frac{1}{20}$	£500		$\frac{1}{20}$	£551..5
	25			27..11..3
$\frac{1}{20}$	525 amount in 1 yr.			578..16..3 amount in 3 years.
	26..5			500..0..0 principal subtract.
	551..5 do. in 2 yrs.			£78..16..3 *Ans.*

(6) What is the compound interest of £57..10..6, for 5 years, 7 months, and 15 days, at £5 per cent per annum?
Ans. £18..3..8$\frac{1}{4}$.

(7) What is the compound interest of £259..10, for 3 years, 9 months, and 10 days, at £4$\frac{1}{2}$ per cent per annum?
Ans. £46..19..10$\frac{1}{2}$.

EQUATION OF PAYMENTS

Is, when several sums are due at different times, to find a mean time for paying the whole debt; to do which, this is the common

RULE. Multiply each term by its time, and divide the sum of the products by the whole debt; the quotient is accounted the mean time.

(1) A owes B £200, whereof £40 is to be paid at 3 months, £60 at 5 months, and £100 at 10 months. At what time may the whole debt be paid together, without prejudice to either?

$$\begin{array}{rcl} & & £ \\ 40 \times 3 & = & 120 \\ 60 \times 5 & = & 300 \\ 100 \times 10 & = & 1000 \\ & & 2|00)\overline{14|20} \end{array}$$

Ans. 7$\frac{1}{10}$ *months.*

(2) B owes C £800, whereof £200 is to be paid at 3 months, £100 at 4 months, £300 at 5 months, and £200 at 6 months; but they agree that the whole shall be paid at once. What is the equated time? *Ans.* 4 *months,* 18$\frac{6}{8}$ *days.*

(3) A debt of £360 was to have been paid as follows: *viz.* £120 at 2 months, £200 at 4 months, and the rest at 5 months; but the parties have agreed to have it paid at one mean time. What is that time? *Ans.* 3 *months,* 13$\frac{1}{3}$ *days.*

(4) A merchant bought goods to the value of £500, to pay £100 at the end of 3 months, £150 at the end of 6 months, and £250 at the end of 12 months; but it was afterwards agreed to discharge the debt at one payment. Required the time. *Ans.* 8 *months,* 12 *days.*

(5) H is indebted to L a certain sum, which is to be paid at 6 different payments; that is, $\frac{1}{4}$ at 2 months, $\frac{1}{8}$ at 3 months, $\frac{1}{8}$ at 4 months, $\frac{1}{4}$ at 5 months, $\frac{1}{8}$ at 6 months, and the rest at 7 months; but they mutually agree that the whole shall be paid at one equated time. What is that time? *Ans.* 4$\frac{1}{4}$ *months.*

(6) A is indebted to B £120, whereof $\frac{1}{2}$ is to be paid at 3 months, $\frac{1}{4}$ at 6 months, and the rest at 9 months. What is the equated time of the whole payment? *Ans.* 5$\frac{1}{4}$ *months.*

BARTER

Is the exchanging of commodities.

Rule. Compute, by the most expeditious method, the value of the article whose quantity is given: then find what quantity of the other, at the rate proposed, may be had for the same money.

Note. Sometimes one tradesman, in bartering, advances his goods above the ready money price. In this case, it will be necessary to proportionate the other's bartering price to his ready money price, by the Rule of Three.

(1) What quantity of chocolate, at 4*s.* per *lb.* must be exchanged for 2 *cwt.* of tea, at 9*s.* per *lb.*? *

(2) A and B barter: A has 20 *cwt.* of prunes, at 4*d.* per *lb.* ready money, but in barter will have 5*d.* per *lb.* and B has hops worth 32*s.* per *cwt.* ready money: what ought B to charge his hops, and what quantity must he give for the 20 *cwt.* of prunes? †

(3) How much tea at 9*s.* per *lb.* can I have in barter for 4 *cwt.* 2 *qrs.* of chocolate, at 4*s.* per *lb.*? *Ans.* 2 *cwt.*

(4) A exchanges with B $23\frac{1}{2}$ *cwt.* of cheese, worth 52*s.* 6*d.* per *cwt.* for 8 pieces of cloth, containing 248 yards, at 4*s.* 4*d.* per yard; the difference to be paid in money. Who receives the balance, and how much? *Ans. A receives* £7..19..1.

(5) How much ginger, at $15\frac{1}{4}$*d.* per *lb.* must be exchanged for $3\frac{1}{2}$ *lb.* of pepper, at $13\frac{1}{2}$*d.* per *lb.*? *Ans.* 3 *lb.* $1\frac{35}{61}$ *oz.*

(6) How many dozen of candles, at 5*s.* 2*d.* per dozen, must be bartered for 3 *cwt.* 2 *qrs.* 16 *lb.* of tallow, at 37*s.* 4*d.* per *cwt.*? *Ans.* 26 *dozen*, $3\frac{54}{62}$ *lb.*

(7) A exchanges with B 608 yards of cloth, worth 14*s.* per

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* 224 × 9 = 2016*s.* the value of the tea.
As 4*s.* : 1 *lb.* : : 2016*s.* : 504 *lb.* of chocolate. *Ans.*

† As 4*d.* : 5*d.* : : 32*s.* : 40*s.* the price per *cwt.* to be charged for the hops.

20 *cwt.* = 2240 *lb.*
5
11200*d.* the value of the prunes.

As 40*s.* : 1 *cwt.* : : 11200*d.* : $\frac{11200}{480}$ = 23 *cwt.* 1 *qr.* $9\frac{16}{48}$ *lb.* *Ans.*
12
480*d.*

yard, for 85 *cwt.* 2 *qrs.* 24 *lb.* of bees' wax, and £125..12 in cash. What was the wax charged per *cwt.*? *Ans.* £3..10.

(8) A barters with B 320 dozen of candles at 4*s.* 6*d.* per dozen, for cotton at 8*d.* per *lb.* and £30 in cash. What was the quantity of cotton? *Ans.* 11 *cwt.* 1 *qr.*

(9) How much cotton, at 1*s.* 2*d.* per *lb.* must be given for 114 *lb.* of tobacco, at 6*d.* per *lb.*? *Ans.* 48$\frac{6}{7}$ *lb.*

PROFIT AND LOSS

Is a Rule by which we discover the gain or loss in the buying and selling of goods; and which enables us to adjust the prices of articles, so as to gain or lose so much per cent, &c.

The questions are solved by the Rule of Three, or Practice.

The *prime cost* means the purchase money; therefore

The prime cost $\left\{\begin{array}{l}\textit{plus}\text{ the gain, or}\\ \textit{minus}\text{ the loss,}\end{array}\right\}$ *equal* the selling price.

The selling price *minus* $\left\{\begin{array}{l}\text{the prime cost }\textit{equal}\text{ the gain.}\\ \text{the gain }\textit{equal}\text{ the prime cost.}\end{array}\right.$

The selling price *plus* the loss *equal* the prime cost.

Gain or loss *per cent* means so much on £100 purchase money, or *prime cost:* therefore, when £20 per cent are gained, £120 is the *selling price per cent;* when £20 per cent are lost, £80 is the *selling price.*

Case 1. Given, the *prime cost* and the *selling price* of an integer or quantity, to find the *gain* or *loss per cent.*

As the *prime cost* given : the *gain* or *loss* : : £100 : the *gain* or *loss per cent.*

Case 2. Given, the *prime cost* as before, with a proposed *gain* or *loss per cent*, to find the *selling price.*

As £100 : $\left\{\begin{array}{l}\text{£100 plus the }\textit{gain}\\ \text{or £100 minus the }\textit{loss}\end{array}\right\}$: : the *prime cost* : the *selling price.*

Case 3. Given, the *selling price* of an integer or quantity, and the *gain* or *loss per cent*, to find the *prime cost.*

$\left.\begin{array}{l}\text{As £100 plus the }\textit{gain}\\ \text{or, £100 minus the }\textit{loss}\end{array}\right\}$: £100 : : the *selling price* : the *prime cost.*

Case 4. Given, the *selling price* of an integer, and the *gain per cent*, to find the *gain per cent* at some *other proposed price.*

As the *selling price* : £100 plus the *gain* : : the *proposed price* : the *selling price per cent*, from which deduct £100 for the *gain per cent required.*

Secondly, To find another *selling price*, at a *different gain per cent.*

As £100 plus the *gain* : the *selling price* : : £100 plus the *proposed gain* : the *selling price* required.

A much greater variety of cases may occur; but it is presumed that the student who attains a due knowledge of these, will easily comprehend the rest.

(1) If 1 yard of cloth costs 11*s*. and is sold for 12*s*. 6*d*. what is the gain per cent?*

(2) If 60 ells of Holland cost £18, what must 1 ell be sold for to gain £8 per cent?†

(3) If 1 *lb*. of tobacco cost 16*d*. and be sold for 20*d*. what is the gain per cent? *Ans*. £25.

(4) If a parcel of cloth be sold for £560, gaining £12 per cent, what is the prime cost? *Ans*. £500.

(5) If a yard of cloth be bought for 13*s*. 4*d*. and sold again for 16*s*. what is the gain per cent? *Ans*. £20.

(6) If 112 *lb*. of iron cost 27*s*. 6*d*. what must 1 *cwt*. be sold for to gain £15 per cent? *Ans*. £1..11..7$\frac{1}{2}$.

(7) If 375 yards of cloth be sold for £490, at £20 per cent profit, what did it cost per yard? *Ans*. £1..1..9$\frac{1}{4}$ $\frac{1}{3}$q.

(8) Sold 1 *cwt*. of hops for £6..15, at the rate of £25 per cent profit. What would have been the gain per cent, if they had been sold for £8 per *cwt*.? *Ans*. £48..2..11$\frac{1}{2}$ $\frac{2}{9}$.

(9) If 90 ells of cambric cost £60, how must I sell it per yard to gain £18 per cent? *Ans*. 12*s*. 7$\frac{18}{450}$*d*.

(10) A plumber sold 10 fothers of lead for £204..15, and gained after the rate of £12..10 per cent. What did it cost him per *cwt*.? *Ans*. 18*s*. 8*d*.

(11) What was the profit on 436 yards of cloth, bought at 8*s*. 6*d*. and sold at 10*s*. 4*d*. per yard? *Ans*. £39..19..4.

(12) Bought 14 tons of steel at £69 per ton, which was retailed at 6*d*. per *lb*. What was the loss sustained?

Ans. £182.

(13) Bought 124 yards of linen for £32. How should the same be retailed per yard, to gain £15 per cent?

Ans. 5*s*. 11$\frac{28}{124}$*d*.

(14) Bought 249 yards of cloth at 3*s*. 4*d*. per yard, and retailed the same at 4*s*. 2*d*. per yard. What was the whole gain, and how much per cent?‡

Ans. £10..7..6 *profit, and* £25 *per cent*.

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* As 11*s*. (*cost*) : 1*s*. 6*d*. (*gain*) : : £100 (*cost*) : $\frac{100 \times 3}{22}$ = £13..12..8$\frac{1}{2}$ $\frac{10}{11}$. *Ans*.
2 2
sixp. 22 3 *sixp*.

† As £100 (*cost*) : £108 (*s. price*) : : £18 (*cost*) : $\frac{108 \times 18}{100}$ = £19..8..9$\frac{1}{2}$ $\frac{4}{10}$, the selling price. And £19..8..9$\frac{1}{2}$ ÷ 60 = 6*s*. 5$\frac{3}{4}$*d*. the price per ell.

‡ For the solving of this question, see Cases 1 and 2.

FELLOWSHIP, OR PARTNERSHIP,

Is a rule by which any number or quantity may be divided into certain proportionate parts. It is applied to determine the respective shares of gain or loss of the several partners in a company, in proportion to their respective shares of the capital employed as a joint stock: also in the division of common lands, and other cases of a similar kind.

FELLOWSHIP WITHOUT TIME.

RULE. As the whole stock, is to the whole gain or loss; so is each individual share, to the correspondent gain or loss.

PROOF. The sum of the shares will be equal to the whole gain or loss.

(1) A and B join in trade. A puts into stock £20, and B £40; they gain £50. What is the share of each?*

(2) A, B, and C, joined in trade: A put in £20; B, £30; and C, £40; and they gained £180. What is each man's part of the gain? *Ans. A,* £40; *B,* £60; *C,* £80.

(3) Four persons, B, C, D, and E, formed a joint stock: B put in £227; C, £349; D, £115; and E, £439: they gained £428. Required each person's share of the gain.

Ans. B, £85..19..6$\frac{3}{4}$ $\frac{69}{113}$. *C,* £132..3..9$\frac{12}{113}$.
D, £43..11..1$\frac{3}{4}$ $\frac{26}{113}$. *E,* £166..5..6$\frac{1}{4}$ $\frac{7}{113}$.

(4) D, E, and F, entered into partnership. D's stock was £750; E's, £460; and F's, £500; and at the end of 12 months they had gained £684. What is each man's particular share of the gain? *Ans. D,* £300; *E,* £184; *and F,* £200.

(5) A tradesman is indebted to B, £275..14; to C, £304..7; to D, £152; and to E, £104..6; but, upon his decease, his estate is found to be worth but £675..15. How must it be divided among his creditors?

Ans. B's share, £222..15..2—6584; *C's,* £245..18..1$\frac{1}{2}$—15750;
D's, £122..16..2$\frac{1}{4}$—12227; *and E's,* £84..5..5—15620.

(6) Four persons trade together with a joint capital; of which A has $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $\frac{1}{6}$; and at the end of 6

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* 20 + 40 = 60

$$\text{As } 60 : 50 :: \begin{cases} 20 : \text{£}16..13..\ 4 = \text{A's share.} \\ 40 : \quad 33..\ 6..\ 8 = \text{B's share.} \end{cases}$$

50.. 0.. 0 Proof.

months they gain £100. What is each person's share of the gain? *Ans. A,* £35..1..9—12; *B,* £26..6..3$\frac{3}{4}$—9; *C,* £21..1..0$\frac{1}{2}$—30; *and D,* £17..10..10$\frac{1}{2}$—6.

(7) Two persons joined in the purchase of an estate yielding £1700 per annum, for £27200, whereof D paid £15000, and E the rest: some time after, they sold it for 24 years' purchase. What was each person's share?*

Ans. D, £22500; *E,* £18300.

(8) D, E, and F form a joint capital of £647. Their respective shares are in proportion to each other as 4, 6, and 8; and the gain is equal to D's stock. Required each person's stock and gain.

Ans. D's stock, £143..15..6$\frac{6}{9}$ *gain,* £31..19..0$\frac{4}{27}$.
E's,....... 215..13..4 ...... 47..18..6$\frac{6}{27}$.
F's,....... 287..11..1$\frac{3}{9}$ ...... 63..18..0$\frac{8}{27}$.

(9) D, E, and F joined in partnership; the amount of their stock was £100: D's gain was £3; E's, £5; and F's, £8. What was each man's stock?

Ans. D's stock, £18..15; *E's,* £31..5; *and F's,* £50.

FELLOWSHIP WITH TIME.

RULE. As the sum of the products of each person's money and time, is to the whole gain or loss; so is each individual product, to the corresponding gain or loss.

(1) D and E enter into partnership: D puts in £40 for three months, and E £75 for four months, and they gain £70. What is each man's share of the gain?†

(2) Three tradesmen joined in company: D put into the joint stock £195..14 for three months; E, £16..9..18..3 for 5 months; and F, £59..14..10 for 11 months: they gained £364..18. What was each man's share of the gain?

Ans. D's, £102..6..4—5008; *E's,* £148..1..1$\frac{1}{2}$—482802; *and F's,* £114..10..6$\frac{1}{4}$—14707.

(3) Three merchants join in company for 18 months: D puts in £500, and at 5 months' end takes out £200, at 10

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* The sale of a property for *so many years' purchase*, is understood to be, for so much present money as the *annual rent* or value × that number of years.

† $40 \times 3 = 120$
$75 \times 4 = 300$
$\overline{420}$

As 42|0 : 7|0 : : $\begin{cases} 120 : £20 = \text{D's share.} \\ 300 : 50 = \text{E's share.} \end{cases}$
$\overline{70}$ Proof.

months' end puts in £300, and at the end of 14 months takes out £130; E puts in £400, and at the end of 3 months £270 more, at 9 months he takes out £140, but puts in £100 at the end of 12 months, and withdraws £99 at the end of 15 months; F puts in £900, and at 6 months takes out £200, at the end of 11 months puts in £500, but takes out that and £100 more at the end of 13 months. They gain £200. Required each man's share of the gain?

Ans. D, £50..7..6—21720; *E*, £62..12..5¼—29859; *and F*, £87..0..0¼—14167.

(4) D, E, and F, hold a piece of ground in common, for which they are to pay £36..10..6: D puts in 23 oxen, 27 days; E, 21 oxen, 35 days; and F, 16 oxen, 23 days. What is each man to pay of the said rent?

Ans. D, £13..3..1½—624; *E*, £15..11..5—1688; *and F*, £7..15..11—1136.

ALLIGATION

Is a rule by which we ascertain the *mean price* of any compound formed by mixing ingredients of *various prices;* or the quantities of the various articles which will form a mixture of a certain *mean* or *average value.* It comprises four distinct cases.

CASE 1. ALLIGATION MEDIAL. The various quantities and prices being given, to find the *mean price* of the mixture.

RULE. Multiply each quantity by its price, and divide the sum of the products by the sum of the quantities.*

(1) A grocer mixed 4 *cwt.* of sugar, at 56*s.* per *cwt.* with 7 *cwt.* at 43*s.* per *cwt.* and 5 *cwt.* at 37*s.* per *cwt.* What is the value of 1 *cwt.* of this mixture? *Ans.* £2..4..4½.

(2) A vintner mixes 15 gallons of canary, at 8*s.* per gallon, with 20 gallons, at 7*s.* 4*d.* per gallon; 10 gallons of sherry,

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EXAMPLE.

* A farmer mixed 20 bushels of wheat, at 5*s.* per bushel, and 36 bushels of rye, at 3*s.* per bushel, with 40 bushels of barley, at 2*s.* per bushel. What is the worth of a bushel of this mixture?

	s.	*s.*
20 × 5 =	100	
36 × 3 =	108	
40 × 2 =	80	
96	96)288(3*s.* *Ans.*	

at 6*s*. 8*d*. per gallon; and 24 gallons of white wine, at 4*s*. per gallon. What is the worth of a gallon of this mixture?

Ans. 6*s*. $2\frac{1}{2}\,\frac{46}{69}d$.

(3) A maltster mixes 30 quarters of brown malt, at 28*s*. per quarter, with 46 quarters of pale, at 30*s*. per quarter, and 24 quarters of high dried ditto, at 25*s*. per quarter. What is one quarter of the mixture worth?

Ans. £1..8..$2\frac{1}{4}\,\frac{6}{10}d$.

(4) A vintner mixes 20 quarts of port, at 5*s*. 4*d*. per quart, with 12 quarts of white wine, at 5*s*. per quart, 30 quarts of Lisbon, at 6*s*. per quart, and 20 quarts of mountain, at 4*s*. 6*d*. per quart. What is a quart of this mixture worth? *Ans*. 5*s*. $3\frac{3}{4}\,\frac{50}{82}d$.

(5) A refiner melts 12 *lb*. of silver bullion, of 6 *oz*. fine, with 8 *lb*. of 7 *oz*. fine, and 10 *lb*. of 8 *oz*. fine: required the fineness of 1 *lb*. of that mixture? *Ans*. 6 *oz*. 18 *dwt*. 16 *gr*.

CASE 2. ALLIGATION ALTERNATE. The *various prices* being given, to find the *quantities* which may be mixed, to bear a *certain average price*.

RULE. Arrange the given prices in one column, with the proposed average price on the left.

Link *each less* than the average with *one greater*.

Place against each term the difference between that with which it is linked and the mean; and the respective differences will be the quantities required.

NOTE. Questions in this rule admit of a great variety of answers, according to the manner of linking them: also by taking other numbers proportional to the answers found.

(1) A vintner would mix four sorts of wine together, of 18*d*. 20*d*. 24*d*. and 28*d*. per quart: what quantity of each sort must he take to sell the mixture at 22*d*. per quart.*

(2) A grocer would mix sugar at 4*d*. 6*d*. and 10*d*. per *lb*.

		Answer.	*Proof.*
*	18	2 of 18*d*. =	36*d*.
22	20	6 of 20*d*. =	120
	24	4 of 24*d*. =	96
	28	2 of 28*d*. =	56
		14	14)308
			22*d*.

		Or thus:	*Proof.*
	18	6 of 18*d*. =	108*d*.
22	20	2 of 20*d*. =	40
	24	2 of 24*d*. =	48
	28	4 of 28*d*. =	112
		14	14)308
			22*d*.

so as to sell the compound for 8*d.* per *lb.* What quantity of each kind must he take?

Ans. 2 *lb. at* 4*d.;* 2 *lb.* at 6*d.; and* 6 *lb. at* 10*d.*

(3) How much tea at 16*s.* 14*s.* 9*s.* and 8*s.* per *lb.* will compose a mixture worth 10*s.* per *lb.?*

Ans. 1 *lb. at* 16*s.;* 2 *lb. at* 14*s.;* 6 *lb. at* 9*s.; and* 4 *lb. at* 8*s*

(4) A farmer would mix as much barley, at 3*s.* 6*d.* per bushel, rye at 4*s.* per bushel, and oats at 2*s.* per bushel, as will make a mixture worth 2*s.* 6*d.* per bushel. How much of each sort? *Ans.* 6 *b. of barley,* 6 *of rye, and* 30 *of oats.*

(5) A tobacconist would mix tobacco at 2*s.* 1*s.* 6*d.* and 1*s.* 3*d.* per *lb.* so that the compound may be worth 1*s.* 8*d.* per *lb.* What quantity of each sort must he take?

Ans. 7 *lb. at* 2*s.;* 4 *lb. at* 1*s.* 6*d.; and* 4 *lb. at* 1*s.* 3*d.*

Case 3. Alligation Partial. This is similar to Case 2, except that *one* of the *quantities is limited.*

Rule. Link the prices, and place the differences as before. Then, as the difference opposite to that whose quantity is given, is to each other difference; so is the given quantity, to each required quantity.

(1) A tobacconist intends to mix 20 *lb.* of tobacco at 15*d.* per *lb.* with others at 16*d.* 18*d.* and 22*d.* per *lb.* How many pounds of each sort must he take to make one pound of the mixture worth 17*d.?**

(2) How much coffee, at 3*s.* at 2*s.* and at 1*s.* 6*d.* per *lb.* with 20 *lb.* at 5*s.* will make a mixture worth 2*s.* 8*d.* per *lb.?*

Ans. 35 *lb. at* 3*s.;* 70 *lb. at* 2*s.; and* 10 *lb. at* 1*s.* 6*d.*

(3) A distiller would mix 10 gallons of French brandy, at 48*s.* per gallon, with British at 28*s.* and spirits at 16*s.* per gallon. What quantity of each sort must he take to afford it for 32*s.* per gallon? *Ans.* 8 *British, and* 8 *spirits.*

(4) What quantity of teas at 12*s.* 10*s.* and 6*s.* must be mixed with 20 *lb.* at 4*s.* per *lb.* that the mixture may be worth 8*s.* per *lb.?* *Ans.* 10 *lb. at* 6*s.;* 10 *lb. at* 10*s.;* 20 *lb. at* 12*s.*

			Answer.	*Proof.*					*lb.*		*lb.*
*	15	5	20 *lb.* at 15*d.* =	300*d.*	As 5	:	1	: :	20	:	4
17	16	1	4 *lb.* at 16*d.* =	64*d.*							
	18	1	4 *lb.* at 18*d.* =	72*d.*	As 5	:	2	: :	20	:	8
	22	2	8 *lb.* at 22*d.* =	176*d.*							
			As 36 *lb.* .	612*d.*	: : 1 *lb.* : 17*d.*						

CASE 4. ALLIGATION TOTAL. This is also similar to Case 2, except that the *whole quantity* of the compound *is limited.*

RULE. Link the prices, and place the differences as before. Then, as the sum of the differences, is to each particular difference; so is the quantity given, to each required quantity.

(1) A grocer has four sorts of sugar at 12*d.* 10*d.* 6*d.* and 4*d.* per *lb.* and would make a composition of 144 *lb.* worth 8*d.* per *lb.* What quantity of each sort must he take?*

(2) A grocer having 4 sorts of tea at 5*s.* 6*s.* 8*s.* and 9*s.* per *lb.* would have a composition of 87 *lb.* worth 7*s.* per *lb.* What quantity must there be of each sort?
Ans. 14½ *lb. of* 5*s.*; 29 *lb. of* 6*s.*; 29 *lb. of* 8*s.*; *and* 14½ *lb. of* 9*s.*

(3) A vintner having 4 sorts of wine, *viz.* white wine at 16*s.* per gallon, Flemish at 24*s.* per gallon, Malaga at 32*s.* per gallon, and Canary at 40*s.* per gallon; would make a mixture of 60 gallons, worth 20*s.* per gallon. What quantity of each sort must he take? *Ans.* 45 *gallons of white wine,* 5 *of Flemish,* 5 *of Malaga, and* 5 *of Canary.*

(4) A jeweller would melt together four sorts of gold, of 24, 22, 20, and 15 carats fine, so as to produce a compound of 42 *oz.* of 17 carats fine. How much must he take of each sort? *Ans.* 4 *oz. of* 24, 4 *oz. of* 22, 4 *oz. of* 20, *and* 30 *oz. of* 15 *carats fine.*

COMPARISON OF WEIGHTS AND MEASURES.

THIS is merely an application of the Rule of Proportion.

(1) If 50 Dutch pence be worth 65 French pence, how many Dutch pence are equal to 350 French pence?†

(2) If 12 yards at London make 8 ells at Paris, how many ells at Paris will make 64 yards at London? *Ans.* $42\frac{2}{3}$.

			Answer.	*Proof.*		*lb.*	*lb.*
*	12	4	48 at 12*d.*	= 576	As 12 : 4 : :	144	: 48
8	10	2	24 at 10*d.*	= 240	As 12 : 2 : :	144	: 24
	6	2	24 at 6*d.*	= 144			
	4	4	48 at 4*d.*	= 192			
	Sum	12	144	144)1152(8*d.*			

† As 65 : 50 or, as 13 : 10 $: : 350 : \frac{3500}{13} = 269\frac{3}{13}$ *Ans.*

(3) If 30 *lb.* at London make 28 *lb.* at Amsterdam, how many *lb.* at London will be equal to 350 *lb.* at Amsterdam? *Ans.* 375.

(4) If 95 *lb.* Flemish make 100 *lb.* English, how many *lb.* English are equal to 275 *lb.* Flemish? *Ans.* $289\frac{9}{19}$.

PERMUTATION

Is the changing or varying of the order of things.

To find the number of changes that may be made in the position of any given number of things.

RULE. Multiply the numbers 1, 2, 3, 4, &c. continually together, to the given number of terms, and the last product will be the answer.

(1) How many changes may be rung upon 12 bells; and in what time would they be rung, at the rate of 10 changes in a minute, and reckoning the year to contain 365 days, 6 hours?

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 =$ 479001600 changes, which $\div$ 10 $=$ 47900160 minutes $=$ 91 *years*, 26 *days*, 6 *hours*.

(2) A young scholar, coming to town for the convenience of a good library, made a bargain with the person with whom he lodged, to give him £40 for his board and lodging, during so long a time as he could place the family (consisting of 6 persons besides himself) in different positions, every day at dinner. How long might he stay for his £40? *Ans.* 5040 *days.*

VULGAR FRACTIONS.

DEFINITIONS.

1. *A Fraction* is a part or parts of a unit, or of any whole number or quantity; and is expressed by two numbers, called the *terms*, with a line between them.

2. The *upper term* is called the *Numerator*, and the *lower term*, the *Denominator*. The Denominator shows into how

many equal parts *unity* is divided; and the Numerator is the number of those equal parts signified by the fraction.*

3. Every fraction may be understood to represent *Division;* the numerator being the *dividend,* and the denominator the *divisor.*†

Fractions are distinguished as follows:

4. A SIMPLE FRACTION consists of *one numerator,* and *one denominator;* as, $\frac{3}{4}$, $\frac{11}{12}$, &c.

5. A COMPOUND FRACTION, or fraction of a fraction, consists of two or more fractions connected by the word *of;* as, $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{7}{12}$, &c. This properly denotes the *product* of the several fractions.

6. A PROPER FRACTION is one which has the numerator *less* than the denominator; as, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{11}{12}$, &c.‡

7. An IMPROPER FRACTION is one which has the numerator either *equal to,* or *greater than,* the denominator; as, $\frac{4}{4}$, $\frac{5}{4}$, $\frac{8}{4}$, $\frac{15}{12}$, &c.‡

8. A MIXED NUMBER is composed of a whole number and a fraction; as, $1\frac{2}{3}$, $17\frac{1}{2}$, $8\frac{21}{97}$, &c.

9. A COMPLEX FRACTION has a *fractional* numerator or denominator: but this denotes *Division of Fractions.* Thus, $\frac{\frac{2}{3}}{\frac{5}{6}}$, two-thirds *divided by* five-sixths, $\frac{8}{1\frac{2}{3}}$ eight *divided by* one and two-thirds.

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* In the fraction *five-twelfths* ($\frac{5}{12}$), the *Denominator* 12 shows that the *unit* or *whole quantity* is supposed to be divided into 12 equal parts: so that if it be one shilling, each part will be one-twelfth of 1*s.* or one penny. The *Numerator* shows that 5 is the number of those twelfth parts intended to be taken: so $\frac{5}{12}$ of a shilling are the same as 5 pence; $\frac{5}{12}$ of a foot, the same as 5 inches.

† The fraction $\frac{5}{12}$ signifies not only $\frac{5}{12}$ of a unit, but 5 units divided into 12 parts, or a twelfth part of 5: and it is obvious that *five twelfth parts* of *one* shilling (or five pence) is the same as *one twelfth part* of *five* shillings. This mode of considering Fractions removes many of the student's difficulties.

‡ A *proper fraction* is always *less than unity:* thus $\frac{3}{4}$ *wants one-fourth,* and $\frac{11}{12}$ *wants one-twelfth* of being equal to 1. But an *improper fraction* is *equal to unity* when the *terms are equal,* and *greater than unity* when the *numerator* is the *greater.*

Thus $\frac{4}{4}$, or $\frac{12}{12}$, or $\frac{73}{73}$, is each $= 1$; and $\frac{5}{4} = 1\frac{1}{4}$; $\frac{8}{4} = 2$; $\frac{69}{20} = 3\frac{9}{20}$.

10. A COMMON MEASURE (or DIVISOR) is a number that will exactly divide *both* the *terms*. When it is the *greatest* number by which they are both divisible, it is called the GREATEST COMMON MEASURE.

NOTE. A *prime* number has no *factor*, except itself and unity.

A *multiple* signifies any product of a number; and is, therefore divisible by the number of which it is a multiple: thus, 14, 21, 28 &c. are *multiples of seven*. Also 14 is a common multiple of 2 and 7, 21, of 3 and 7, &c.

REDUCTION

Is the method of changing the form of fractional numbers or quantities, without altering the value.

Case 1. *To reduce a fraction to its lowest terms.*

RULE. Divide both the terms by *any common measure* that can be discovered by inspection; which will produce an equivalent fraction in *lower* terms. Treat the new fraction in a similar manner; repeating the operation till the *lowest terms* are obtained.*

When the object cannot be accomplished by this process, divide the greater term by the less, and that divisor by the remainder, and so on till nothing remains. The last divisor will be the *greatest common measure;* by which divide both terms of the fraction, and the quotients will be the *lowest terms*.

(1) Reduce $\frac{30}{125}$ to its lowest terms. *Ans.* $\frac{6}{25}$.

(2) Reduce $\frac{208}{684}$ to its lowest terms. *Ans.* $\frac{52}{171}$.

(3) Reduce $\frac{192}{576}$ to the least terms. *Ans.* $\frac{1}{3}$.

(4) Reduce $\frac{825}{960}$ to the least terms. *Ans.* $\frac{55}{64}$.

(5) Abbreviate $\frac{5184}{6912}$ as much as possible. *Ans.* $\frac{3}{4}$.

(6) Reduce $\frac{12540}{21945}$ to its lowest terms. *Ans.* $\frac{4}{7}$.

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* This first method of *abbreviating* fractions is, when practicable, always to be preferred; and, in the application of it, the following observations will be found exceedingly useful.

An *even* number is divisible by 2.

A number is divisible by 4, when the *tens* and *units* are so; and by 8, when the *hundreds*, *tens*, and *units*, are divisible by 8.

A number is a multiple of 3, or of 9, when the *sum of its digits* is a multiple of 3, or of 9.

A 5 or a 0, in the units' place, admits of division by 5; *one* cipher admits of division by 10; *two*, by 100, &c.

(7) Reduce $\frac{99715}{113960}$ to the lowest terms. *Ans.* $\frac{7}{8}$.
(8) What are the lowest terms of $\frac{3108}{3552}$? *Ans.* $\frac{7}{8}$.

Case 2. *To reduce an improper fraction to its equivalent number.*

RULE. Divide the upper term by the lower.

This is evident from Definition 3.

(1) Reduce $\frac{129}{7}$ to a mixed number. $\frac{129}{7}=18\frac{3}{7}$. *Ans.*
(2) Reduce $\frac{69}{5}$ to its equivalent number. *Ans.* $13\frac{4}{5}$.
(3) Reduce $\frac{245}{9}$ to its equivalent number. *Ans.* $27\frac{2}{9}$.
(4) Reduce $\frac{1246}{22}$ to its equivalent number. *Ans.* $56\frac{13}{22}$.
(5) Reduce $\frac{3849}{21}$ to its equivalent number. *Ans.* $183\frac{2}{7}$.
(6) Reduce $\frac{1151}{16}$ to its equivalent number. *Ans.* $71\frac{15}{16}$.

Case 3. *To reduce a mixed number to an improper fraction.**

RULE. Multiply the whole number by the denominator of the fraction, and to the product add the numerator for the numerator required, which place over the denominator.

NOTE. Any whole number may be expressed in a fractional form, by putting 1 for the denominator: thus $11=\frac{11}{1}$.

(1) Reduce $18\frac{3}{7}$ to the form of a fraction.†
(2) Reduce $56\frac{13}{22}$ to an improper fraction. *Ans.* $\frac{1245}{22}$.

A number is a multiple of 11, when the *sum* of the 1st, 3d, 5th, &c. digits = *that* of the 2d, 4th, 6th, &c. digits, after retrenching the *elevens* contained in each.

A multiple of both 2 and 3 is, of course, a multiple of 6; and a multiple of 3 and 4 may be divided by 12.

All *prime* numbers, except 2 and 5, have 1, 3, 7, or 9, in the units' place: all others are *composite.*

EXAMPLES.

(1) Reduce $\frac{1260}{1620}$ to the *least terms possible.*

$\div 10 \quad \div 9 \quad \div 2$

$\frac{1260}{1620}=\frac{126}{162}=\frac{14}{18}=\frac{7}{9}$. *Ans.*

(2) Reduce $\frac{12540}{21945}$ to the lowest terms.

$\div 5 \quad \div 3 \quad \div 11$

$\frac{12540}{21945}=\frac{2508}{4389}=\frac{836}{1463}=\frac{76}{133}$.

Now, because we cannot easily discover a common measure, proceed thus:

```
76)133(1     then 19)76
   76             ------- = 4/7. Ans.
   --             19)133
   57)76(1
      57
      --
greatest com. meas. 19)57(3
                       57
                       --
```

* This is the converse of Case 2.

† $18\frac{3}{7}=\frac{18\times 7+3}{7}=\frac{129}{7}$. *Ans.*

(3) Reduce $183\frac{5}{21}$ to an improper fraction. *Ans.* $\frac{3848}{21}$.
(4) Reduce $13\frac{4}{5}$ to its equivalent fraction. *Ans.* $\frac{69}{5}$.
(5) Reduce $27\frac{2}{9}$ to its equivalent fraction. *Ans.* $\frac{245}{9}$.
(6) Reduce $514\frac{5}{16}$ to a fractional form. *Ans.* $\frac{8229}{16}$.

Case 4. *To reduce a fraction to another of the same value, having a certain proposed numerator or denominator.*

RULE. As the present numerator, is to the denominator; so is the proposed numerator, to its denominator. Or, as the present denominator, is to the numerator; so is the proposed denominator, to its numerator.

(1) Reduce $\frac{2}{3}$ to a fraction of the same value, whose numerator shall be 12. As 2 : 3 : : 12 : 18. *Ans.* $\frac{12}{18}$.

(2) Reduce $\frac{5}{7}$ to a fraction of the same value, whose numerator shall be 25. *Ans.* $\frac{25}{35}$.

(3) Reduce $\frac{5}{7}$ to a fraction of the same value, whose numerator shall be 47. *Ans.* $\frac{47}{65\frac{4}{5}}$.

(4) Reduce $\frac{2}{3}$ to a fraction of the same value, whose denominator shall be 18. *Ans.* $\frac{12}{18}$.

(5) Reduce $\frac{5}{7}$ to a fraction of the same value, whose denominator shall be 35. *Ans.* $\frac{25}{35}$.

(6) Reduce $\frac{8}{9}$ to a fraction of the same value, whose denominator shall be 19. *Ans.* $\frac{16\frac{8}{9}}{19}$.

Case 5. *To reduce complex and compound fractions to a simple form.*

RULE. For a *complex* fraction, reduce both terms to simple fractions: then, by *inverting* the lower fraction, they may be considered as the terms of a *compound* fraction. And to reduce a *compound* fraction, arrange all the numerators above a line, and the denominators below, with the *signs* of multiplication interserted: divide all the upper and lower terms that are *commensurable*,* cancelling them with a dash, and placing their quotients above and below them respectively. Do the same with the quotients: then the products of the uncancelled numbers will give the *single fraction* in its *lowest terms*.†

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* That is, having a *common divisor*.

† This rule is of the *highest importance* as tending *to expedite* the bus

(1) Reduce $\frac{36\frac{2}{3}}{48}$ to a simple fraction. *Ans.* $\frac{55}{72}$.

(2) Reduce $\frac{23\frac{5}{7}}{38}$ to a simple fraction. *Ans.* $\frac{83}{133}$.

(3) Reduce $\frac{47}{65\frac{4}{5}}$ to a simple fraction. *Ans.* $\frac{5}{7}$.

(4) Reduce $\frac{19}{44\frac{1}{3}}$ to a simple fraction. *Ans.* $\frac{3}{7}$.

(5) Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a simple fraction. *Ans.* $\frac{1}{4}$.

(6) Reduce $\frac{5}{9}$ of $\frac{4}{7}$ of $\frac{11}{12}$ to a simple fraction. *Ans.* $\frac{55}{189}$.

---

siness of computation, by *abbreviating to the utmost* all fractional expressions, as we proceed.

EXAMPLES.

(1) Reduce the *complex* fraction $\frac{7\frac{1}{3}}{8\frac{5}{9}}$ to a *simple* form.

$$\frac{7\frac{1}{3}}{8\frac{5}{9}}=\frac{\frac{22}{3}}{\frac{77}{9}}=\frac{\cancel{22}^{\,2}\times\cancel{9}^{\,3}}{\cancel{3}_{\,1}\times\cancel{77}_{\,7}}=\frac{6}{7}.\ \textit{Ans.}$$

(2) Reduce $\frac{9}{10}$ of $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{5}{14}$ to a simple fraction.

$$\frac{9}{10}\text{ of }\frac{2}{3}\text{ of }\frac{7}{8}\text{ of }\frac{5}{14}=\frac{\cancel{9}\times\cancel{2}\times\cancel{7}\times 5}{\cancel{10}\times\cancel{3}\times\cancel{8}\times\cancel{14}}=\frac{3}{16}.\ \textit{Ans.}$$

(3) Reduce the annexed fractional expression to its proper quantity.

$$\frac{16}{11}\text{ of }\frac{111\frac{9}{13}}{108}\text{ of }\frac{10\frac{2}{7}}{\frac{18}{25}}\text{ of }\frac{6\frac{8}{15}}{24\frac{8}{13}}\text{ of }£8\tfrac{55}{64}=\frac{16}{11}\times\frac{\frac{1452}{13}}{\frac{108}{1}}\times\frac{\frac{72}{7}}{\frac{18}{25}}\times\frac{\frac{98}{15}}{\frac{320}{13}}\times\frac{567}{64}$$

$$=\frac{\cancel{16}\times\cancel{1452}\times1\times\cancel{7}\times\cancel{21}\times\cancel{98}\times\cancel{13}\times\cancel{567}}{\cancel{11}\times\cancel{108}\times\cancel{13}\times\cancel{18}\times\cancel{147}\times\cancel{320}\times\cancel{15}\times\cancel{64}}=\frac{£77}{32}=£2\tfrac{13}{32}$$

$=£2..8\frac{1}{8}s.=£2..8..1\frac{1}{2}.$ *Ans.*

(7) Reduce $\frac{11}{12}$ of $\dfrac{13\frac{3}{5}}{14\frac{4}{6}\frac{2}{5}}$ of $\dfrac{28}{38\frac{2}{3}}$ to a simple fraction. *Ans.* $\frac{143}{282}$.

(8) Reduce $\frac{3}{4}$ of $\frac{6}{91}$ of $11\frac{3}{8}$ to a single fraction. *Ans.* $\frac{9}{16}$.

(9) Reduce $\frac{7}{128}$ of $37\frac{1}{3}$ of 5 to its equivalent number. *Ans.* $112\frac{7}{24}$.

(10) Reduce $\dfrac{1\frac{1}{2}}{14}$ of $\frac{236}{3}$ of $\dfrac{\frac{1}{2}}{\frac{4}{7}}$ to its equivalent number. *Ans.* $7\frac{3}{8}$.

Case 6. *To reduce a fractional quantity of a given denomination, to an equivalent fraction of another denomination.*

RULE. Consider what numbers would reduce the *greater* denomination to the *less;* then to reduce to a *greater* name, multiply the *denominator* by those numbers; and to reduce to a *less* name, multiply the *numerator:* the compound thus produced, when reduced to a simple form, will be the fraction required.

(1) Reduce $\frac{7}{8}$ of a penny to the fraction of a pound.*

(2) Reduce $\frac{3}{4}d.$ to the fraction of a crown. *Ans.* $\frac{1}{80}$ *cr.*

(3) Reduce $\frac{4}{5}$ *dwt.* to the fraction of a *lb.* troy. *Ans.* $\frac{1}{300}$ *lb.*

(4) Reduce $\frac{4}{7}$ *lb.* avoirdupois to the fraction of a *cwt.* *Ans.* $\frac{1}{196}$ *cwt.*

(5) Reduce $\frac{7}{1920}$ of a pound to the fraction of a penny.†

(6) Reduce £$\frac{1}{320}$ to the fraction of a penny. *Ans.* $\frac{3}{4}d.$

(7) Reduce $\frac{1}{300}$ of a pound troy to the fraction of a pennyweight. *Ans.* $\frac{4}{5}$ *dwt.*

(8) Reduce $\frac{1}{196}$ *cwt.* to the fraction of a *lb.* *Ans.* $\frac{4}{7}$ *lb.*

Case 7. *To find the proper value of a fractional quantity.*

RULE. Reduce the numerator to such lower denomination as may be necessary, and divide by the denominator; abbreviating as much as possible in valuing the remainders.

NOTE. It is evident, from Definition 3, that this Case is precisely that of Compound Division.

(1) Reduce $\frac{3}{4}$ of a pound sterling to its proper value.‡

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* $\frac{7}{8}d. = \dfrac{7}{8\times12\times20} = £\frac{7}{1920}$. *Ans.*

† $£\frac{7}{1920} = \dfrac{7\times20\times12}{1920} = \dfrac{7\times12}{96} = \frac{7}{8}d.$ *Ans.*

‡ $£\frac{3}{4} = \dfrac{3\times20}{4} = 3\times5 = 15s.$ *Ans.*

(2) Reduce $\frac{2}{5}s.$ to its proper value. *Ans.* 4*d.* $3\frac{1}{5}$ *qrs.*

(3) Reduce $\frac{4}{7}$ of a *lb.* avoirdupois to its proper value.
Ans. 9 *oz.* $2\frac{2}{7}$ *dr.*

(4) Reduce $\frac{7}{9}$ *cwt.* to its proper value. *Ans.* 3 *qrs.* $3\frac{1}{9}$ *lb.*

(5) Reduce $\frac{3}{5}$ of a *lb.* troy to its proper value.
Ans. 7 *oz.* 4 *dwts.*

(6) Reduce $\frac{14}{25}$ of an ell English to its proper value.
Ans. 2 *qrs.* $3\frac{1}{5}$ *nails.*

(7) What is the value of £$\frac{5719}{5760}$? *Ans.* 19*s.* $10\frac{1}{4}\ \frac{1}{6}$.

(8) Reduce $\frac{285}{352}$ of a mile to its proper value.
Ans. 6 *fur.* 105 *yds.*

(9) Reduce $\frac{73}{48}$ of an acre to its proper value.
Ans. 1 *a.* 2 *r.* $3\frac{1}{3}$ *per.*

(10) Find the value of $\frac{15487}{34160}$ *cwt.* *Ans.* 1 *qr.* 22 *lb.* $\frac{237}{305}$

Case 8. *To reduce any given quantity to the fraction of a greater denomination.*

Rule. Reduce the given quantity (if compound) to the lowest denomination mentioned, that it may assume a *simple* form: then multiply the *denominator* as in Case 6.

(1) Reduce 15*s.* to the fraction of a pound sterling.
15*s.* = £$\frac{15}{20}$ = £$\frac{3}{4}$. *Ans.*

(2) Reduce 4*d.* $3\frac{1}{5}$ *qrs.* to the fraction of a shilling. *Ans.* $\frac{2}{5}$.

(3) Reduce 9 *oz.* $2\frac{2}{7}$ *dr.* to the fraction of a *lb.* avoirdupois.
Ans. $\frac{4}{7}$ *lb.*

(4) Reduce 3 *qrs.* $3\frac{1}{9}$ *lb.* to the fraction of a *cwt.*
Ans. $\frac{7}{9}$ *cwt.*

(5) Reduce 7 *oz.* 4 *dwts.* to the fraction of a *lb.* troy.
Ans. $\frac{3}{5}$ *lb.*

(6) Reduce 2 *qrs.* $3\frac{1}{9}$ *nails,* to the fraction of an English ell.
Ans. $\frac{5}{9}$ *ell.*

(7) Reduce 14*s.* $6\frac{1}{2}$*d.* $\frac{2}{11}$ to the fraction of a pound.
Ans. £$\frac{8}{11}$.

(8) Reduce 4*d.* $1\frac{11}{13}$ *qrs.* to the fraction of a crown.
Ans. $\frac{29}{390}$ *cr.*

(9) What fraction of an acre are 3 roods, 32 perches?
Ans. $\frac{19}{20}$ *a.*

(10) What part of a shilling are $\frac{2}{3}$ of 2*d.* *Ans.* $\frac{1}{9}$*s.*

Case 9. *To find the least common multiple of two or more numbers.*

Rule. Arrange the given numbers in a line (omitting any one that is a *factor* of one of the others), and divide any two or more of them by *a common divisor*, placing the quotients and undivided numbers below; proceed with these in the same manner, and repeat the process till there remain not any two numbers *commensurable:* the continued product of the divisors, quotients, and undivided numbers, will be the least common multiple.

(1) Required the least common multiple of 2, 3, 4, 5, 6, 7, 8, 9, and 10.*

(2) Find the least number divisible by 3, 4, 5, 6, 7, and 8. *Ans.* 840.

(3) What is the least common multiple of 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12? *Ans.* 27720.

Case 10. *To reduce fractions to a common denominator.*

Rule 1. Multiply each numerator into all the denominators, except its own, for a numerator; and all the denominators for a common denominator. Or,

Rule 2. Find the *least common multiple* of the denominators, which will be the *least common denominator.* Divide this by each denominator, and multiply the several quotients by the respective numerators for the required numerators.

(1) Reduce $\frac{2}{4}$ and $\frac{4}{7}$ to a common denominator.†

(2) Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$, to a common denominator. *Ans.* $\frac{32}{64}$, $\frac{48}{64}$, *and* $\frac{40}{64}$; *or,* $\frac{4}{8}$, $\frac{6}{8}$, *and* $\frac{5}{8}$.

(3) Reduce $\frac{7}{8}$, $\frac{4}{6}$, $\frac{6}{10}$, and $\frac{6}{7}$, to a common denominator. *Ans.* $\frac{735}{840}$, $\frac{560}{840}$, $\frac{504}{840}$, *and* $\frac{720}{840}$.

(4) Reduce $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{1}{7}$, to a common denominator. *Ans.* $\frac{42}{70}$, $\frac{35}{70}$, *and* $\frac{10}{70}$.

(5) Reduce $\frac{8}{11}$, $\frac{4}{7}$, and $\frac{3\frac{1}{2}}{15}$ of 2, to a common denominator. *Ans.* $\frac{315}{1155}$, $\frac{660}{1155}$, *and* $\frac{539}{1155}$.

(6) Reduce $1\frac{1}{4}$, $2\frac{1}{5}$, and $\frac{1}{3}$ of $1\frac{1}{4}$, to a common denominator. *Ans.* $\frac{75}{60}$, $\frac{132}{60}$, *and* $\frac{25}{60}$.

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* 2 and 4, being factors of 8, 3 a factor of 9, and 5 a factor of 10, may be omitted. Thus,

$$\begin{array}{r|l} 2 & 6,\ 7,\ 8,\ 9,\ 10 \\ 3 & 3,\ 7,\ 4,\ 9,\ 5 \\ & 1,\ 7,\ 4,\ 3,\ 5 \end{array}$$

Then $2 \times 3 \times 7 \times 4 \times 3 \times 5 = 42 \times 60 = 2520$, the least number divisible by all the given numbers.

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† $\left.\begin{array}{l} 2 \times 7 = 14 \\ 4 \times 4 = 16 \end{array}\right\}$ numerators. *Ans.* $\frac{14}{28}$ and $\frac{16}{28}$.
$4 \times 7 = 28$ the denominator.

ADDITION.

RULE. Reduce the given fractions to a common denominator, over which place the *sum* of the numerators.

(1) Add $\frac{2}{3}$ and $\frac{5}{7}$ together. $\frac{2}{3} \times \frac{5}{7} = \frac{14}{21} + \frac{15}{21} = \frac{29}{21} = 1\frac{8}{21}$. *Ans.*

(2) Add $\frac{3}{4}$, $\frac{4}{7}$, and $\frac{5}{6}$.

(3) Add $\frac{1}{5}$, $4\frac{1}{3}$, and $\frac{2}{5}$.*

(4) Add $7\frac{2}{3}$ and $\frac{3}{5}$ together.

(5) Add $\frac{2}{7}$, and $\frac{2}{3}$ of $\frac{3}{4}$.

(6) Add $5\frac{2}{3}$, $6\frac{7}{8}$, and $4\frac{1}{2}$.

(7) Add $1\frac{4}{9}$, $3\frac{1}{7}$, and $\frac{1}{2}$ of 7.

(8) Add $\frac{9}{10}$ of $6\frac{7}{8}$, and $\frac{4}{7}$ of $7\frac{1}{2}$.

(9) Add $\frac{1}{6}$ of $9\frac{3}{8}$, and $\frac{2}{3}$ of $4\frac{5}{8}$.

Fractional quantities may be reduced to their proper values, and the sum found by Compound Addition.

(10) Add $\frac{3}{8}$ of a pound to $\frac{5}{6}$ of a shilling. *Ans.* 8*s.* 4*d.*

(11) Add $\frac{1}{2}$*d.* $\frac{3}{8}$*s.* and £$\frac{2}{3}$. *Ans.* 14*s.*

(12) Add $\frac{3}{5}$ *lb.* troy, $\frac{1}{6}$ *oz.* and $\frac{5}{8}$ *oz.*
Ans. 7 *oz.* 19 *dwts.* 20 *gr.*

(13) Add $\frac{4}{7}$ of a ton to $\frac{5}{8}$ of a cwt. *Ans.* 12 *cwt.* 1 *qr.* $1\frac{1}{3}$ *lb*

(14) What is the sum of $\frac{2}{3}$ of £17..7..6*d.* $\frac{4}{5}$ of £$1\frac{2}{3}$, and $\frac{5}{8}$ of a crown? *Ans.* £13..0..$2\frac{1}{2}$.

(15) Add $\frac{4}{5}$ of 3 *a.* 1 *r.* 20 *p.* $\frac{1}{8}$ of an acre, and $\frac{3}{4}$ of 3 roods, 15 perches. *Ans.* 3 *a.* 2 *r.* $33\frac{1}{4}$ *p.*

SUBTRACTION.

RULE. Reduce the given fractions to a common denominator, over which place the *difference* of the numerators.

When the numerator of the fractional part in the *subtrahend* is greater than the other numerator, borrow a fraction *equal* to *unity*, having the common denominator; then subtract, and carry one to the *integer* of the subtrahend.

(1) From $\frac{3}{4}$ take $\frac{5}{7}$. $\frac{3}{4} - \frac{5}{7} = \frac{21}{28} - \frac{20}{28} = \frac{1}{28}$. *Ans.*

(2) From $\frac{5}{6}$ take $\frac{3}{5}$.

(3) From $5\frac{2}{3}$ take $\frac{9}{10}$ of $\frac{5}{9}$.

(4) From $\frac{38}{54}$ take $\frac{5}{8}$ of $\frac{1}{3}$.

(5) From $\frac{19}{20}$ take $\frac{1}{4}$ of $\frac{2}{3}$.

(6) From $64\frac{1}{4}$ take $\frac{2}{3}$ of $\frac{3}{4}$.

(7) From $15\frac{1}{8}$ take $12\frac{7}{15}$.

(8) Subtract $\frac{33}{35}$ from $1\frac{2}{5}$.

(9) Subtract $\frac{17}{21}$ from $\frac{1}{8}$ of 9.

Fractional quantities may be reduced to their proper values, as directed in Addition.

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* When there are *integers* among the given numbers, first find the *sum of the fractions*, to which add the *integers.*
Thus in Ex. 3, $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$; then $\frac{3}{5} + \frac{1}{3} = \frac{9}{15} + \frac{5}{15} = \frac{14}{15}$; and $4 + \frac{14}{15} = 4\frac{14}{15}$ *Ans.*

(10) From $\frac{3}{8}$ of a pound take $\frac{3}{8}$ of a shilling. *Ans* 7*s*. $1\frac{1}{2}$*d*.
(11) From $1\frac{2}{3}$*s*. take $\frac{2}{3}$ of $7\frac{1}{2}$*d*. *Ans.* 1*s*. 3*d*.
(12) What is the difference between $\frac{3}{8}$ of £$1\frac{6}{24}$, and $\frac{5}{16}$ of £$1\frac{33}{80}$? *Ans.* 2*d*. $3\frac{1}{4}$ *qrs.*
(13) Subtract $\frac{5}{6}$ *cwt.* from $\frac{4}{7}$ *ton.* *Ans.* 10 *cwt.* 2 *qrs.* $10\frac{2}{3}$ *lb*
(14) From $\frac{2}{3}$ of 5 *lb.* troy subtract $\frac{5}{9}$ of $3\frac{1}{2}$ *oz.*
Ans. 3 *lb.* 2 *oz.* 1 *dwt.* $2\frac{2}{3}$ *gr.*
(15) Subtract $7\frac{131}{220}$ furlongs from $1\frac{5}{11}$ mile.
Ans. 4 *fur.* 9 *yds.*

MULTIPLICATION.

RULE. Prepare the given numbers (if they require it) by the rules of Reduction: then multiply all the numerators together for the numerator of the *product*, and all the denominators for the denominator.

(1) Multiply $\frac{3}{4}$ by $\frac{3}{5}$. $\frac{3}{4}\times\frac{3}{5}=\frac{9}{20}$. *Ans.*
(2) Multiply $\frac{7}{8}$ by $\frac{2}{3}$.
(3) Multiply $48\frac{3}{5}$ by $13\frac{5}{6}$.
(4) Multiply $430\frac{2}{5}$ by $18\frac{3}{7}$.
(5) Multiply $\frac{18}{21}$ by $\frac{3}{4}$ of $\frac{5}{7}$.
(6) Multiply $\frac{1}{4}$ of $\frac{3}{8}$ by $\frac{5}{7}$.
(7) Multiply $5\frac{6}{7}$ by $\frac{6}{8}$.
(8) Multiply 24 by $\frac{2}{3}$.
(9) Multiply $\frac{3}{4}$ of 9 by $\frac{7}{8}$.
(10) Multiply £3..15..$9\frac{1}{4}\frac{1}{5}$ by $\frac{9}{11}$ of 5. *Ans.* £15..9..$11\frac{3}{4}\frac{5}{11}$.
(11) Multiply $3\frac{14}{44}$ miles by $\frac{4}{7}$ of $4\frac{37}{98}$.
Ans. 8 *m.* 2 *fur.* $188\frac{4}{7}$ *yds.*
(12) Required the product, in square feet, of 14 *ft.* 7 *in.* by 8 *ft.* 9 *in* *Ans.* $127\frac{29}{48}$ *sq. ft.*

DIVISION.

RULE. Prepare the given numbers (if they require it) by the rules of Reduction: then *invert the divisor*, and proceed as in Multiplication.*

(1) Divide $\frac{9}{20}$ by $\frac{3}{5}$.†
(2) Divide $\frac{14}{27}$ by $\frac{5}{8}$.
(3) Divide $672\frac{9}{30}$ by $13\frac{5}{6}$.
(4) Divide $7935\frac{24}{70}$ by $18\frac{3}{7}$.
(5) Divide 16 by 24.
(6) Divide $\frac{9}{16}$ by $4\frac{1}{2}$.
(7) Divide $\frac{441}{936}$ by $\frac{9}{10}$ of $\dfrac{10\frac{1}{2}}{\frac{3}{16}}$.
(8) Divide $9\frac{1}{6}$ by $\frac{1}{2}$ of 7.
(9) Divide $\frac{3}{8}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$.
(10) Divide $\frac{3}{8}$ of 16 by $\frac{2}{7}$ of $\frac{3}{4}$.

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* A number *inverted* becomes the *reciprocal* of that number; which is the quotient arising from dividing *unity* by the given number: thus $1\div 7=\frac{1}{7}$, the *reciprocal* of 7; $1\div\frac{3}{5}=\frac{5}{3}$, the *reciprocal* of $\frac{3}{5}$.

† $\frac{9}{20}\div\frac{3}{5}=\dfrac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{20}}}\times\dfrac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{3}}}=\frac{3}{4}$. *Ans.*

(11) Divide £$^{1}\frac{1}{2}\frac{2}{7}\frac{1}{6}^{3}$ by $\frac{3^{2}}{5}$ of $1\frac{2}{3}$. *Ans.* £3..17..$10\frac{1}{4}$ $\frac{2}{3}$.

(12) Divide 1*s.* $4\frac{1}{2}$*d.*$\frac{5}{8}$ by $\frac{2}{3}$ of $\frac{1^{8}}{5}$. *Ans.* 6*d.* $3\frac{6}{7}\frac{1}{2}$ *qrs.*

(13) Divide 3 *qrs.* $24\frac{1}{1}\frac{5}{6}$ *lb.* by $\frac{7}{15}$ of $1\frac{1}{2}$, in the fraction of a *cwt.*; and value the quotient. *Ans.* 1 *cwt.* 1 *qr.* $15\frac{5}{8}$ *lb.*

(14) What must £7..14..6 be multiplied by, to produce £21..17..9? *Ans.* $2\frac{5}{6}$.

THE RULE OF THREE.

RULE. Prepare the terms, previous to stating, so that no subsequent Reduction will be necessary: then, having stated the question, as previously directed, *invert* the *dividing term*, and the continued product of the three will be the answer.

(1) If $\frac{3}{4}$ of a yard cost £$\frac{5}{8}$, what will $\frac{9}{10}$ of a yard cost?*

(2) If $\frac{6}{8}$ *yd.* cost £$\frac{2}{3}$, what will $\frac{1}{1}\frac{1}{2}$ *yd.* cost? *Ans.* 14*s.* 8*d.*

(3) If $\frac{3}{4}$ of a yard of lawn cost 7*s.* 3*d.* what will $10\frac{1}{3}$ yards cost? *Ans.* £4..19..$10\frac{1}{2}$ $\frac{2}{3}$.

(4) If $\frac{7}{8}$ *lb.* cost $\frac{3}{4}$*s.* how much will $\frac{8}{9}$*s.* buy? *Ans.* $1\frac{1}{27}$ *lb.*

(5) If 48 men can build a wall in $24\frac{1}{4}$ days, how many men can do the same in 192 days? *Ans.* $6\frac{1}{16}$ *men.*

(6) If $\frac{3}{4}$ of a yard of Holland cost £$\frac{1}{3}$, what will $12\frac{2}{3}$ ells cost at the same rate? *Ans.* £7..0..$8\frac{3}{4}$ $\frac{8}{9}$.

(7) If $3\frac{3}{4}$ yards of cloth, that is $1\frac{1}{5}$ yard wide, be sufficient to make a cloak, how much that is $\frac{4}{5}$ of a yard wide, will make another of the same size? *Ans.* $4\frac{7}{8}$ *yards.*

(8) If $12\frac{1}{2}$ yards of cloth cost 15*s.* 9*d.* what will $48\frac{1}{4}$ yards cost at the same rate? *Ans.* £3..0..$9\frac{1}{2}$ $\frac{4}{25}$.

(9) If $25\frac{2}{7}$*s.* will pay for the carriage of 1 *cwt.* $145\frac{1}{4}$ miles, how far may $6\frac{1}{2}$ *cwt.* be carried for the same money? *Ans.* $22\frac{9}{26}$ *miles.*

(10) If $\frac{9}{10}$ of a *cwt.* cost £14..4, what is the value of $7\frac{1}{2}$ *cwt.*? *Ans.* £118..6..8.

(11) If $\frac{3}{4}$ *lb.* of cochineal cost £1..5, what will $36\frac{7}{10}$ *lb.* come to? *Ans.* £61..3..4.

(12) How much in length that is $7\frac{8}{13}$ inches broad, will make a foot square? *Ans.* $20\frac{4}{13}$ *inches.*

(13) What is the value of 4 pieces of broad cloth, each $27\frac{3}{7}$ yards, at $15\frac{5}{8}$*s.* per yard? *Ans.* £85..14..$3\frac{1}{4}$ $\frac{4}{7}$.

$$* \text{ As } \overset{yd.}{\tfrac{3}{4}} : \overset{£}{\tfrac{5}{8}} :: \overset{yd.}{\tfrac{9}{10}} : \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} = \overset{£}{\tfrac{3}{4}} = 15s. \textit{ Ans.}$$

(14) If a penny white loaf weigh 7 *oz.* when a bushel of wheat costs 5*s.* 6*d.* what is the bushel worth when a penny white loaf weighs but $2\frac{1}{2}$ *oz.*? *Ans.* 15*s.* 4*d.* $3\frac{1}{5}$ *qrs.*

(15) What quantity of shalloon that is $\frac{3}{4}$ of a yard wide will line $7\frac{1}{2}$ yards of cloth that is $1\frac{1}{2}$ yard wide? *Ans.* 15 *yards.*

(16) Bought $3\frac{1}{2}$ pieces of silk, each containing $24\frac{3}{8}$ ells, at 6*s.* $0\frac{3}{4}$*d.* per ell. How must I sell it per yard, to gain £5 by the bargain? *Ans.* 5*s.* $9\frac{1}{4}$*d.* $\frac{369}{433}$.

THE DOUBLE RULE OF THREE.

(1) If a carrier receive £$2\frac{1}{10}$ for the carriage of 3 *cwt.* 150 miles, how much ought he to receive for the carriage of 7 *cwt.* $3\frac{1}{2}$ *qrs.* 50 miles? *Ans.* £1..16..9.

(2) If £100 in 12 months gain £$5\frac{1}{4}$ interest, what principal will gain £$3\frac{3}{8}$ in 9 months? *Ans.* £85..14..$3\frac{1}{4}$ $\frac{4}{7}$.

(3) If 9 students spend £$10\frac{7}{8}$ in 18 days, how much will 20 students spend in 30 days? *Ans.* £39..18..$4\frac{20}{81}$.

(4) Two persons earned $4\frac{5}{8}$*s.* for one day's labour: how much would 5 persons earn in $10\frac{1}{2}$ days, at the same rate? *Ans.* £6..1..$4\frac{3}{4}$ $\frac{1}{2}$.

(5) If £50 in 5 months gain £$2\frac{37}{144}$, what time will £$13\frac{1}{3}$ require to gain £$1\frac{1}{12}$? *Ans.* 9 *months.*

(6) If the carriage of 60 *cwt.* 20 miles, cost £$14\frac{1}{2}$, what weight can I have carried 30 miles for £$5\frac{7}{16}$? *Ans.* 15 *cwt.*

DECIMAL FRACTIONS.

In Decimal Fractions, the unit is supposed to be divided into tenths, hundredths, thousandth parts, &c.; consequently the denominator is always 10, or 100, or 1000, &c.

In our system of Notation, the figures of a *whole number* follow each other in a *decimal* (or *tenfold*) proportion. Hence, the *numerator* of a decimal Fraction is written as a whole number, only distinguished by a *separating point prefixed* to it. Thus ·5 for $\frac{5}{10}$, ·25 for $\frac{25}{100}$, ·123 for $\frac{123}{1000}$.

The *denominator* is, therefore, not expressed; being always understood to be 1, with *as many ciphers affixed* as there are *places* in the *numerator.*

The different values of figures will be evident in the annexed Table

Integers.								Decimal parts.						
7	6	5	4	3	2	1	·	2	3	4	5	6	7,	&c.
Millions.	Hundreds of thous.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenth parts of *unity*.	Hundredth parts.	Thousandth parts.	Ten thous. parts.	Hund. thous. parts.	Millionth parts.	

From this it plainly appears, that the figures of the *decimal fraction* decrease successively from left to right in a tenfold proportion, precisely as those of the *whole number*.*

Ciphers *on the right* of other decimals do not alter their value; for $\cdot 2 = \frac{2}{10}$, $\cdot 20 = \frac{20}{100}$, $\cdot 200 = \frac{200}{1000}$, are all equal. But one cipher *on the left* diminishes the value *ten times*, two ciphers *one hundred times*, &c.; for $\cdot 02 = \frac{2}{100}$, $\cdot 002 = \frac{2}{1000}$, &c.

A vulgar fraction having a denominator compounded of 2 or 5, or of both, when converted into its *equivalent decimal fraction*, will be finite; that is, will terminate at some certain number of places. All others are *infinite;* and, because they have one or more figures continually repeated without end, they are called *Circulating Decimals*. The repeating figures are called *repetends*.

One repeating figure is called a *single repetend;* as ·222, &c.; generally written thus, ·2′; or thus, $\cdot\dot{2}$. But when more than one repeat, the decimal is a *compound repetend;* as ·36 36, &c. or ·142857 142857, &c. These may be written ·‘36′, and ·‘142857′; or $\cdot\dot{3}\dot{6}$, and $\cdot\dot{1}4285\dot{7}$.

Pure repetends consist of the repeating figures alone; but *mixed repetends* have other figures before the circulating decimal begins; as, ·045′, ·96‘354′.

Finite decimals may be considered as infinite, by making ciphers to recur, which do not alter the value.

Circulating decimals having the *same number of repeating figures* are called *similar* repetends, and those which have an *unequal* number are *dissimilar*. *Similar* and *conterminous* repetends *begin* and *terminate* at the *same places*.

* The first, second, third, fourth, &c. places of decimals are called *primes*, *seconds*, *thirds*, *fourths*, &c. respectively; and decimals are read thus: 57·57 fifty-seven, and *five*, *seven*, of a decimal; that is, fifty-seven, and fifty-seven hundredths. 206·043 two hundred and six, and *nought*, *four*, *three*; that is, 206, and forty-three thousandths.

ADDITION.

RULE. Place the numbers so that the decimal points may stand in a perpendicular line: then will units be under units, &c according to their respective values. Then add as in integers.

(1) Add 72·5 + 32·071 + 2·1574 + 371·4 + 2·75.
(2) Add 30·07 + 2·0071 + 59·432 + 7·1.
(3) Add 3·5 + 47·25 + 927·01 + 2·0073 + 1·5.
(4) Add 52·75 + 47·21 + 724 + 31·452 + ·3075.
(5) Add 3275 + 27·514 + 1·005 + 725 + 7·32.
(6) Add 27·5 + 52 + 3·2675 + ·5741 + 2720.

SUBTRACTION.

RULE. Place the *subtrahend* under the *minuend*, with the decimal points as in Addition; and subtract as in integers.

(1) From ·2754 take ·2371.	(5) From 571 take 54·72.
(2) From 2·37 take 1·76.	(6) From 625 take 76·91.
(3) From 271 take 215·7.	(7) From 23·415 take ·3742.
(4) From 270·2 take 75·4075.	(8) From ·107 take ·0007.

MULTIPLICATION.

RULE. Place the factors, and multiply them, as in whole numbers; and in the product point off as many decimal places as there are in both factors together. When there are not so many figures in the product, supply the defect with ciphers on the left.

(1) Multiply 2·071 by 2·27.	(7) 27·35 × 7·70011.
(2) Multiply 27·15 by 24·3.*	(8) 57·21 × ·0075.
(3) Multiply ·2365 by ·2435.	(9) ·007 × ·007.
(4) Multiply 72347 by 23·15.	(10) 20·15 × ·2705.
(5) Multiply 17105 by ·3257.	(11) ·907 × ·0025.
(6) Multiply 17105 by ·0327.	(12) ·3409803 × ·0016218.

When the multiplier is 10, 100, 1000, &c. it is only removing the separating point in the multiplicand so many places towards the right as there are ciphers in the multiplier: thus ·578 × 10 = 5·78, ·578 × 100 = 57·8, ·578 × 1000 = 578, and ·578 × 10000 = 5780.

CONTRACTED MULTIPLICATION.

RULE. Write the multiplier under the multiplicand in an *inverted* order, the units' figure under that place which is intended to be retained in the product.

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* The 2d example may be multiplied in *two* products, first by 3, and that product by 8 for 24. The 3d, 6th, 7th, and 12th, may be contracted in a similar way.

In multiplying, begin with that figure of the multiplicand which stands over the multiplying figure, rejecting all on the right of that; and set down the first figures of all the products in a perpendicular row.

Increase the first figure of each product by *carrying to it* what would arise from multiplying the *two next* rejected figures on the right, at the rate of *one* from 5 to 14 inclusive, *two* from 15 to 24, *three* from 25 to 34 inclusive, &c.

NOTE. If perfect accuracy as far as the last decimal figure be desired, it will be eligible to find one figure more in the product than is actually wanted.

(13) Multiply 384·672158 by 3·683, and let there be only four places of decimals in the product.*

(14) Multiply 3·141592 by 52·7438, retaining only 4 places of decimals in the product. *Ans.* 165·6995.

(15) Multiply 238·645 by 8217·5, retaining only the integers in the product. *Ans.* 1961065.

(16) Multiply 375·13758 by 16·7324, and reserve only one place of decimals; and again, reserving three places. *Ans.* 6276·9, *and* 6276·951.

(17) Multiply 395·3756 by ·75642, retaining only 4 places of decimals. *Ans.* 299·0700.

DIVISION.

DIVIDE as in integers; and the first figure of the quotient will be of the same value as that figure of the dividend which stands *over the units* in the *first product* of the divisor: so that the *point* must be placed accordingly; ciphers being prefixed, when necessary.

NOTE 1. After proceeding through the dividend, to ascertain if the quotient is correctly *pointed*, observe that the decimal places in the divisor and quotient together, must equal in number those of the dividend.

2. When there are *fewer* decimal places in the dividend than in the divisor, *equalise* them by *affixing ciphers;* and the quotient, to *that extent*, will be a *whole* number.

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* *Contracted method*	*Common method.*
384·672158	384·672158
386·3	3·683
11540165	11540 | 16474
2308033	307737 | 7264
307738	2308032 | 948
11540	11540164 | 74
1416·7476	1416·7475 | 57914

3. Ciphers may be subjoined to the decimal part of the dividend, or brought down as if they were subjoined; in order to continue the operation to any degree of exactness desired.

(1) Divide 217·75 by 65.
(2) Divide 709 by 2·574.
(3) Divide 125 by ·1045.
(4) Divide 48 by 144.
(5) Divide 5·714 by 8275.
(6) Divide 715 by ·3075.
(7) 7382·54 ÷ 6·4252.
(8) ·0851648 ÷ 423.
(9) 267·15975 ÷ 13·25.
(10) 72·1564 ÷ ·1347.
(11) 85643·825 ÷ 6·321.
(12) 1 ÷ 3·1416.

To divide by 10, 100, 1000, &c. remove the separating point in the dividend so many places towards the left, as there are ciphers in the divisor, and the thing is accomplished.

Thus 5784 ÷ 10 = 578·4, 5784 ÷ 100 = 57·84, 5784 ÷ 1000 = 5·784, 5784 ÷ 10000 = ·5784.

(13) 3719 ÷ 10.
(14) 3·74 ÷ 100.
(15) 130·7 ÷ 1000.
(16) 34·012 ÷ 10000.

CONTRACTED DIVISION.

Ascertain the value of the first quotient figure: from which it will be known what number of figures in the quotient will serve the purpose required. Use *that number* of the figures in the divisor, (rejecting the others on the right) and a *sufficient number* of the dividend, to find the first figure of the quotient; make each remainder a new dividual, and for each succeeding figure reject another from the divisor: but observe to carry to each product from the rejected figures as in Contracted Multiplication.

Note. When there are *fewer* figures in the divisor than the number wanted in the quotient, proceed by the common rule till those in the divisor are just *as many* as remain to be found in the quotient, and then use the contraction.

(17) Divide 70·23 by 7·9863, to three places of decimals.*

(18) Divide 721·17562 by 2·257432, to the extent of only three places of decimals in the quotient.

(19) Divide 25·1367 by 217·35, to the fourth decimal.

.... * *Contracted Method.*	*Common Method.*
7·9863)70·230(8·793	7·9863)70·2300(8·793
63 890	638904
6340	6339 | 60
5590	5590 | 41
750	749 | 190
719	718 | 767
31	30 | 4230
24	23 | 9589
7	6 | 4641

(20) Divide 51·47542 by ·123415, to the second decimal.
(21) Divide 27·104 by ·3712, the integral quotient only.

CIRCULATING DECIMALS.

To reduce a circulate to a vulgar fraction.

RULE 1. For a *pure* repetend, make the *circulating* figures the *numerator*, to *as many nines* for the *denominator*.

2. For a *mixed* repetend, subtract the *finite part* from the *whole*, and make the *difference* the *numerator*; the *denominator* to which will consist of *as many nines* as there are *repetends*, with *as many ciphers* subjoined as there are *finite* figures.

EXAMPLES.

(1) Reduce ·1′, ·3′, ·9′, ·`01′, and ·`142857′, to their equivalent vulgar fractions.

·1′ = $\frac{1}{9}$; ·3′ = $\frac{3}{9} = \frac{1}{3}$; ·9′ = $\frac{9}{9} = 1$; ·`01′ = $\frac{1}{99}$; and ·`142857′ $= \frac{142857}{999999} = \frac{15873}{111111} = \frac{5291}{37037} = \frac{481}{3367} = \frac{1}{7}$.

(2) Reduce ·03`45′ and 3·5`126′ to equivalent vulgar fractions.

·03`45′ $= \frac{345-3}{9900} = \frac{342}{9900} = \frac{38}{1100} = \frac{19}{550}$.

3·5`126′ $= \frac{35126-35}{9990} = \frac{35091}{9990} = \frac{3899}{1110} = 3\frac{569}{1110}$. Or thus:

3·5`126′ $= 3 + \frac{5126-5}{9990} = 3\frac{5121}{9990} = 3\frac{569}{1110}$.

In ADDITION and SUBTRACTION of *Circulating Decimals*, make them *similar* and *conterminous*, and carry to the figures on the right whatever would arise from the *repetends being continued*.

NOTE. In *all cases*, when the repetend is 9, make it a *cipher*, and add 1 to the *next figure*: for ·999, &c. = 1.

In MULTIPLICATION, carry to the product of the right hand figure what would arise from the product of the *repetends continued*; and, in finding the sum of the products, observe what is directed in ADDITION.

In DIVISION, it is only necessary to observe that the operation may be carried on with the *repeating* figures of the dividend, to any extent required.

NOTE. When the *Multiplier* or the *Divisor* is a circulate, the most convenient method is, to change it into a *common fraction*.

EXAMPLES.

(3) What is the sum of 25·`142857′, 10·3`90′, 12·035′, and 4·02`567′?

		Similar.		*Similar and conterminous.*
25·`142857′	=	25·`142857′	=	25·14`285714′
10·3`90′	=	10·3`909090′	=	10·39`090909′
12·035′	=	12·03`555555′	=	12·03`555555′
4·02`567′	=	4·02`567567′	=	4·02`567567′
			Sum	51·59`499746′

(4) What is the difference between 567·`367′ and 55·0`9729′? Also, between 57, and 49·8`53′?

567·`367′	=	567·3`673673673673′
55·0`9729′	=	55·0`972997299729′
	difference	512·2`700676373943′

	57·
	49·8`53′
diff.	7·1`46′

(5) Multiply 65·316′ by ·753.

65·316′
·753
195950
3265833′
45721666′
product 49·183450

(6) Multiply 13·`45′ by 3·`36′.

$3\cdot`36' = 3\frac{36}{99} = 3\frac{4}{11} = \frac{37}{11}$.

13·`45′
37
94`18′
403`63′
11)497·`81′
product 45·256198, &c.

(7) Divide 150·9`045′ by 33.

3)150·9`045′
11)50·3`015′
4·5`728637′ quotient.

(8) Divide 17·8054′ by 3·6′.

$3\cdot6' = 3\frac{6}{9} = 3\frac{2}{3} = \frac{11}{3}$.

$17\cdot8054' \times \frac{3}{11} = \frac{53\cdot4163'}{11} =$ 4·856`03′ quotient.

(9) What are the equivalents to ·004`354′ and 65·00063`648′?
Ans. $\frac{29}{6660}$, and $65\frac{471}{740000}$.

(10) What is the sum of 57·575 + 3·59`163′ + 210·16′ + ·06`3759′? *Ans.* 271·397`057674235892′.

(11) Required the difference between 36·30`45207′ and 47·280`43′. *Ans.* 10·975`9135982268′.

(12) Multiply 4·`428571′ by 347; and 17·0`54′ by 6·`148′. *Ans.* 1536·`714285′; and 104·85`387205′.

(13) Divide 1536·`714285′ by 347; and 104·85`387205′ by 6·`148′. *Ans.* 4·`428571′; and 17·0`54′.

REDUCTION.

To reduce a Vulgar Fraction to a Decimal.

RULE. Add ciphers to the numerator, and divide by the denominator: the quotient will be the decimal fraction required.

(1) Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{3}{8}$, to decimals.
Ans. ·25, ·5, ·75, and ·375.

(2) Reduce $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, and $\frac{1}{9}$, to decimals.
Ans. ·3′, ·2, ·`142857′, and ·1′.

(3) Reduce $\frac{5}{26}$ to a decimal. *Ans.* ·1`923076′.

(4) Reduce $\frac{11}{14}$ of $\frac{10}{13}$ to a decimal. *Ans.* ·6`043956′.

To reduce a given quantity to the Decimal of any denomination required.

RULE. Reduce those of the lowest denomination to decimal parts of the next superior, on the left of which place the given quantity of that denomination; reduce this to the next, and proceed as before, till it is of the denomination required.

(5) Reduce 5*s.** 9*s.* and 16*s.* to the decimals of a pound.
Ans. £·25, £·45, and £·8.

(6) Reduce 8*s.* 4*d.* to the decimal of a £. *Ans.* £·416′

(7) Reduce 16*s.* 7$\frac{3}{4}$*d.* to the decimal of a £.†

(8) Reduce 19*s.* 5$\frac{1}{2}$*d.* to the decimal of a £. *Ans.* £·972916′.

(9) Reduce 12 grains to the decimal of a *lb.* Troy.
Ans. lb. ·002083′.

(10) Reduce 12$\frac{13}{64}$ drams to the decimal of a *lb.* avoirdupois.
Ans. lb. ·047668+.

(11) Reduce 2*qrs.* 14$\frac{1}{4}$ *lb.* to the decimal of a *cwt.*
Ans. cwt. ·62723+.

(12) Reduce 2 furlongs, 161$\frac{1}{4}$ yards to the decimal of a mile. *Ans.* ·341761`36′ *mile.*

(13) Reduce 5$\frac{14}{15}$ pints to the decimal of a gallon.
Ans. ·7416′ *gal.*

(14) Reduce 4$\frac{1}{2}$ gallons of wine to the decimal of a hogshead. *Ans.* ·0`714285′.

---

* 20) 5·00 *s.*
£·25 *Ans.*

† 4 | 3·00 *qrs.*
12 | 7·75 *d.*
20 | 16·[illegible] *s.*
£·[illegible] *Ans.*

(15) Required the mixed decimal number equivalent to £3..9..1$\frac{163}{1250}$*d*. *Ans.* £3·45471.

(16) Express 7 weeks, 3 days, in the decimal of a year. *Ans. yr.* ·142465+.

To find the proper value of a Decimal Fraction of any Integer.

RULE. Multiply the given decimal by the proper number to reduce it to the next inferior denomination, pointing off the given number of decimals in the product; reduce these to the next, and so on to the lowest; and the whole numbers on the left (being collected together) will be the value required.

A decimal of a £ may be thus valued by inspection. Double the *tenths* for *shillings*, and call the number in the *second* and *third*, *farthings*, abating *one* above 12, and *two* above 37. But if the *second* is 5, or upwards, call the 5 *one shilling*, and reckon only the excess above five with the third.

By reversing these directions, any given sum in shillings, &c. may be expressed in the decimal of a £.—Thus, half the shillings are tenths, and an odd shilling, 5 hundredths; the rest (in farthings) add into the second and third places, increasing *one* above 11 farthings, and *two* above 36.

(17) What is the value of ·8322916 of a £.*

(18) Reduce £·740596 to its proper value. *Ans.* 14*s.* 9*d.*.2·97216 *qrs.*

(19) What is the value of ·082084 of a *lb.* Troy? *Ans.* 19 *dwts.* 16·80384 *grains.*

(20) What is the value of ·4909375 *lb.* avoirdupois? *Ans.* 7 *oz.* 13·68 *drams.*

(21) What is the value of £·19895? *Ans.* 3*s.* 11*d.*.2·992 *qrs.*

(22) What is the value of ·625 of a *cwt.*? *Ans.* 2 *qrs.* 14 *lb.*

(23) What is the value of ·071428 of a hogshead of wine? *Ans.* 4 *gal.* 1·999856 *qts.*

(24) What is the value of ·0625 of a barrel of beer? *Ans.* 2 *gallons*, 1 *quart.*

(25) What is the value of ·142465 of a year? *Ans.* 51·999725 *days.*

---

* £ ·8322916
20
s. 16·645832
12
d. 7·749984
4
qrs. 2·999936

s. *d.*
Ans. 16..7$\frac{3}{4}$ very nearly.

By inspection.

£		*s.* *d.*
·8	=	16..0
·032	=	0 .7$\frac{3}{4}$
·832	=	16 .7$\frac{3}{4}$

Decimal Tables of Coin, Weight, and Measure.

TABLE I.

STERLING MONEY.

£1 the Integer.

s.	*dec.*	*s.*	*dec.*
19	·95	9	·45
18	·9	8	·4
17	·85	7	·35
16	·8	6	·3
15	·75	5	·25
14	·7	4	·2
13	·65	3	·15
12	·6	2	·1
11	·55	1	·05
10	·5		

6*d.*	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

3*qrs.*	·003125
2	·0020833
1	·0010416

TABLE II.

ENG. COIN. 1*s.*

Long Meas. 1 Foot the Integer.

Pence or Inches.	*Decimals.*
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

3*qrs.*	·0625
2	·041666
1	·020833

TABLE III.

TROY WEIGHT.

1 *lb.* the Integer.

Ounces the same as Pence in Table II.

Dwts.	*Decimals.*
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

12*gr.*	·002083
11	·001910
10	·001736
9	·001562
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

1 *oz.* the Integer.

Penny-weights the same as Shillings in the first Table.

Grains.	*Decimals.*
12	·025
11	·022916
10	·020833
9	·01875
8	·016666
7	·014583
6	·0125
5	·010416
4	·008333
3	·00625
2	·004166
1	·002083

TABLE IV.

AVOIR. WEIGHT.

1 *cwt.* the Integer.

Qrs.	*Decimals.*
3	·75
2	·5
1	·25

14 *lbs.*	·125
13	·116071
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

8 *oz.*	·004464
7	·003906

Decimal Tables of Coin, Weight, and Measure.

6 *oz.*	·003348
5	·002790
4	·002232
3	·001674
2	·001116
1	·000558
¾	·000418
½	·000279
¼	·000139

TABLE V.

AVOIR. WEIGHT.

1 *lb.* the Integer.

Ounces.	*Decimals.*
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625

8 *dr.*	·03125
7	·027343
6	·023437
5	·019531
4	·015625
3	·011718
2	·007812
1	·003906

TABLE VI.

LIQUID MEASURE.

1 Tun the Integer.

Gallons.	*Decimals.*
100	·396825
90	·357142
80 *g.*	·317460
70	·277777
60	·238095
50	·198412
40	·158730
30	·119047
20	·079365
10	·039682
9	·035714
8	·031746
7	·027777
6	·023809
5	·019841
4	·015873
3	·011904
2	·007936
1	·003968

4 *pts.*	·001984
3	·001488
2	·000992
1	·000496

1 Hogshead the Integer.

Gallons.	*Decimals.*
30	·476190
20	·317460
10	·158730
9	·142857
8	·126984
7	·111111
6	·095238
5	·079365
4	·063492
3	·047619
2	·031746
1	·015873
3 *pts.*	·005952
2	·003968
1	·001984

TABLE VII.

MEASURES.

Liquid. Dry.

1 *Gal.* 1 *Qr.*

the Integer.

Pts.	*Dec.*	*Bush.*
4	·5	4
3	·375	3
2	·25	2
1	·125	1
¾	·09375	3 *p.*
½	.0625	2
¼	·03125	1

Decimals.	*Qr. Pks.*
·0234375	3
·015625	2
·0078125	1
·005859	3 *pts.*
·003906	2
·001953	1

TABLE VIII.

LONG MEASURE.

1 Mile the Integer.

Yards.	*Decimals.*
1000	·568182
900	·511364
800	·454545
700	·397727
600	·340909

Decimal Tables of Coin, Weight, and Measure.

500*yd.*	·284091
400	·227272
300	·170454
200	·113636
100	·056818
90	051136
80	·045454
70	·039773
60	·034091
50	·028409
40	·022727
30	·017045
20	·011364
10	·005682
9	·005114
8	·004545
7	·003977
6	·003409
5	·002841
4	·002273
3	·001704
2	·001136
1	·000568
2 *ft.*	·0003787
1	·0001894
6 *in.*	·0000947
3	·0000474
2	·0000315
1	·0000158

TABLE IX.

TIME.

1 Year the Integer.

Months the same as Pence in Table II.

Days.	*Decimals.*
300	·821918
200	·547945
100	·273973
90	·246575
80 *d.*	·219178
70	·191781
60	·164383
50	·136986
40	·109589
30	·082192
20	·054794
10	·027397
9	·024657
8	·021918
7	·019178
6	·016438
5	·013698
4	·010959
3	·008219
2	·005479
1	·002739

1 Day the Integer.

12 *hrs*	·5
11	·458333
10	·416666
9	·375
8	·333333
7	·291666
6	·25
5	·208333
4	·166666
3	·125
2	·083333
1	·041666
30 *m.*	·020833
20	·013888
10	·006944
9	·00625
8	·005555
7	·004861
6	·004166
5	·003472
4	·002777
3	·002083
2	·001388
1	·000694

TABLE X.

CLOTH MEASURE.

1 Yard the Integer.

Qrs. the same as Table IV.

Nails.	*Decimals.*
3	·1875
2	·125
1	·0625

TABLE XI.

LEAD WEIGHT.

A Foth the Integer.

Hund.	*Decimals.*
10	·512820
9	·461538
8	·410256
7	·358974
6	·307692
5	·256410
4	·205128
3	·153846
2	·102564
1	·051282
3 *qrs.*	·038461
2	·025641
1	·012820
14 *lbs.*	·0064102
13	·0059523
12	·0054945
11	·0050366
10	·0045787
9	·0041208
8	·0036630
7	·0032051
6	·0027472
5	·0022893
4	·0018315
3	·0013736
2	·0009157
1	·0004578

THE RULE OF THREE.

(1) If $26\frac{1}{2}$ yards cost £3..16..3, what will $32\frac{1}{4}$ *yds.* cost?*

(2) If $7\frac{1}{4}$ yards of cloth cost £2..12..9, what will $140\frac{1}{2}$ yards of the same cost? *Ans.* £47..16..$3\frac{1}{4}$.

(3) If a chest of sugar, weighing 7 *cwt.* 2 *qrs.* 14 *lb.* cost £36..12..9, what will 2 *cwt.* 1 *qr.* 21 *lb.* of the same cost? *Ans.* £11..14..$2\frac{3}{4}$.

(4) What will $326\frac{1}{4}$ *lb.* of coffee be worth when $1\frac{1}{2}$ *lb.* is sold for 3*s.* 6*d.*? *Ans.* £38..1..3.

(5) What is the value of 19 *oz.* 3 *dwts.* 5 *grs.* of gold, at £2..19 per *oz.*? *Ans.* £56..10..5..2·3 *qrs.*

(6) What is the charge for $827\frac{1}{4}$ yards of painting, at $10\frac{1}{2}d.$ per yard? *Ans.* £36..4..3..1·5 *qrs.*

(7) If I lent my friend £34 for $\frac{3}{4}$ of a year, how much ought he to lend me for $\frac{5}{12}$ of a year? *Ans.* £51.

(8) If $\frac{3}{4}$ of a yard of cloth, that is $2\frac{1}{4}$ yards broad, make a garment, how much of $\frac{4}{5}$ of a yard wide will make a similar one? *Ans.* 2 *yds.* 1·75 *nail.*

(9) If 1 *oz.* of silver is worth 5*s.* 6*d.* what is the price of a tankard that weighs 1 *lb.* 10 *oz.* 10 *dwts.* 4 *grs.*? *Ans.* £6..3..9..2·2 *qrs.*

(10) What is the value of 15 *cwt.* 1 *qr.* 19 *lb.* of cotton, at 15*d.* per *lb.*? *Ans.* £107..18..9.

(11) If 1 *cwt.* of currants cost £2..9..6, what will 45 *cwt.* 3 *qrs.* 14 *lb.* cost at the same rate? *Ans.* £113..10..$9\frac{3}{4}$.

(12) Bought 6 chests of sugar, each 6 *cwt.* 3 *qrs.* at £2..16 per *cwt.* What do they come to? *Ans.* £113..8.

(13) Bought a tankard for £10..12, at the rate of 5*s.* 4*d.* per *oz.* What was the weight? *Ans.* 39 *oz.* 15 *dwts.*

(14) Gave £187..3..3 for 25 *cwt.* 3 *qrs.* 14 *lb.* of coffee: at what rate did I buy it per *lb.*? *Ans.* 1*s.* $3\frac{1}{2}d.$

(15) Bought 29 *lb.* 4 *oz.* of snuff, for £10..11..3. What is the value of 3 *lb.*? *Ans.* £1..1..8.

(16) If I give 1*s.* 1*d.* for $3\frac{1}{2}$ *lb.* of rags, what will be the value of 1 *cwt.*? *Ans.* £1..14..8.

---

* As 26·5 *yds.* : 3·8125 £ : : 32·25 *yds.* : 4·63974 £
32·25

26·5)122·953125(4·63974 = £4..12..$9\frac{1}{2}$. *Ans.*

EXCHANGE

Is the act of bartering the money of one place for that of another, by means of a written instrument called a *Bill of Exchange.*

The operations in this Rule consist in finding the quantity of *one sort* of money that will be equal to a given sum of *the other,* according to the existing *Course of Exchange.*

Par of Exchange signifies the *equality* in the *intrinsic value* of two sums of money of different countries; and shows how much of the one is worth a constant sum (or piece of coin) of the other.

Course of Exchange is the comparative value between the money of two different countries at any particular time; which often fluctuates above or below the *Par.*

Agio is a difference of so much per cent in the value of the *Bank-money* and the *Current-money* of some foreign countries, the former being of superior value.

To change Foreign Money into British Sterling Money, or Sterling into Foreign; according to a given Course of Exchange.

RULE. As the quantity of Foreign mentioned in the given course of exchange, is to the quantity of Sterling; so is any other sum of the Foreign, to its corresponding value in Sterling money.

And by mutually changing the words Foreign and Sterling, the Rule will serve for changing Sterling into Foreign money.

I. FRANCE.

Accounts are kept at Paris, Lyons, and Rouen, in livres, sols, and deniers; and exchange is made by the écu, or crown = 4*s.* 6*d.* at par.

TABLE.	12 deniers make	1 sol.
	20 sols	1 livre.
	3 livres	1 écu, or crown.

(1) How many crowns must be paid at Paris, to receive in London £180, exchange at 4*s.* 6*d.* per crown?*

---

	s. d.	cr.	£	
* As	4..6	: 1 :	: 180	
	2		40	
	9 sixp.		9)7200	sixp.
			800	crowns. *Ans.*

(2) How much sterling must be paid in London, to receive in Paris 758 crowns, exchange at 4*s.* 8*d.* per crown?
Ans. £176..17..4.

(3) A merchant in London remits £176..17..4. to his correspondent at Paris: what is the value in French crowns, at 4*s.* 8*d.* per crown? *Ans.* 758 *crowns.*

(4) Change 725 crowns, 17 sols, 7 deniers, at 4*s.* $6\frac{1}{2}$*d.* per crown, into sterling money. *Ans.* £164..14..$0\frac{1}{4}$. $\frac{319}{360}$.

(5) Change £164..14..$0\frac{1}{2}$ sterling into French crowns, exchange at 4*s.* $6\frac{1}{2}$*d.* per crown.
Ans. 725 *crowns,* 17 *sols,* $7\frac{41}{109}$ *deniers.*

II. SPAIN.

Accounts are kept at Madrid, Cadiz, and Seville, in dollars, rials, and maravedies; and exchange is made by the piece of eight = 4*s.* 6*d.* at par.

TABLE.	34 maravedies make	1 rial.
	8 rials	1 piastre, or piece of eight.
	10 rials	1 dollar.

(6) A merchant at Cadiz remits to London 2547 pieces of eight, at 4*s.* 8*d.* per piece: how much sterling is the sum?
Ans. £594..6.

(7) How many pieces of eight, at 4*s.* 8*d.* each, will answer a bill of £594..6 sterling? *Ans.* 2547.

(8) If I pay here a bill of £2500, for what Spanish money may I draw my bill at Madrid, exchange at 4*s.* $9\frac{1}{2}$*d.* per piece of eight? *Ans.* 10434 *pieces of eight,* 6 *rials,* $8\frac{20}{23}$ *mar.*

III. ITALY.

Accounts are kept at Genoa and Leghorn, in livres, sols, and deniers; and exchange is made by the piece of eight, or dollar = 4*s.* 6*d.* at par.

TABLE.	12 deniers make	1 sol.
	20 sols	1 livre.
	5 livres	1 piece of eight at Genoa.
	6 livres	1 piece of eight at Leghorn.

N.B. The exchange at Florence is by ducatoons; at Venice by ducats.

TABLE.	6 solidi make	1 gross.
	24 gross	1 ducat.

(9) How much sterling money may a person receive in London, if he pay in Genoa 976 dollars at 4*s.* 5*d.* per dollar?
Ans. £215..10..8.

(10) A factor has sold goods at Florence, for 250 ducatoons, at 4*s*. 6*d*. each: what is the value in pounds sterling? *Ans.* £56..5.

(11) If 275 ducats, at 4*s*. 5*d*. each, be remitted from Venice to London, what is the value in pounds sterling? *Ans.* £60..14..7.

(12) A traveller would exchange £60..14..7 sterling for Venice ducats, at 4*s*. 5*d*. each: how many must he receive? *Ans.* 275.

IV. PORTUGAL.

Accounts are kept at Oporto and Lisbon, in reas, and exchange is made by the milrea = 6*s*. $8\frac{1}{2}$*d*. at par.

TABLE. 1000 reas make 1 milrea.

(13) A gentleman being desirous to remit to his correspondent in London, 2750 milreas, exchange at 6*s*. 5*d*. per milrea; for how much sterling will he be creditor in London? *Ans.* £882..5..10.

(14) A merchant at Oporto remits to London 4366 milreas, 183 reas, at 5*s*. $5\frac{5}{8}$*d*. exchange per milrea. How much sterling must be paid in London for this remittance? *Ans.* £1193..17..6..3·0375 *qrs*.

(15) If I pay a bill in London of £1193..17..6..3·0375 *qrs*. what must I draw for on my correspondent in Lisbon, exchange at 5*s*. $5\frac{5}{8}$*d*. per milrea? *Ans.* 4366 *milreas*, 183 *reas*.

V. HOLLAND, FLANDERS, AND GERMANY.

At Antwerp, Amsterdam, Brussels, Rotterdam, and Hamburgh, some accounts are kept in pounds, shillings, and pence, as in England; others in guilders, stivers, and pennings: exchange with London, at from 33*s*. to 36*s*. or 38*s*. Flemish per pound sterling.

TABLE.	8 pennings make	1	groat.
	2 groats, or 16 pennings	1	stiver.
	20 stivers	1	guilder, or florin.
Also,	12 groats, or six stivers, make	1	schelling.
	20 schellings, or 6 guilders	1	pound.

(16) Remitted from London to Amsterdam, a bill of £754..10 sterling: how many pounds Flemish is the sum, the exchange at 33*s*. 6*d*. Flemish per pound sterling? *Ans.* £1263..15..9 *Flemish*.

(17) A merchant in Rotterdam remits £1263..15..9 Flemish to be paid in London: how much sterling money must he

draw for, the exchange being at 33*s.* 6*d.* Flemish per pound sterling? *Ans.* £754..10.

(18) If I pay in London £852..12..6 sterling, how many guilders must I draw for at Amsterdam, exchange at 34 schellings, $4\frac{1}{2}$ groats Flemish per pound sterling?

Ans. 8792 *guild.* 13 *stiv.* 1 *gr.* $6\frac{1}{2}$ *pennings.*

(19) What must I draw for in London, if I pay in Amsterdam 8792 guild. 13 stiv. $14\frac{1}{2}$ pennings, exchange at 34 schellings, $4\frac{1}{2}$ groats per pound sterling? *Ans.* £852..12..6.

To convert Bank Money into Currency; and the contrary.

As 100 : 100 *plus the agio* : : the Bank-money : the Currency.

As 100 *plus the agio* : 100 : : the Currency : the Bank-money.

(20) Change 794 guilders, 15 stivers, Current money, into Bank florins, agio $4\frac{3}{8}$ per cent.

Ans. 761 *guilders,* 8 *stivers,* $11\frac{7}{8}\frac{3}{3}\frac{6}{5}$ *pennings.*

(21) Change 761 guilders, 9 stivers Bank, into Current money, agio $4\frac{3}{8}$ per cent.

Ans. 794 *guilders,* 15 *stivers,* $4\frac{3}{10}$ *pennings.*

VI. IRELAND.

The *par of Exchange,* long established with Ireland, was £108..6..8 Irish = £100 English. That is, £1..1..8 Irish = £1 English; or 13*d.* Irish = 1*s.* English.

But the English and Irish currency are now assimilated.

(22) A gentleman remitted to Ireland £575..15 sterling: what would he receive there, the exchange being at £10 per cent? *Ans.* £633..6..6.

(23) What would be paid in London for a remittance of £633..6..6 Irish, exchange at £10 per cent? *Ans.* £575..15.

CONJOINED PROPORTION, OR COMPOUND ARBITRATION OF EXCHANGE,

Is the method of comparing the coins, weights, or measures of one country with those of another, when the comparison is to be made *through the medium* of those of other countries.

Case 1. When it is required to find *how many* of the *first* sort mentioned are equal to a *given quantity* of the *last.*

Rule. Place the terms alternately, *antecedents* and *consequents*, in two columns, *left* and *right*. The last term, being an antecedent, will stand on the left.

Divide the product of the antecedents by the product of the consequents for the answer.

Proof. By as many single statements as the question requires.

(1) If 20 *lb.* at London make 23 *lb.* at Antwerp, and 155 *lb.* at Antwerp make 180 *lb.* at Leghorn, how many *lb.* at London are equal to 72 *lb.* at Leghorn?*

(2) If 12 *lb.* at London make 10 *lb.* at Amsterdam, and 100 *lb.* at Amsterdam 120 *lb.* at Toulouse, how many *lb.* at London are equal to 40 *lb.* at Toulouse? *Ans.* 40 *lb.*

(3) If 140 braces at Venice be equal to 156 braces at Leghorn, and 7 braces at Leghorn equal to 4 ells English, how many braces at Venice are equal to 16 ells English?

Ans. $25\frac{5}{39}$.

(4) If 40 *lb.* at London make 36 *lb.* at Amsterdam, and 90 *lb.* at Amsterdam make 116 *lb.* at Dantzic, how many *lb.* at London are equal to 130 *lb.* at Dantzic? *Ans.* $112\frac{2}{29}$.

Case 2. When it is required to find *how many* of the *last* sort mentioned are equal to a *given quantity* of the *first*.

Rule. Place the *antecedent* and *consequent* terms as before. But the last term, being a *consequent*, will stand on the right. Divide the product of the consequents by that of the antecedents.

(5) If 12 *lb.* at London make 10 *lb.* at Amsterdam, and 100 *lb.* at Amsterdam 120 *lb.* at Toulouse, how many *lb.* at Toulouse are equal to 40 *lb.* at London? *Ans.* 40 *lb.*

(6) If 40 *lb.* at London make 36 *lb.* at Amsterdam, and 90 *lb.* at Amsterdam 116 *lb.* at Dantzic, how many *lb.* at Dantzic are equal to 122 *lb.* at London? *Ans.* $141\frac{13}{25}$.

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* *Antecedents.*		*Consequents.*
20 *lb.* London	=	23 *lb.* Antwerp.
155 *lb.* Antwerp	=	180 *lb.* Leghorn.
72 *lb.* Leghorn	=	how many London?

$$\frac{\overset{1}{\cancel{20}} \times 155 \times \overset{8}{\cancel{72}}}{23 \times \underset{\underset{1}{\cancel{9}}}{\cancel{180}}} = \frac{1240}{23} = 53\tfrac{21}{23}\ lb.\ \mathit{Ans.}$$

INVOLUTION

Is the method of finding the *powers* of numbers.

Any number is the *first power* of itself, and the *root* of all its powers: and when the root is multiplied by itself, the product is the *second* power; the *second* multiplied by the *first* produces the third, &c.

The second power is commonly called the *square;* and the third power, the *cube.*

Numbers called *indices,* or *expónents,* are placed on the right a little above the line, to denote the respective powers. Thus, 3^2 signifies the *square,* or *second power,* of 3; the smal figure ^{2} being the *index,* or *expónent.*

To involve a number to any power.

RULE. Multiply the given number (or root) by itself continually *one time less* than the *index* of the power: that is, once for the second, twice for the third power, &c.

Observe, that *any* two or more powers multiplied together, will produce a power whose index is the *sum* of their indices. Thus, the *seventh* power is the *product* of the *fourth* and the *third;* because the sum of the indices, $4 + 3 = 7$.

ILLUSTRATIONS.

The first power of 3 is 3^1, or 3.

The second power of 5 is $5^2 = 5 \times 5 = 25$.

The third power of 4 is $4^3 = 4 \times 4 \times 4 = 64$.

The fourth power of ·05 is $\cdot05^4 = \cdot05 \times \cdot05 \times \cdot05 \times \cdot05 =$ ·00000625.

The fifth power of $\frac{2}{3}$ is $\overline{\frac{2}{3}}|^5 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{32}{243}$.

(1) Required the squares of 43, 2174, 4·3, and ·2174.

Ans. 1849, 4726276, 18·49, *and* ·04726276.

(2) Cube 111, 1·11, $\frac{3}{4}$, and $2\frac{5}{8}$.

Ans. 1367631, 1·367631, $\frac{27}{64}$, *and* $18\frac{45}{512}$.

(3) Involve 9 to the ninth power. *Ans.* 387420489.

(4) Find the third, fifth, and eighth powers (without finding the fourth, sixth, and seventh) of 1·7.

Ans. 4·913, 14·19857, *and* 69·75757441.

(5) What are the third and sixth powers of ·05?

Ans. ·000125 *and* ·000000015625.

EVOLUTION

Is the method of extracting the *roots* of powers. It is, therefore, the reverse of *Involution:* by referring to which it will be obvious, that the *Square Root* of a number multiplied by itself, will produce that number; and that the *Cube Root*, multiplied *twice* by itself, will produce the number (or power) of which it is the root.

NOTE. The roots of *complete* powers are called *rational;* and those which cannot be *completely* extracted, are called *surds*, or *irrational roots:* thus $\sqrt{4}=2$, is *rational;* but $\sqrt{5}$ is a *surd*. The surd roots may, however, be found to any extent proposed.

SQUARE ROOT.

RULE 1. Place *points* over the units, hundreds, &c. so as to form *periods* of *two figures* each.

2. From the first period on the left, subtract the *greatest square* contained in it; put the *root* on the right, as a quotient; annex the succeeding period to the remainder, and call that number the *Résolvend.*

3. Divide the resolvend, exclusive of the units, by double the root; annex the quotient to the root, and also to the right of the divisor to complete it: then multiply the divisor by that quotient figure, and subtract the product from the resolvend.

4. The remainder, with the next period joined, will form a new resolvend; and double the root, a new divisor; with which proceed as before.

NOTE 1. When the number of figures in the *Integer* is uneven, the first period will consist of but one figure. When there is an odd number of *decimals*, a cipher must be added to complete the periods.

2. When the figures of the whole number are exhausted, periods of ciphers may be used at pleasure, to continue the extraction in decimals. In all cases, the root will consist of as many figures as there are periods, whether integral or decimal.

ROOTS.	1.	2.	3.	4.	5.	6.	7.	8.	9.
SQUARES.	1.	4.	9.	16.	25.	36.	49.	64.	81.

(1) What is the square root of 119025?*

(2) What is the square root of 106929? *Ans.* 327.

(3) What is the square root of 22071204? *Ans.* 4698.

---

* 119025 (345, the root.
9
64) 290
256
685) 3425
3425

345
345
1725
1380
1035
Proof 119025

(4) What is the square root of 2268741? *Ans.* 1506·23+.
(5) What is the square root of 7596796? *Ans.* 2756·228+.
(6) What is the square root of 4·372594? *Ans.* 2·091+.
(7) What is the square root of 2·2710957? *Ans.* 1·50701+.
(8) What is the square root of ·00032754? *Ans.* ·01809+.
(9) What is the square root of 1·270059? *Ans.* 1·1269+.

To find the Roots of Fractional Numbers.

RULE. When the *terms* of a Fraction are *complete powers*, extract their roots for the *corresponding terms* of the root.

When they are *surds*, find an equivalent fraction, by multiplying *both terms* by the *denominator;* or by the *least number* that will make the denominator a *square*. Then divide the root of the numerator by the root of the denominator for the answer.—Or, reduce the fraction to a decimal, and extract its root.

Mixed numbers may either be reduced to their equivalent fractions, or into a decimal form.

(10) What is the square root of $\frac{2304}{5184}$? *Ans.* $\frac{2}{3}$.
(11) What is the square root of $\frac{9216}{12544}$? *Ans.* $\frac{6}{7}$.
(12) What is the square root of $51\frac{21}{25}$? *Ans.* $7\frac{1}{5}$.
(13) What is the square root of $27\frac{9}{16}$? *Ans.* $5\frac{1}{4}$.
(14) What is the square root of $9\frac{43}{49}$? *Ans.* $3\frac{1}{7}$.
(15) What is the square root of $\frac{275}{341}$? *Ans.* ·89802+.
(16) What is the square root of $\frac{478}{549}$? *Ans.* ·93309+.
(17) What is the square root of $85\frac{14}{15}$? *Ans.* 9·27+.
(18) What is the square root of $8\frac{5}{7}$? *Ans.* 2·9519+.

To find a mean proportional between any two given numbers.

RULE. Extract the square root of their product.

(19) What is the mean proportional between 3 and 12?

$\sqrt{3\times12}=\sqrt{36}=6$, *the mean proportional. Ans.*

(20) What is the mean proportional between 4276 and 842? *Ans.* 1897·4+.

To find the side of a square equal in area to any given surface.

RULE. Extract the square root of the given area for the side of the square sought.

(21) If the content of a given circle be 160, what is the side of the square equal? *Ans.* 12·64911.

(22) If the area of a circle is 750, what is the side of the square equal? *Ans.* 27·38612.

The area of a circle given, to find the diameter.

RULE. As 355 : 452, or, as 1 : 1·273239 : : the area : the square of the diameter:—or, multiply the square root of the area by 1·12837, and the product will be the diameter.

(23) What length of cord must be tied to a cow's tail, the other end fixed in the ground, to enable her to eat just an acre of grass, and no more; supposing the cow and tail to measure 5½ yards? *Ans.* 33·75 *yards.*

The area of a circle given, to find the circumference.

RULE. As 113 : 1420, or, as 1 : 12·56637 : : the area : the square of the circumference:—or, multiply the square root of the area by 3·5449, and the product will be the circumference.

(24) When the area is 12, what is the circumference?
Ans. 12·279.

(25) When the area is 160, what is the circumference?
Ans. 44·839.

Two sides of a right-angled triangle being given, to find the third side.

Case 1. *The base and perpendicular being given, to find the hypotenuse.*

RULE. The square root of the sum of the squares of the base and perpendicular, is the length of the hypotenuse.

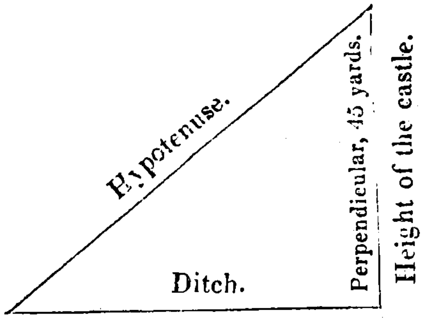
Case 2. *The hypotenuse and perpendicular being given, to find the base.*

RULE. The square root of the difference of the squares of the hypotenuse and perpendicular, is the length of the base.

Case 3. *The base and hypotenuse being given, to find the perpendicular.*

RULE. The square root of the difference of the squares of the hypotenuse and base, is the height of the perpendicular.

(26) The top of a castle from the ground is 45 yards high, and it is surrounded with a ditch 60 yards broad: what length must a ladder be to reach from the outside of the ditch to the top of the castle? *Ans.* 75 *yards.*



(27) The wall of a town is 25 feet high, and is surrounded by a moat of 30 feet in breadth: required the length of a ladder that will reach from the outside of the moat to the top of the wall. *Ans.* 39·05 *feet.*

N.B. These two questions may be varied for examples to the second and third cases.

(28) In an army consisting of 331776 men, how many must be in rank and file to form a solid square? *Ans.* 576.

(29) A certain square pavement contains 48841 equal square stones. How many are contained in one of the sides? *Ans.* 221.

CUBE ROOT.

RULE 1. Point every *third figure* of the given number, beginning at the units' place; find the greatest cube in the first period, and subtract it therefrom; put the root in the quotient, and bring down the figures in the next period to the remainder, for a *Resolvend.*

2. Multiply the square of the root found by 300, for a *Divisor,* and annex to the root the number of times which that is contained in the *Resolvend.*

3. Add 30 times the preceding figure (or figures) multiplied by the last, and the square of the last, to the divisor; and multiply the sum by the last, for a *Subtrahend:* subtract it from the *Resolvend,* and repeat the process as far as necessary.*

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* The subjoined Theorems (deduced from Problem 91, page 266, *Emerson's Algebra*) are very convenient approximations for the Cube Root.

$$\tfrac{1}{2}r + \tfrac{1}{2}\sqrt{\left(\frac{4n - r^3}{3r}\right)} = \tfrac{1}{2}r + \sqrt{\left(\frac{n}{3r} - \tfrac{1}{12}r^2\right)} =$$

NOTE. As the units must always be pointed, there will be sometimes only one or two figures in the first period.—The decimals must always consist of so many figures as will constitute *complete periods*, as in the Square Root.—Also, what is observed in Note 2, Square Root, will hold good in this Rule.

ROOTS.	1.	2.	3.	4.	5.	6.	7.	8.	9.
CUBES.	1.	8.	27.	64.	125.	216.	343.	512.	729.

(1) What is the cube root of 99252847?*
(2) What is the cube root of 389017? *Ans.* 73.
(3) What is the cube root of 5735339? *Ans.* 179.
(4) What is the cube root of 32461759? *Ans.* 319.
(5) What is the cube root of 84604519? *Ans.* 439.
(6) What is the cube root of 27054036008? *Ans.* 3002.
(7) What is the cube root of 673373097125? *Ans.* 8765.
(8) What is the cube root of 12·977875? *Ans.* 2·35.
(9) What is the cube root of ·001906624? *Ans.* ·124.

R, the *required root, nearly.* In which n denotes the given number; and r, an assumed root found by trial.

The second *formula*, which is more convenient than the other, because it contains no higher power than the *square* of r, may be thus expressed.

Divide the given number by three times the assumed root, and from the quotient subtract $\frac{1}{12}$ of the square of the assumed root: the square root of the remainder, added to half the assumed root, will give the root required. See also the method of extracting ANY ROOT *by approximation.*

```
                          .  .  .
                       * 99252847 ( 463 the root.
               4³ = 64
    4² × 300 = 4800) 35252 resolvend.
                      720 = 4 × 30 × 6
                       36 = 6²
                     4800 divisor.
                     5556
                        6
                    33336 subtrahend.
46² × 300 = 634800) 1916847 resolvend.
                       4140 = 46 × 30 × 3
                          9 = 3²
                     634800
                     638949
                          3
                    1916847 subtrahend.
```

(10) What is the cube root of 36155·02756? *Ans.* 33·06+.
(11) What is the cube root of 33·230979937? *Ans.* 3·215+.
(12) What is the cube root of 15926·972504? *Ans.* 25·16+.

To find the Roots of Fractional numbers.

RULE. When the *terms* of a fraction are *complete powers* extract their roots for the *corresponding terms* of the root.

When they are *surds*, if both terms be multiplied by the *square of the denominator*, an equal fraction will be produced, the denominator of which will be a *cube*. Then divide the root of the numerator by the root of the denominator for the answer.—Or, the fraction may be reduced to a decimal, and its root extracted.

Mixed numbers may be reduced as in the Square Root.

(13) What is the cube root of $\frac{250}{686}$? *Ans.* $\frac{5}{7}$.
(14) What is the cube root of $\frac{324}{1500}$? *Ans.* $\frac{3}{5}$.
(15) What is the cube root of $12\frac{19}{27}$? *Ans.* $2\frac{1}{3}$.
(16) What is the cube root of $31\frac{15}{343}$? *Ans.* $3\frac{1}{7}$.
(17) What is the cube root of $405\frac{28}{125}$? *Ans.* $7\frac{2}{5}$.
(18) What is the cube root of $\frac{4}{7}$? *Ans.* ·8298265+.
(19) What is the cube root of $\frac{5}{9}$? *Ans.* ·8220707+.
(20) What is the cube root of $7\frac{1}{5}$? *Ans.* 1·930978+.
(21) What is the cube root of $9\frac{1}{6}$? *Ans.* 2·092845+.
(22) What is the cube root of $8\frac{5}{7}$? *Ans.* 2·0578352+.

(23) A water cistern in the form of a cube contains 60 cubic feet, 143 inches: what is the length of the side?
Ans. 47 *inches.*

(24) There is an excavation made for a cellar equal in length, breadth, and depth; which required 4913 cubic feet of earth to be dug out. What is the length of the side?
Ans. 17 *feet.*

(35) There is a building of cubic form, which contains 389017 solid feet: what is the superficial content of one of its sides? *Ans.* 5329 *sq. feet.*

Between two numbers given, to find two mean proportionals.

RULE. Divide the greater extreme by the less, and the cube root of the quotient multiplied by the less extreme gives the *less mean;* multiply the said cube root by the less mean, and the product will be the *greater mean proportional.*

(26) What are the two mean proportionals between 6 and 162? *Ans.* 18 *and* 54.

(27) What are the two mean proportionals between 4 and 108? *Ans.* 12 *and* 36.

To find the side of a cube, equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.

RULE. The cube root of the solid content given, is the side of a cube of equal solidity.

(28) If the solid content of a globe is 10648, what is the side of a cube of equal solidity? *Ans.* 22.

The side of a cube being given, to find the side of a cube that shall be double, treble, &c. in quantity to the cube given.

RULE. Cube the side given, and multiply it by 2, 3, &c. the cube root of the product will be the side sought.

(29) There is a cubical vessel, whose side is 12 inches, and it is required to find the side of another vessel, that will contain three times as much. *Ans.* 17·30699 *inches.*

BIQUADRATE ROOT.

RULE. Extract the square root of the given number, and then the square root of that square root; which will be the biquadrate root required.

(1) What is the biquadrate root of 531441? *Ans.* 27.
(2) What is the biquadrate root of 33362176? *Ans.* 76.
(3) What is the biquadrate root of 5719140625? *Ans.* 275.

A GENERAL RULE FOR EXTRACTING THE ROOTS OF ALL POWERS.

1. PREPARE the given number, by pointing it into *periods* of *two figures* each for the square root, *three* for the cube root, &c.

2. Find the first figure of the root, and subtract its power from the first period.

3. Bring down the first figure in the next period to the remainder, and call that the *dividend.*

4. Involve the root to the next inferior power to the given one, and multiply it by the index of the given power, for a *divisor.*

5. Find a quotient figure by common division, and annex it to the root; then involve the whole root to the given power for a *subtrahend,* which subtract from the first *two periods.*

6. To the remainder bring down the first figure of the next period for a new *dividend;* find a new *divisor* and a new *subtrahend* as before; subtract from *three periods,* and proceed thus to the end.

Otherwise. To find ANY ROOT by approximation.

RULE. Let g denote the given number or power; n, the index of the power; a, an assumed power nearly equal to g; r, its root, and R, the required root.

Then, as $\frac{(n+1)a+(n-1)g}{2} : a \backsim g :: r : R \backsim r$;*

which *difference* or *correctional number*, being added or subtracted (as required) will give R: and by repeating the process, any degree of accuracy may be obtained.

(1) What is the square root of 141376?†

(2) What is the cube root of 53157376? *Ans.* 376.

(3) What is the fourth root of 19987173376? *Ans.* 376.

(4) Required the fifth root of 2508·4746156142 40625. *Ans.* 4·785.

(5) Required the sixth root of 3·1416. *Ans.* 1·210201 +

SINGLE POSITION

Is the method of using *one supposed number*, and working with it as the true one, to find the *real number* required.‡

RULE. As the result from the supposition, is to the true result; so is the supposed number, to the true one required.

PROOF. Add the several parts together, according to the conditions of the question.

(1) A schoolmaster being asked how many scholars he had, said, "If I had as many, half as many, and one quarter as many more, I should have 88." How many had he?§

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* This for the Cube Root will be, As $2a+g : a \backsim g :: r : R \backsim r$.

† $\dot{1}4\dot{1}37\dot{6}$(376 the root.
9
$3\times2 =$ 6)51 dividend.
$37^2 =$ 1369 subtrahend.
$37\times2 =$ 74)447 dividend.
$376^2 =$ 141376 subtrahend.

‡ Questions belonging to this Rule have the results proportional to their suppositions: the conditions requiring the number sought to be increased by the addition of itself, or of some known multiple or part thereof; or to be diminished by the subtraction of such part.

§ Suppose he had 40.—Then $40+40+20+10=110$.

And, as $110 : 88 :: 40 : \frac{88\times4|0}{11|0} = \frac{352}{11} = 32$ *Ans.*

(2) A person who had a certain number of antique coins, said, "If the third, fourth, and sixth parts of the number were added together, they would make 54." How many had he?
Ans. 72.

(3) A chaise, a horse, and harness, cost £60; the horse being double the price of the harness, and the chaise double the price of the horse and harness. What was given for each?
Ans. Horse, £13..6..8; *harness,* £6..13..4; *chaise,* £40.

(4) What sum of money will amount to £300, in ten years, at £6 per cent per annum, simple interest?
Ans. £187..10.

(5) A, B, and C, dividing a quantity of goods, which cost £120, mutually agreed that B should have a third part more than A, and C a fourth part more than B. What must each man pay? *Ans. A,* £30; *B,* £40; *C,* £50.

(6) A gentleman bought a house, with a garden, and a horse in the stable, for £500. He paid four times the price of the horse for the garden, and five times the price of the garden for the horse. What were their respective prices?
Ans. Horse, £20; *garden,* £80; *house,* £400.

DOUBLE POSITION

REQUIRES the use of *two supposed numbers* to find the *true one required.**

RULE. Work with the two supposed numbers, and mark the *errors* in the results with + or —, according as they *exceed* or *fall short* of the *true* result: then place the *errors* against their respective *positions,* and multiply them *cross-wise.*

If the errors be of *like* kinds, i.e. *both greater,* or *both less* than the given number, take their difference for a divisor, and the difference of the products for a dividend. But if *unlike,* take their sum for a divisor, and the sum of their products for a dividend: the quotient will be the answer.†

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* Questions belong to this Rule which require the addition or subtraction of a number, &c. which is not any known part of the number required. The results are, therefore, not proportional to their suppositions.

† The following Rule will, in some cases, be found more eligible:
Multiply the *difference* of the supposed numbers by the *less error;* and divide the product by the *difference* of the errors when they are of

(1) A, B, and C, would divide £200 among them, so that B may have £6 more than A, and C £8 more than B. How much must each have?*

(2) A man had two silver cups of unequal weight, having one cover to both of 5 ounces. Now if the cover is put on the less cup, it will double the weight of the greater; and put on the greater cup, it will be thrice as heavy as the less. What is the weight of each?

Ans. 3 *ounces the less, and* 4 *the greater.*

(3) Three persons conversing about their ages; says K, "My age is equal to that of H, and $\frac{1}{4}$ of L's;" and L says, "I am as old as both of you together." Required the ages of K and L; H's being 30. *Ans. K,* 50; *and L,* 80.

(4) D, E, and F, playing at cards, staked 324 crowns; but, disputing about the tricks, each man seized as many as he could: E got 15 more than D; and F got a fifth part of both their sums added together. How many did each person get?

Ans. D, $127\frac{1}{2}$; *E,* $142\frac{1}{2}$; *and F,* 54.

(5) A gentleman meeting with some ladies, said to them, "Good morning to you, ten fair maids."—"Sir, you mistake,"

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like kinds, or by their *sum*, when unlike: the quotient will be a *correctional* number; which being *added* to the *nearest supposition* when *defective*, or *subtracted* from it when *excessive*, will give the number required.

	£		£
* 1st. Suppose A's share =	40	2nd. Suppose A's share =	70
then B's =	46	then B's =	76
and C's =	54	and C's =	84
Sum	140	Sum	230

Therefore the error is — 60, or 60 too little.

Here the error is + 30, or 30 too much.

	sup.		*err.*
	40	×	60
	70		30
	4200		1200
divisor.	1200		

60 + 30 = 9|0) 5400|0 dividend.

£60 = A's share.

	£
A	60
B	66
C	74
	200 Proof.

Or, by the Rule in the Note.

$$\frac{70-40\times 30}{60+30} = \frac{30\times 30}{90} = 10 \text{ the correctional number.}$$

Then 70 — 10 = 60 = A's share, as before.

answered one of them, "We are not ten: but if we were three times as many as we are, we should be as many above ten as we are now under." How many were they? *Ans.* 5.

ARITHMETICAL PROGRESSION.

An *Arithmetical Progression* is a series of numbers increasing or decreasing uniformly by a *continued equal difference.* Thus,

$\left.\begin{matrix} 1, & 2, & 3, & 4, & 5, \&c. \\ 2, & 5, & 8, & 11, & 14, \&c. \end{matrix}\right\}$ are *increasing* Arithmetical Series.

$\left.\begin{matrix} 9, & 8, & 7, & 6, & 5, \&c. \\ 16, & 12, & 8, & 4, & 0, \&c. \end{matrix}\right\}$ are *decreasing* Arithmetical Series.

Observe, that the *terms* of the first series are formed by adding successively the common difference 1, and the second by the common difference 3. The terms of the third and the fourth diminish continually by the subtraction of 1 and 4 respectively.

In an odd number of terms, the double of the *mean* (or middle term) is equal to the *sum* of the *extremes,* or of any *two terms equidistant* from the mean. Thus, in 1, 2, 3, 4, 5, the double of $3 = 1 + 5 = 2 + 4 = 6$.

In an even number of terms, the *sum* of the *two means* is equal to the *sum* of the *extremes,* or of any *two equidistant* terms. Thus, in 2, 4, 6, 8, 10, 12; $6 + 8 = 2 + 12 = 4 + 10 = 14$.

To give *Theorems* or *Rules* for the solution of the various cases, the terms are represented by symbols, or letters.

Thus, let a denote the *less extreme,* or *least term;*
z the *greater extreme,* or *greatest term;*
d the common *difference;*
n the *number* of terms; and
s the *sum* of all the terms.

Any three being given, the others may be found.

Note. The *twenty cases* in this Rule may be resolved by the following *Theorems.*

$$a = z-(n-1)d = \frac{2s}{n} - z = \frac{s}{n} - \tfrac{1}{2}d(n-1) = \tfrac{1}{2}d + \sqrt{(\tfrac{1}{2}d+z)^2 - 2ds}.$$

$$z = a+(n-1)d = \frac{2s}{n} - a = \frac{s}{n} + \tfrac{1}{2}d(n-1) = \sqrt{(\tfrac{1}{2}d-a)^2 + 2ds} - \tfrac{1}{2}d.$$

$$d = \frac{z-a}{n-1} = \frac{s-an}{n-1} \cdot \frac{2}{n} = \frac{nz-s}{n-1} \cdot \frac{2}{n} = \frac{(z+a).(z-a)}{2s-a-z}.$$

$$n=\frac{z-a}{d}+1=\frac{2s}{a+z}=\frac{\frac{1}{2}d-a+\sqrt{(\frac{1}{2}d-a)^2+2ds}}{d}=$$

$$\frac{\frac{1}{2}d+z-\sqrt{(\frac{1}{2}d+z)^2-2ds}}{d}.$$

$$s=\tfrac{1}{2}n(a+z)=\frac{a+z}{2}\cdot\frac{z-a+d}{d}=\tfrac{1}{2}n.\overline{2a+d(n-1)}=\tfrac{1}{2}n.\overline{2z-d(n-1)}.$$

Moreover, when the least term $a=$ *nothing*, the *Theorems* become, $z=d(n-1)$, and $s=\frac{1}{2}nz$.

Case 1. *The two extremes, and the number of terms being given, to find the sum.*

RULE. Multiply the sum of the extremes by the number of terms, and half the product will be the answer.*

(1) How many strokes does the hammer of a clock strike in 12 hours?†

(2) A man bought 17 yards of cloth, and gave for the first yard 2*s.* and for the last 10*s.* What was the price of the 17 yards? *Ans.* £5..2.

(3) If 100 eggs be placed in a right line, exactly a yard from each other, and the first a yard from a basket, how far must a person travel to gather them all up singly, and return with every egg to put it into the basket?

Ans. 5 *miles,* 1300 *yards.*

Case 2. *The same three terms given, to find the common difference.*

RULE. Divide the difference of the extremes by the number of terms less 1; and the quotient will be the answer.

(4) A man had eight sons, whose ages were in arithmetical progression; the youngest being 4 years old, and the eldest 32. What was the common difference of their ages?‡

(5) A man travelling from London to a certain place, went 3 miles the first day, and increased every day by an equal excess, making the twelfth day's journey 58 miles.

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* The learner should find each of these cases among the preceding *Theorems.* Thus, the present Rule will be found designated by $s=\frac{1}{2}n(a+z)$, &c.

† $\overline{12+1}\times 6=13\times 6=78$. *Ans.*

‡ $\overline{32-4}\div\overline{8-1}=28\div 7=4$ *years. Ans.*

What was the daily increase, and how far did he travel in 12 days?

Ans. *5 miles daily increase, the whole distance* 366 *miles.*

Case 3. *The two extremes and the common difference being given, to find the number of terms.*

RULE. Divide the difference of the extremes by the common difference, and the quotient increased by *unity* is the number sought.

(6) A person travelling into the country, went 3 miles the first day, and increased every day 5 miles, till at last he went 58 miles in one day. How many days did he travel? *Ans.* 12.

(7) A man being asked how many sons he had, said, that the youngest was 4 years old, and the eldest 32; and that his family had increased one in every 4 years. How many had he? *Ans.* 8.

Case 4. *The greater extreme, the number of terms, and the common difference being given, to find the less extreme.*

RULE. Multiply the common difference by the number of terms less 1; subtract the product from the greater extreme, and the difference will be the less extreme.

(8) A man went from London to a certain town in the country in 10 days; every day's journey exceeding the former by 4 miles, and the last being 46 miles. What was the first?

Ans. 10 *miles.*

(9) A man took out of his pocket, at 8 several times, so many different numbers of shillings, every one exceeding the former by 6, the last being 46. What was the first? *Ans.* 4.

Case 5. *The common difference, the number of terms, and the sum being given, to find the less extreme.*

RULE. Divide the sum by the number of terms: from the quotient subtract half the product of the common difference into the number of terms less 1; and the remainder will be the less extreme.

(10) A man is to receive £360, at 12 several payments, each payment to exceed the former by £4, and is willing to bestow the first payment on any one that can tell him what it is. What will that person have for his pains? *Ans.* £8.

Case 6. *The less extreme, the common difference, and the number of terms being given, to find the greater extreme.*

RULE. Multiply the number of terms less 1 by the common difference; to this product add the less extreme, and the sum will be the greater extreme.

(11) What is the last number of an arithmetical progression, beginning at 6, and continuing by the increase of 8 to 20 places? *Ans.* 158.

GEOMETRICAL PROGRESSION.

A *Geometrical Progression* is a series of numbers increasing or decreasing uniformly by a *common ratio;* that is, by the continual multiplication or division of some particular number. Thus,

1, 2, 4, 8, 16, 32, &c. is an *increasing Geometrical Series,* in which the terms are formed by multiplying successively by the ratio 2.

81, 27, 9, 3, 1, $\frac{1}{3}$, &c. is a *decreasing Geometrical Series,* in which the terms are formed by dividing successively by the ratio 3. It is evident that either of these may be continued without end.

In an odd number of terms, the *square* of the *mean* is equal to the *product* of the *extremes,* or of any *two terms equidistant* from the *mean.* Thus, in 3, 6, 12, 24, 48; $12 \times 12 = 3 \times 48 = 6 + 24 = 144$.

In an even number of terms, the *product* of the *two means* is equal to the *product* of the *extremes,* or of any *two equidistant terms.* Thus, in 32, 16, 8, 4, 2, 1; $8 \times 4 = 32 \times 1 = 16 \times 2 = 32$.

To give *Theorems,* or Rules expressed in symbols, for the solution of the various cases, as in Arithmetical Progression, let a denote the *less extreme;*

z the *greater extreme;*
r the *ratio;*
n the *number* of terms; and
s the *sum* of all the terms.

Any three being given, the others may be found.

NOTE. The twenty *Theorems* following, solve all the possible cases in Geometrical Progression.

Theor. I. $r^{n-1} = \frac{z}{a}$*; or, $\frac{\text{Log. } z - \text{log. } a}{\text{Log. } r} + 1 = n$.

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* In this case, if the *quotient* of $\frac{z}{a}$ be divided *continually* by r, till nothing remains; the *number of divisions* + 1 will give n.

II. $\frac{rz-a}{r-1}=s$; or, $z+\frac{z-a}{r-1}=s$.

III. $\left(\frac{z}{a}\right)^{\frac{1}{n-1}}=r$; or, $\overline{\text{Log. } z-\text{log. } a}\div\overline{n-1}=\text{Log. } r$.

IV. $z+\frac{z-a}{\left(\frac{z}{a}\right)^{\frac{1}{n-1}}-1}=s$. V. $\frac{s-a}{s-z}=r$.

VI. $\left(\frac{s-a}{s-z}\right)^{n-1}=\frac{z}{a}$; from which n may be found as in Theorem I.

or, $\frac{\text{Log. } z-\text{log. } a}{\text{Log. } (s-a)-\text{log. } (s-z)}+1=n$. VII. $ar^{n-1}=z$.

VIII. $\frac{a(r^n-1)}{r-1}=s$. IX. $\frac{(r-1)s+a}{r}=s-\frac{s-a}{r}=z$.

X. $r^n=\frac{(r-1)s}{a}+1$; or, $\frac{\text{Log. } \overline{(r-1)s+a}-\text{log. } a}{\text{Log. } r}=n$.

XI. $z\times\overline{s-z}|^{n-1}=a\times\overline{s-a}|^{n-1}$; whence z may be found by Double Position. XII. $\frac{sr}{a}-r^n=\frac{s-a}{a}$; whence r may be found in the same manner.

XIII. $\frac{z}{r^{n-1}}=a$. XIV. $\frac{z(r^n-1)}{r^n-r^{n-1}}=s$.

XV. $rz-(r-1)s=a$. XVI. $r^{n-1}=\frac{z}{rz-(r-1)s}$; or,

$$\frac{\text{Log. } z-\text{log. } \overline{rz-(r-1)s}}{\text{Log. } r}+1=n.$$

XVII. $a\times\overline{s-a}|^{n-1}=z\times\overline{s-z}|^{n-1}$. XVIII. $\frac{r^{n-1}s}{s-z}-r^n=\frac{z}{s-z}$.

From the two preceding, a and r are to be found by Double Position.

XIX. $\frac{(r-1)s}{r^n-1}=a$. XX. $\frac{r^n-r^{n-1}}{r^n-1}\times s=z$.

In a Geometical series decreasing *ad infinitum*, a becomes $=0$, and n is infinite, or greater than any assignable number.

Hence the three following will exhibit all the various cases of such a series.

$$\text{I. } s=\frac{rz}{r-1}=z+\frac{z}{r-1}=\frac{z^2}{z-\frac{z}{r}}.\quad \text{II. } z=\frac{s(r-1)}{r}.\quad \text{III. } r=\frac{s}{s-z}.$$

NOTE. In these cases, when the *ratio* is *a proper fraction*, r must be taken = the *reciprocal* of the fraction. Thus, when the ratio is $\frac{3}{4}$, $r=\frac{4}{3}$.

Case 1. *The less extreme, the ratio, and the number of terms, being given, to find the greater extreme (or any remote term) without producing all the intermediate terms.*

RULE. 1. When the last term is *equal* to the *ratio*. Write down a few of the leading terms of the series, and over them the arithmetical series, 1, 2, 3, 4, &c. as *indices* or *exponents*. Find which of the *indices added together* will give the *index* of the *term sought;* and the continual product of the terms standing under those indices, will be the term sought.

2. When the least term is *not equal* to the *ratio*. Write down the leading terms as before; and over them the *indices*, 0, 1, 2, 3, 4, &c. Examine which of *these added together* will give an *index one less* than the *number* of the *term sought;* multiply the terms under such indices into each other, dividing the product of every two by the first term, and the last quotient will be the term required.*

Otherwise. By Theorem VII.

(1) A man agrees, for 12 peaches, to pay only the price of the last; reckoning a farthing for the first, and a halfpenny for the second, &c. doubling the price to the last. What must he give for them?†

(2) A farmer who went to a fair to buy some oxen, met with a drover who had 23; for which he asked him £16 a piece.—After a great deal of dodging between the parties, it was finally agreed that the farmer should pay the price of the last ox only, reckoning a farthing for the first, and doubling it to the last. How much would they cost him?

Ans. £4369..1..4.

---

* If the least term is *unity*, there will (of course) be no division.

† Here $a=1$, $r=2$, and $n=12$; a and r being *unequal*.

Indices,	0, 1, 2, 3, 4.	Then $4+4+3=11=n-1$.
Geom. series,	1, 2, 4, 8, 16.	

Hence $16\times16\times8=2048$ *qrs.* $=z$. And 2048 *qrs.* = £2..2..8. *Ans.*

(3) A sum of money is to be divided among 8 persons, the first to have £20, the second £60, and so on in triple proportion. What will the last have?* *Ans.* £43740.

(4) A gentleman dying left nine sons, to whom and to his executors he bequeathed his estate in the manner following: to his executors £50, to his youngest son twice as much as to the executors, to the next double that sum, and so on to the eldest. What was his fortune? *Ans.* £25600.

Case 2. *The less extreme, the ratio, and the number of terms, being given, to find the sum of all the terms.*

RULE. Find the greater extreme as before, and divide the *difference* between the *extremes* by the *ratio less* 1: to the quotient add the *greater extreme*, for the *sum* required. This is Theorem II. Or, by Theorem VIII; without finding z.

(5) A young man conversant with numbers, agreed with a gentleman to serve him twelve months, provided he would give him a farthing for his first month's service, a penny for the second, and 4*d.* for the third, &c. What did his wages amount to?† *Ans.* £5825..8..5$\frac{1}{4}$.

(6) A man bought a horse, and by agreement was to give a farthing for the first nail, three for the second, &c. Now supposing there were 8 nails in each of his four shoes, what was the price of the horse? *Ans.* £965114681693..13..4.

(7) A person whose daughter was married on new-year's day, gave her husband 1*s.* towards her portion; promising to double the sum on the first day of every month during the year. What was her portion? *Ans.* £204..15.

(8) A laceman, well versed in numbers, agreed with a gentleman to sell him 22 yards of rich brocaded gold lace, for 2 pins the first yard, 6 pins the second, &c. in triple proportion. What was the price of the lace, valuing the pins

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* Indices 0, 1, 2, 3. Geom. series 20, 60, 180, 540. } Then $3+3+1=7=n-1$.

Hence $\dfrac{540\times540\times60}{20\times20}=540\times27\times3=43740=z.$

Otherwise, $ar^{n-1}=20\times3^7=43740=z.$

† Here $a=1$, $r=4$, and $z=12$. Therefore $s=\dfrac{4^{12}-1}{4-1}=\dfrac{16777215}{3}$ 5592405 *qrs.*

at 100 for a farthing? Also, what did the laceman gain, supposing the lace to have cost him £7 per yard?

Ans. The lace sold for £326886..0..9. *Gain* £326732..0..9.

Case 3. *The first term and the ratio being given, to find the sum of an infinite decreasing series.*

RULE. Divide the square of the first term by the difference between the first and second.*

(9) What is the sum of the circulating decimal ·9′, or the series $\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}$, &c. continued *ad infinitum*? *Ans.* 1.

(10) Required the sum of the infinite series $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, &c.; also of the series $\frac{1}{3}+\frac{1}{9}+\frac{1}{27}$, &c. *Ans.* 1, and $\frac{1}{2}$.

(11) Suppose a body to be put in motion by a force which gives it a velocity of ten miles the first minute (or any given space of time), 9 miles in the second equal space, and so on in the ratio of $\frac{9}{10}$; how many miles would it pass over, if continued in motion for ever? *Ans.* 100 *miles.*

SIMPLE INTEREST, BY DECIMALS.

To give *Theorems* for the solution of the different cases in *Simple Interest*, let p denote the *principal*, r the *ratio*, t the *time* (in years), i the *interest*, and a the *amount*.

NOTE. The Ratio is the interest of £1 for one year, at the rate per cent proposed, and may be found by Proportion: thus, at £5 per cent per annum, say,

As £100 : £5 : : £1 : £·05, the *ratio*.

Therefore the ratio at

3 per cent is	·03	$4\frac{1}{2}$ per cent is	·045
$3\frac{1}{2}$	·035	5	·05
4	·04	$5\frac{1}{2}$	·055, &c.

Case 1. *When the principal, rate per cent, and time are given, to find the interest.*

RULE. Multiply the principal, ratio, and time together, and the product will be the interest required.

That is, $prt = i$.

(1) What is the interest of £945..10 for 3 years, at £5 per cent per annum?†

* See the third *formula*, Theorem I. for infinite series, page 130.

† $prt =$ £945·5 × ·05 × 3 = £141·825. = £141..16..6. *Ans.*

(2) What is the interest of £547..14, at £4 per cent per annum, for 6 years? *Ans.* £131..8..11..2·08 *qrs.*

(3) What is the interest of £796..15, at £4½ per cent per annum, for 5 years? *Ans.* £179..5..4½.

(4) What is the interest of £397..9..5, for 2½ years, at £3½ per cent per annum? *Ans.* £34..15..6..3·55 *qrs.*

(5) What is the interest of £554..17..6, for 3 years, 8 months, at 4½ per cent per annum? *Ans.* £91..11..1·05*d.*

(6) What is the interest of £236..18..8, for 3 years, 8 months, at £5½ per cent per annum?

Ans. £47..15..7..2·293 *qrs.*

When the interest is for any number of days only.

RULE. Multiply the interest of £1 for a day, at the given rate, by the principal and the number of days; and the product will be the answer.

The Interest of £1 *for one day,*

At £2 per cent	=£·00005479452*	at £4	=·00010958904
2½	= ·00006849315	4½	=·00012328767
3	= ·00008219178	5	=·00013698630
3½	= ·00009589041	5½	=·00015068493, &c.

(7) What is the interest of £240, for 120 days, at £4 per cent per annum?†

(8) What is the interest of £364..18, for 154 days, at £5 per cent per annum? *Ans.* £7..13..11¼.

(9) What is the interest of £725..15, for 74 days, at £4 per cent per annum? *Ans.* £5..17..8½.

(10) What is the interest of £100, from the 1st of June, 1826, to the 9th of March following, at £5 per cent per annum? *Ans.* £3..16..11¼.

Case 2. When p, r, and t are given, to find a.

RULE. $prt + p = a$.

(11) What will £279..12 amount to in 7 years, at £4½ per cent per annum?‡

(12) What will £320..17 amount to in 5 years, at £3½ per cent per annum? *Ans.* £376..19..11..2·8 *qrs.*

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* The table is formed thus:
As 365 days : £·02 : : 1 day : £·00005479452, &c.

† ·00010958904 × 240 × 120 = £3·156164352 = £3..3..1¼. *Ans.*

‡ 279·6 × ·045 × 7 + 279·6 = £367·674 = £367..13..5..3·04 *qrs.*

(13) What will £926..12 amount to in $5\frac{1}{2}$ years, at £4 per cent per annum? *Ans.* £1130..9..0..1·92 *qrs.*

(14) What will £273..18 amount to in 4 years, 175 days, at £3 per cent per annum? *Ans.* £310..14..1..3·3512 *qrs.*

Case 3. When a, r, and t are given, to find p.

RULE. $\frac{a}{rt+1} = p$.

(15) What principal, being put to interest, will amount to £367..13..5..3·04 *qrs.* in 7 years, at £$4\frac{1}{2}$ per cent per annum?*

(16) What principal will amount to £376..19..11..2·8 *qrs.* in 5 years, at £$3\frac{1}{2}$ per cent per annum? *Ans.* £320..17.

(17) What principal will amount to £1130..9..0..1·92 *qrs.* in $5\frac{1}{2}$ years, at £4 per cent per annum? *Ans.* £926..12.

(18) What principal will amount to £310..14..1..3·3512 *qrs.* in 4 years, 175 days, at £3 per cent per annum?

Ans. £273..18.

Case 4. When a, p, and t are given, to find r.

RULE. $\frac{a-p}{pt} = r$.

(19) At what rate per cent per annum will £279..12 amount to £367..13..5..3·04 *qrs.* in 7 years?†

(20) At what rate per cent per annum will £320..17 amount to £376..19..11..2·8 *qrs.* in 5 years?

Ans. £$3\frac{1}{2}$ *per cent.*

(21) At what rate per cent per annum will £926..12 amount to £1130..9..0..1·92 *qrs.* in $5\frac{1}{2}$ years?

Ans. £4 *per cent.*

(22) At what rate per cent per annum will £273..18 amount to £310..14..1..3·3512 *qrs.* in 4 years, 175 days?

Ans. £3 *per cent.*

Case 5. When a, p, and r are given, to find t.

RULE. $\frac{a-p}{pr} = t$.

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* $\cdot045 \times 7 + 1 = 1\cdot315$; then $367\cdot674 \div 1\cdot315 = £279\cdot6 =$ £279..12. *Ans.*

† $\frac{367\cdot674 - 279\cdot6}{279\cdot6 \times 7} = \frac{88\cdot074}{1957\cdot2} = \cdot045$, or £$4\frac{1}{2}$ per cent. *Ans.*

(23) In what time will £279..12 amount to £367..13..5.. 3·04 *qrs.* at £4½ per cent per annum?*

(24) In what time will £320..17 amount to £376..19..11.. 2·8 *qrs.* at £3½ per cent per annum? *Ans.* 5 *years.*

(25) In what time will £926..12 amount to £1130..9..0.. 1·92 *qrs.* at £4 per cent per annum? *Ans.* 5½ *years.*

(26) In what time will £273..18 amount to £310..14..1.. 3·3512 *qrs.* at £3 per cent per annum?

Ans. 4 *years,* 175 *days.*

ANNUITIES.

An annuity is a yearly income or rent. *Perpetual Annuities* are those which are to continue for ever; *Terminable Annuities* are to cease within a limited time; and *Life Annuities* are to continue during the term of life of one or more persons.

The Amount of Annuities in Arrears.

Let u denote the *annuity,* r, t, and a, as before.

Case 1. Given, u, r, and t, to find a.

Rule. $\left(\frac{\overline{t-1}\,.\,r}{2}+1\right)\times tu=a.$

(27) If a salary of £150 be forborne 5 years, at £5 per cent per annum, what will be the amount?†

(28) If £250 yearly pension be forborne 7 years, what will it amount to at £4 per cent per annum? *Ans.* £1960.

(29) There is a house let upon lease for 5½ years, at £60 per annum, what will be the accumulated rent, allowing interest at £4½ per cent per annum? *Ans.* £363..8..3.

(30) Suppose an annual pension of £28 remain unpaid for 8 years, what would it amount to at £5 per cent per annum? *Ans.* £263..4.

Note. When the annuity is payable half-yearly or quarterly, u will denote a single payment; r, the interest of £1 for that interval of time; and t, the number of payments.

(31) If a salary of £150, payable every half year, remain unpaid for 5 years, what will it amount to in that time, allowing interest at £5 per cent per annum? *Ans.* £834..7..6.

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* 367·674 — 279·6 = 88·074, and 279·6 × ·045 = 12·582; then 88·074 ÷ 12·582 = 7 years, *Ans.*

† $\left(\frac{4\times\cdot05}{2}+1\right)\times5\times150=(\overline{2\times\cdot05}+1)\times5\times150=1\cdot1\times5\times150=$ £825. *Ans.*

(32) If a salary of £150, payable every quarter, were left unpaid for 5 years, what would it amount to in that time, at £5 per cent per annum? *Ans.* £839..1..3.

NOTE. It may be observed, by comparing the results of the 27th, 31st, and 32d examples, that half-yearly payments are more advantageous than yearly, and quarterly more than half-yearly.

Case 2. When a, r, and t are given, to find u.

RULE. $$\frac{2\,a}{(r.(t-1)+2)\times t}=u.$$

(33) If a salary amounted to £825 in 5 years, at £5 per cent per annum, what was the salary?*

(34) If a house has been let upon lease for $5\frac{1}{2}$ years, and the amount in that time is £363..8..3, at £$4\frac{1}{2}$ per cent per annum, what is the yearly rent? *Ans.* £60.

(35) If a pension amounted to £1960 in 7 years, at £4 per cent per annum, what was the pension? *Ans.* £250.

(36) Suppose the amount of a pension was £263..4 in 8 years, at £5 per cent per annum, what was the pension? *Ans.* £28.

(37) If the amount of a salary, payable half-yearly, be £834..7..6 in 5 years, at £5 per cent per annum, what is the salary per year?† *Ans.* £150.

(38) If the amount of an annuity, payable quarterly, was £839..1..3 in 5 years, at £5 per cent per annum, what was the annuity? *Ans.* £150.

Case 3. When u, a, and t are given, to find r.

RULE. $$\frac{(a-ut)\times 2}{(t-1)\times ut}=r.$$

(39) If a salary of £150 per annum, amounts to £825 in 5 years, what is the rate per cent?‡

(40) If a house has been let upon lease for $5\frac{1}{2}$ years, at £60 per annum, at what rate per cent would it amount to £363..8..3? *Ans.* £$4\frac{1}{2}$ *per cent.*

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* $\frac{825\times 2}{(\cdot 05\times 4+2)\times 5}=\frac{1650}{2\cdot 2\times 5}=\frac{1650}{11}=$ £150. *Ans.*

† See Note, p. 135.

‡ $\frac{(825-\overline{150\times 5})\times 2}{4\times 150\times 5}=\frac{825-750}{2\times 150\times 5}=\frac{75}{150\times 10}=\frac{1}{20}=\cdot 05=r$: therefore the rate is £5 per cent.

(41) If a pension of £250 per annum amounts to £1960 in 7 years, what is the rate per cent? *Ans.* £4 *per cent.*

(42) Suppose the amount of a yearly pension of £28 be £263..4 in 8 years, what is the rate per cent per annum? *Ans.* £5 *per cent.*

(43) If a salary of £150 per annum, payable half-yearly, amount to £834..7..6 in 5 years, what is the rate per cent?* *Ans.* £5 *per cent.*

(44) If an annuity of £150 per annum, payable quarterly, amount to £839..1..3 in 5 years, what is the rate per cent? *Ans.* £5 *per cent.*

Case 4. When u, a, and r are given, to find t.

RULE. $$\frac{\sqrt{8r\frac{a}{u}+(2-r)^2}-(2-r)}{2r}=t.$$

Or, put $\frac{1}{r}-\frac{1}{2}=m$; then $\sqrt{\left(\frac{2a}{ur}+m^2\right)}-m=t$.

(45) In what time will a salary of £150 per annum, amount to £825, at £5 per cent?†

(46) If a house is let upon lease at £60 per annum, till it amount to £363..8..3, at £4½ per cent per annum, for what term of years was it let? *Ans.* 5½ *years.*

(47) If a pension of £250 per annum, having been forborne a certain time, amount to £1960 at £4 per cent, how long has been the time of forbearance? *Ans.* 7 *years.*

(48) In what time will a yearly pension of £28 amount to £263..4, at £5 per cent per annum? *Ans.* 8 *years.*

(49) If an annuity of £150 per annum, payable half-yearly, amounted to £834..7..6, at £5 per cent, what time was the payment forborne?* *Ans.* 5 *years.*

(50) If a yearly pension of £150, payable quarterly, amounts to £839..1..3, at £5 per cent per annum, what has been the time of forbearance? *Ans.* 5 *years.*

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* See Note, p. 135.

† $$\frac{\sqrt{8\times\cdot05\times\frac{825}{150}+(2-\cdot05)^2}-(2-\cdot05)}{2\times\cdot05}=$$

$$\frac{\sqrt{\cdot40\times5\cdot5+3\cdot8025}-1\cdot95}{\cdot1}=\frac{2\cdot45-1\cdot95}{\cdot1}=\cdot5\div\cdot1=5 \text{ years.}$$ *Ans.*

A Table by which the Interest of any sum from £1 to £30000, may be easily computed, for any number of days, at any rate per cent.

No.	£	*s.*	*d.*	*qrs.*	*No.*	*s.*	*d.*	*qrs.*	*No.*	*qrs.*
30000	82	3	10	0·11	200	10	11	2·03	1	2·63
20000	54	15	10	2·74	100	5	5	3·01	0·9	2·37
10000	27	7	11	1·37	90	4	11	0·71	0·8	2·10
9000	24	13	1	3·23	80	4	4	2·41	0·7	1·84
8000	21	18	4	1·10	70	3	10	0·11	0·6	1·58
7000	19	3	6	2·96	60	3	3	1·81	0·5	1·32
6000	16	8	9	0·82	50	2	8	3·51	0·4	1·05
5000	13	13	11	2·68	40	2	2	1·21	0·3	0·79
4000	10	19	2	0·55	30	1	7	2·90	0·2	0·53
3000	8	4	4	2·41	20	1	1	0·60	0·1	0·26
2000	5	9	7	0·27	10	0	6	2·30	0·09	0·24
1000	2	14	9	2·14	9	0	5	3·67	0·08	0·21
900	2	9	3	3·12	8	0	5	1·04	0·07	0·18
800	2	3	10	0·11	7	0	4	2·41	0·06	0·16
700	1	18	4	1·10	6	0	3	3·78	0·05	0·13
600	1	12	10	2·08	5	0	3	1·15	0·04	0·11
500	1	7	4	3·07	4	0	2	2·52	0·03	0·08
400	1	1	11	0·05	3	0	1	3·89	0·02	0·05
300	0	16	5	1·04	2	0	1	1·26	0·01	0·03

The above Table is thus constructed: As 365 days : £1 : : 1 day : 2·63 *qrs.* &c. Hence it appears, that the several tabular sums are those which answer to the respective numbers of days, at the rate of £1 per year.

In a similar Table in *Rees's Cyclopædia*, there are no fewer than 16 errors. In Dr. Hutton's Table, Arith. page 84, 12th edition, there is *one* error. The above may be depended upon as accurate.

RULE. Multiply the principal by the rate, both in pounds; and the product by the number of days: divide the last product by 100, collect from the Table the several sums answering to the several parts of the quotient, and the aggregate amount will be the interest required.

Example 1. What is the interest of £370..10 for 220 days, at £4½ per cent per annum?

		£	*s.*	*d.*	*qrs.*
370·5	Against 3000 stands	8	4	4	2·41
4·5	600	1	12	10	2·08
18525	60	0	3	3	1·81
14820	7	0	0	4	2·41
1667·25	0·9	0	0	0	2·37
220	0·05	0	0	0	0·13
3334500	3667·95	10	0	11	3·21 *Ans.*
333450	True to the last decimal.				
3667|95 00					

Example 2. Taking Ex. 8, page 133, we have

364·9
5
1824·5
154
72980
273675
2809|73·0

	£	s.	d.	qrs.	
Against 2000 stands	5	9	7	0·27	
800	2	3	10	0·11	
9	0	0	5	3·67	
0·7	0	0	0	1·84	
0·03	0	0	0	0·08	
2809·73	7	13	11	1·97	*Ans.*

Example 3. What is the interest of £17..10 for 117 days, at £4¾ per cent per annum.

17·5
4·75
875
1225
700
83·125
117
581875
914375
97|25·625

	£	s.	d.	qrs.	
Against 90 stands	0	4	11	0·71	
7	0	0	4	2·41	
0·2......	0	0	0	0·53	
0·05	0	0	0	0·13	
97·25	0	5	3	3·78	*Ans.*

To find the amount of a yearly income or salary, &c. for a number of days.

Multiply the number of pounds per year by the number of days; collect the tabular sums answering to the product, as before, and their aggregate will be the answer.

Example. What will a person receive for 45 days, at the rate of £105 per annum?

105
45
525
420
4725

	£	s.	d.	qrs.	
Against 4000 stands	10	19	2	0·55	
700	1	18	4	1·10	
20	0	1	1	0·60	
5	0	0	3	1·15	
4725	12	18	10	3·40	*Ans.*

NOTE. Any of the preceding examples *of interest for days*, in page 133, or examples 20 and 21, page 70, may be worked by this method.

A Table showing the number of days from any day in the month to the same day in any other month, through the year.

From \ To	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

DISCOUNT.

LET s represent the sum to be discounted, r the *ratio,* t the *time* (in years), and p the *present worth.*

Case I. Given s, r, and t, to find p.

RULE: $\frac{s}{rt+1} = p$.

(1) What is the present worth of £357..10, to be paid 9 months hence, at £5 per cent per annum?*

(2) What is the present worth of £275..10, due 7 months hence, at £5 per cent per annum? *Ans.* £267..13..10·152*d.*

(3) What is the present worth of £875..5..6, due 5 months hence, at £4½ per cent per annum?

Ans. £859..3..3..3·01824 *qrs.*

(4) How much ready money can I receive for a note of £75, due 15 months hence, at £5 per cent per annum?

Ans. £70..11..9·1764*d.*

* $357·5 \div \overline{·05 \times ·75 + 1} = 344·5783$ = £344..11..6..3 168 *qrs. Ans.*

Case 2. When p, r, and t are given, to find s.

RULE. $prt + p = s$.

(5) If the present worth of a sum of money due 9 months hence, allowing £5 per cent per annum, be £344..11..6..3·168 *qrs.* what was the sum due?*

(6) A person owing a certain sum, payable 7 months hence, agrees with the creditor to pay him down £267..13..10·152*d.* allowing £5 per cent per annum, for present payment: what is the debt? *Ans.* £275..10.

(7) A person receives £859..3..3..3·01824 *qrs.* for a sum of money due 5 months hence, allowing the debtor £4½ per cent per annum, for present payment: what was the sum due? *Ans.* £875..5..6.

(8) A person paid £70..11..9·1764*d.* for a debt due 15 months hence, being allowed £5 per cent per annum for the discount. How much was the debt? *Ans.* £75.

Case 3. When s, p, and t are given, to find r.

RULE. $\frac{s-p}{pt} = r$.

(9) At what rate per cent per annum, will £357..10, payable 9 months hence, produce £344..11..6..3·168 *qrs.* for present payment?†

(10) At what rate per cent per annum, will £275..10, payable 7 months hence, be worth £267..13..10·152*d.* for present payment? *Ans.* £5 *per cent.*

(11) At what rate per cent per annum, will £875..5..6, payable 5 months hence, produce the present payment of £859..3..3..3 01824 *qrs.*? *Ans.* £4½ *per cent.*

(12) At what rate per cent per annum, will £75, payable 15 months hence, produce the present payment of £70..11..9·1764*d.* *Ans.* £5 *per cent.*

Case 4. When s, p, and r are given, to find t.

RULE. $\frac{s-p}{pr} = t$.

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* $344{\cdot}5783 \times {\cdot}05 \times {\cdot}75 + 344{\cdot}5783 =$ £357..10. *Ans.*

† $\frac{357{\cdot}5 - 344{\cdot}5783}{344{\cdot}5783 \times {\cdot}75} = {\cdot}05$ or £5 *per cent.* *Ans.*

(13) The present worth of £357..10, due at a certain time to come, is £344..11..6..3·168 *qrs.* at £5 per cent per annum. In what time should the sum have been paid without any discount?*

(14) The present worth of £275..10, due at a certain time to come, is £267..13..10·152*d.* at £5 per cent per annum. In what time should the sum have been paid without discount? *Ans.* 7 *months.*

(15) A person receives £859..3..3..3·01824 *qrs.* for £875..5..6, due at a certain time to come, allowing £4½ per cent per annum discount. In what time should the debt have been discharged without any discount? *Ans.* 5 *months.*

(16) I have received £70..11..9·1764*d.* for a debt of £75, allowing the person £5 per cent per annum for prompt payment. When would the debt have been payable without discount? *Ans.* 15 *months.*

EQUATION OF PAYMENTS.

To find the equated time for the payment of a sum of money due at several times.

RULE. Find the present worth of each payment for its respective time, by Case 1, Discount, page 140, thus:

$$\frac{s}{rt+1} = p.$$

Add all the present worths together; call their sum p', and the sum of all the payments s': then by Case 4, Discount, p. 141.†

$$\frac{s'-p'}{p'r} = e, \text{ the equated time.}$$

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* $\dfrac{357{\cdot}5 - 344{\cdot}5783}{344{\cdot}5783 \times {\cdot}05} = {\cdot}75 = 9$ *months. Ans.*

† The above is Kersey's Rule. It produces a result something less than the *precise truth;* but sufficiently accurate for any purpose of real utility. The only Rule that is *strictly true* for the *equation of two payments at Simple Interest,* is that given by Malcolm; which is founded on the principle, that the *interest* of the money *withheld after it becomes due,* ought to be equal to the *discount* of that which *is paid before it is due.* But Malcolm's Rule, though it has been simplified in expression by Bonnycastle and others, and is capable of further simplification than I have yet seen in print, is at best very operose, and may be regarded, as Mr. Keith justly observes, as a useless curiosity. EDITOR.

(1) D owes E £200, whereof £40 is to be paid at 3 months, £60 at 6 months, and £100 at 9 months. At what time may the whole debt be paid together, discount being allowed at £5 per cent per annum?*

(2) D owes E £800, whereof £200 is to be paid in 3 months, £200 at 4 months, and £400 at 6 months: but they agree to have the whole paid at once, allowing discount at the rate of £5 per cent per annum. The equated time is required. *Ans.* 4 *months,* 22 *days.*

(3) E owes F £1200, which is to be paid as follows: £200 down, £500 at the end of ten months, and the rest at the end of 20 months; but they agree to have only one payment of the whole, discounting at £3 per cent per annum. The equated time is required. *Ans.* 1 *year,* 11 *days.*

COMPOUND INTEREST.†

The same symbols are adopted in this as in Simple Interest, and denote the same things; except that the *ratio* (r), which in Simple Interest denotes the *interest* of £1, signifies in this Rule the *amount* of £1 for a year. It may be thus found by Proportion.

As £100 : £105 : : £1 : £1·05, the *ratio* at £5 per cent per annum. The *ratios* are, therefore,

at 3 per cent	1·03	at $4\frac{1}{2}$ per cent	1·045
$3\frac{1}{2}$	1·035	5	1·05
4	1·04	$5\frac{1}{2}$	1·055, &c.

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* $\frac{40}{1\cdot0125} = 39\cdot5061$; $\frac{60}{1\cdot025} = 58\cdot5365$; $\frac{100}{1\cdot0375} = 96\cdot3855$;

then $200 - \overline{39\cdot5061 + 58\cdot5365 + 96\cdot3855} = 5\cdot5719$;

and $\frac{5\cdot5719}{194\cdot4281 \times \cdot05} = \cdot57315 =$ 6 *months*, 26 *days. Ans.*

† The law of England does not allow the lender to receive *Compound Interest* for his money, when the receipt of the Interest has been forborne. But in the granting or purchasing of Annuities, Leases, &c. either immediate or in reversion, it is customary, and indeed necessary, to compute them on the principles of *Compound Interest;* for otherwise the calculation would involve most egregious injustice and absurdity.

A Table of the Amount of £1, for years.

yrs.	3 *per Cent.*	3½ *per Cent.*	4 *per Cent.*	4½ *per Cent.*	5 *per Cent.*
1	1·0300000	1·0350000	1·0400000	1·0450000	1·0500000
2	1·0609000	1·0712250	1·0816000	1·0920250	1·1025000
3	1·0927270	1·1087179	1·1248640	1·1411661	1·1576250
4	1·1255088	1·1475230	1·1698586	1·1925186	1·2155062
5	1·1592741	1·1876863	1·2166529	1·2461819	1·2762816
6	1·1940523	1·2292553	1·2653190	1·3022601	1·3400956
7	1·2298739	1·2722793	1·3159318	1·3608618	1·4071004
8	1·2667701	1·3168090	1·3685690	1·4221006	1·4774554
9	1·3047732	1·3628974	1·4233118	1·4860951	1·5513282
10	1·3439164	1·4105988	1·4802443	1·5529694	1·6288946
11	1·3842339	1·4599697	1·5394541	1·6228530	1·7103394
12	1·4257609	1·5110687	1·6010322	1·6958814	1·7958563
13	1·4685337	1·5639561	1·6650735	1·7721961	1·8856491
14	1·5125897	1·6186945	1·7316764	1·8519449	1·9799316
15	1·5579674	1·6753488	1·8009435	1·9352824	2·0789282
16	1·6047064	1·7339860	1·8729812	2·0223702	2·1828746
17	1·6528476	1·7946756	1·9479005	2·1133768	2·2920183
18	1·7024331	1·8574892	2·0258165	2·2084788	2·4066192
19	1·7535060	1·9225013	2·1068492	2·3078603	2·5269502
20	1·8061112	1·9897889	2·1911231	2·4117140	2·6532977
21	1·8602946	2·0594315	2·2787681	2·5202412	2·7859626
22	1·9161034	2·1315116	2·3699188	2·6336520	2·9252607
23	1·9735865	2·2061145	2·4647155	2·7521663	3·0715237
24	2·0327941	2·2833285	2·5633042	2·8760138	3·2250999
25	2·0937779	2·3632450	2·6658363	3·0054345	3·3863549
26	2·1565913	2·4459586	2·7724698	3·1406790	3·5556727
27	2·2212890	2·5315671	2·8833686	3·2820096	3·7334563
28	2·2879677	2·6201720	2·9987033	3·4297000	3·9201291
29	2·3565655	2·7118780	3·1186514	3·5840365	4·1161356
30	2·4272625	2·8067937	3·2433975	3·7453181	4·3219424
31	2·5000803	2·9050315	3·3713334	3·9138574	4·5380395
32	2·5750827	3·0067076	3·5080587	4·0899810	4·7649414
33	2·6523352	3·1119423	3·6483811	4·2740302	5·0031885
34	2·7319053	3·2208603	3·7943163	4·4663615	5·2533479
35	2·8138624	3·3335904	3·9460890	4·6673478	5·5160153
36	2·8982783	3·4502661	4·1039325	4·8773785	5·7918161
37	2·9852266	3·5710254	4·2680898	5·0968605	6·0814069
38	3·0747834	3·6960113	4·4388134	5·3262192	6·3854773
39	3·1670269	3·8253717	4·6163660	5·5658991	6·7047511
40	3·2620378	3·9592597	4·8010206	5·8163645	7·0399887

These tabular numbers are the successive powers of r; thus, $1·05^2 = 1·025$, &c.*

* The amount of £1 in t years, is the *last term* of an increasing geometrical series, of which the *first term* = the *ratio*, and the number of terms = t; because *the first year's amount is identical with the ratio*: and as $1 : r :: r : r^2$ = the amount in 2 years; as $1 : r :: r^2 :: r^3$ = the amount in 3 years, &c. The successive amounts, r, r^2, r^3, &c. are

Case 1. When p, r, and t are given, to find a.

RULE. $pr^t = a$. Or, $\log. r \times t + \log. p = \log. a$.

Or by the Table. Multiply the tabular amount of £1 by the principal, and the product will be the amount required.

(1) What will £225 amount to in 3 years, at £5 per cent per annum?*

(2) What will £200 amount to in 4 years, at £5 per cent per annum? *Ans.* £243..2..0..1·2 *qrs.*

(3) What will £450 amount to in 5 years, at £4 per cent per annum? *Ans.* £547..9..10..2·0538368 *qrs.*

(4) What will £500 amount to in 4 years, at £4½ per cent per annum? *Ans.* £596..5..2·232075*d.*

Case 2. When a, r, and t are given, to find p.

RULE. $\frac{a}{r^t} = p$. Or, $\overline{\log. a - \log. r \times t} = \log. p$.

(5) What principal being put to interest will amount to £260..9..3¾ in 3 years, at £5 per cent per annum?†

(6) What principal being put to interest will amount to £243..2..0..1·2 *qrs.* in 4 years, at £5 per cent per annum? *Ans.* £200.

(7) What principal will amount to £547..9..10..2·0538368 *qrs.* in 5 years, at £4 per cent per annum? *Ans.* 450.

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evidently in geometrical progression, and the amount in t years is $= r^t$; because the *index* always corresponds with the time. By refering to *Theorem* VII. *Geometrical Progression*, it will also be seen that such *last term* $= r \times r^{t-1} = r^t$, when $a = r$.

The immense increase of money accumulating at Compound Interest for a long period, is sufficient to astonish the human mind, and to stagger the credibility of persons who are not in some degree conversant with the properties of Geometrical Progression. The amount of a farthing, placed out at Compound Interest at the commencement of the Christian era, and continued to the conclusion of the eighteenth century, would be 144035 quintillions of pounds. But of the magnitude of this sum, spoken of in the abstract, no just conception can be formed. When, however, by a further calculation, we have ascertained that to coin such a quantity of money (were it possible) in sovereigns of the present weight and fineness, would require 60,308170 solid globes of gold, each as large as the earth, we are enabled to entertain a more adequate idea of the sum, whose vastness, without having recourse to this adscititious assistance, placed it almost beyond the reach of our limited understandings.

The amount at *Simple Interest* for the same period, would be only 1*s.* 10¾*d.* EDITOR.

* $1·05^3 \times 225 = 1·57625 \times 225 = 260·465625 =$ £260..9..3¾. *Ans.*

† $\frac{260·465625}{1·05^3} = \frac{260·465625}{1·157625} =$ £225. *Ans.*

(8) What principal will amount to £596..5..2·232075*d.* in 4 years, at £4½ per cent per annum? *Ans.* £500.

Case 3. When p, a, and t are given, to find r.

RULE. $\frac{a}{p} = r^t$ the root of which being extracted will give r.

Or, $\overline{\log. a - \log. p} \div t = \log. r$.

(9) At what rate per cent per annum, will £225 amount to £260..9..3¼ in 3 years?*

(10) At what rate per cent per annum, will £200 amount to £243..2..0..1·2 *qrs.* in 4 years? *Ans.* £5 *per cent.*

(11) At what rate per cent per annum, will £450 amount to £547..9..10..2·0538368 *qrs.* in 5 years? *Ans.* £4 *per cent.*

(12) At what rate per cent per annum, will £500 amount to £596·2593003125 in 4 years? *Ans.* £4½ *per cent.*

Case 4. When p, a, and r are given, to find t.

RULE. $\frac{a}{p} = r^t$ which being continually divided by r till nothing remains, the *number of the divisions* will be equal to t.

Or, $\overline{\log. a - \log. p} \div \log. r = t$.†

(13) In what time will £225 amount to £260..9..3¼, at £5 per cent per annum?‡

(14) In what time will £200 amount to £243..2·025*s.* at £5 per cent per annum? *Ans.* 4 *years.*

(15) In what time will £450 amount to £547..9..10..2·0538368 *qrs.* at £4 per cent per annum? *Ans.* 5 *years.*

(16) In what time will £500 amount to £596·2593003125, at £4½ per cent per annum? *Ans.* 4 *years.*

THE AMOUNT OF ANNUITIES IN ARREARS.

NOTE. u represents the annuity, pension, or yearly rent; a, r, and t, as before.

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* $\frac{260·465625}{225} = 1·157625$, and $\sqrt[3]{1·157625} = 1·05$, or £5 *per cent. Ans.*

† In all cases of this nature, t cannot be found without Logarithms, unless it be a *whole number.*

‡ $\frac{260·465625}{225} = 1·157625$; $\frac{1·157625}{1·05} = 1·1025$; $\frac{1·1025}{1·05} = 1·05$; $\frac{1·05}{1·05} = 1$; the number of divisions being *three*, which gives the time sought = 3 years. *Ans.*

A Table of the Amount of £1 Annuity for Years.

yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	1·0000000	1·0000000	1·0000000	1·0000000	1·0000000
2	2·0300000	2·0350000	2·0400000	2·0450000	2·0500000
3	3 0909000	3·1062250	3·1216000	3·1370250	3·1525000
4	4·1836270	4·2149429	4·2464640	4·2781911	4·3101250
5	5·3091358	5·3624659	5·4163226	5·4707097	5·5256312
6	6·4684099	6 5501522	6 6329755	6·7168916	6 8019128
7	7·6624622	7·7794075	7·8982945	8·0191517	8·1420084
8	8 8923361	9·0516868	9·2142263	9·3800135	9·5491088
9	10·1591062	10·3684958	10·5827953	10·8021141	11·0265642
10	11·4638794	11·7313932	12·0061071	12·2882092	12·5778924
11	12 8077958	13·1419920	13·4863514	13·8411786	14·2067870
12	14·1920297	14·6019617	15·0258055	15·4640316	15·9171264
13	15·6177906	16·1130304	16·6268377	17·1599130	17·7129827
14	17·0863243	17·6769865	18·2919112	18·9321091	19·5986318
15	18·5989140	19·2956810	20·0235876	20·7840540	21·5785634
16	20·1568814	20·9710298	21·8245311	22·7193364	23·6574916
17	21·7615878	22·7050158	23·6975123	24·7417066	25·8403662
18	23·4144354	24·4996914	25·6454128	26·8550834	28·1323845
19	25·1168685	26 3571806	27·6712293	29·0635622	30·6390037
20	26:8703745	28·2796819	29·7780785	31·3714225	33 0659539
21	28·6764857	30·2694708	31·9692016	33·7831365	35·7192516
22	30·5367803	32·3289023	34·2479697	36·3033777	38 5052142
23	32·4528837	34·4604139	36·6178885	38 9370297	41·4304749
24	34·4264702	36·6665284	39·0826040	41·6891960	44 5019986
25	36·4592643	38·9498569	41·6459082	44·5652098	47·7270985
26	38·5530422	41·3131019	44 3117445	47·5706443	51·1134534
27	40·7096335	43·7590605	47·0842143	50·7113233	54 6691261
28	42·9309225	46·2906276	49·9675829	53·9933329	58·4025824
29	45·2188502	48 9107996	52·9662862	57·4230329	62·3227115
30	47 5754157	51·6226776	56·0849376	61·0070694	66·4388471
31	50·0026782	54·4294713	59·3283351	64·7523875	70·7607895
32	52·5027585	57·3345028	62·7014685	68·6662449	75·2988290
33	55·0778412	60·3412104	66·2095272	72·7562259	80·0637704
34	57 7301764	63·4531527	69·8579083	77·0302561	85·0669589
35	60·4620817	66·6740130	73·6522245	81·4966176	90·3203068
36	63·2759441	70·0076034	77 5983136	86·1639654	95·8363221
37	66·1742224	73·4578695	81·7022461	91·0413439	101·6281382
38	69·1594490	77 0288949	85·9703359	96·1382044	107 7095451
39	72 2342324	80·7249062	90·4091493	101·4644236	114·0950224
40	75 4012593	84·5502779	95·0255153	107·0303227	120·7997735

NOTE. The preceding Table is formed thus: the first year's amount is £1; and 1×1·05 + 1 = 2·05, the second year's amount; 2·05 × 1·05 + 1 = 3·1525, the third year's amount, &c.

Case 1. When u, t, and r are given, to find a.

RULE. $\frac{r^t - 1}{r - 1} \times u = a$.

Or, by the Table. Multiply the tabular amount of £1 annuity by the given annuity, and the product will be the amount required.

(17) What will an annuity of £50 per annum amount to in 4 years, at £5 per cent per annum?*

(18) What will a pension of £45 per annum, payable yearly, amount to in 5 years, at £5 per cent per annum?
Ans. £248..13*s*..3·27 *qrs.*

(19) If an annual salary of £40 be forborne 6 years, at £4 per cent per annum, what is the amount?
Ans. £265..6..4..2·25775616 *qrs.*

(20) If an annuity of £75, payable yearly, be omitted to be paid for 10 years, what is the amount at £5 per cent per annum? *Ans.* £943..6..10·0656*d.* +

Case 2. When a, r, and t are given, to find u.

RULE. $\frac{r - 1}{r^t - 1} \times a = u$.

(21) What annuity, being forborne 4 years, will amount to £215..10..1½, at £5 per cent per annum?†

(22) What pension, forborne 5 years, will amount to £248..13*s*..3·27 *qrs.* at £5 per cent per annum? *Ans.* £45.

(23) What salary, being omitted to be paid 6 years, will amount to £265..6..4..2·25775616 *qrs.* at £4 per cent per annum? *Ans.* £40.

(24) If the payment of an annuity, being forborne 10 years, amount to £943..6..10·0656*d.* at £5 per cent per annum, what is the annuity? *Ans.* £75.

Case 3. When u, a, and r are given, to find t.

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* $\frac{1·05^4 - 1}{·05} \times 50 = (1·21550625 - 1) \times 1000 = 215·50625 =$ £215..10..1½. *Ans.*

Or, by the Table thus: 4·310125 × 50 = £215·50625, as before.

† $\frac{·05 \times 215·50625}{1·05^4 - 1} = \frac{·05 \times 215·50625}{·21550625} = ·05 \times 1000 =$ £50. *Ans.*

RULE. $\frac{(r-1)a}{u}+1=r^t$ which being continually divided by r till nothing remains, the *number of divisions* will be equal to t.

(25) In what time will £50 per annum amount to £215..10..1½, at £5 per cent per annum, for non-payment?*

(26) In what time will £45 per annum amount to £248..13s..3·27 *qrs.* allowing £5 per cent per annum, for forbearance of payment? *Ans.* 5 *years.*

(27) In what time will £40 per annum amount to £265..6..4..2·25775616 *qrs.* at £4 per cent per annum? *Ans.* 6 *years.*

(28) In what time will £75 per annum amount to £943..6..10 0656*d.* allowing £5 per cent per annum, for forbearance of payment? *Ans.* 10 *years.*

NOTE. The examples relating to the *Present Worth of Annuities at Simple Interest* are now expunged from this work; because, being entirely useless, except as *a mere arithmetical exercise*, it is presumed that the judicious teacher will prefer the substitution of other matter of more *real utility*, which is introduced to supply their place. The *Theorems* are, however, retained in a Note, page 150; in order that the ingenious student, who may wish to calculate any example both ways, may have an opportunity of indulging his curiosity, and of comparing the true and the false results. That the *principle* of computing their value by *Simple Interest* is *erroneous* and *absurd*, will be manifested by the following observations.

The present worth of an annuity of £150, to continue only 40 years, calculated at £5 per cent per annum, *Simple Interest* (by Theorem 1, Note, page 150), would be £3950. But this sum, put out at *the same rate*, will produce £197..10, annual interest (or £47..10 a year more than the proposed annuity), *for ever*. If computed on the *true principle* (by the Theorem, Case 1, page 151), the present value is £2573..17..3.

The present value of *any perpetual annuity* (great or small), computed at *Simple Interest*, is an *unlimited*, or *infinite sum*. But by using *Compound Interest*, we shall obtain a *rational result*. For instance, an annuity of £150, to continue *for ever*, will (by Case 1, Perpetual Annuities, page 154), at £5 per cent, be worth £3000 purchase: which, it is evident, is the sum that *will yield* £150, annual interest.

EDITOR.

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* $\frac{0·5 \times 215·50625}{50}+1 = ·001 \times 215·50625 + 1 = 1·21550625$; which being continually divided by 1·05, the number of divisions will be 4, the years required. *Ans.*

THE PRESENT WORTH OF ANNUITIES.*

A Table of the present Worth of £1 Annuity for Years

yrs.	3 per Cent.	3½ per Cent.	4 per Cent.	4½ per Cent.	5 per Cent.
1	0·9708738	0·9661836	0·9615385	0·9569378	0·9523810
2	1·9134697	1·8996943	1·8860947	1·8726677	1·8594105
3	2·8286114	2·8016370	2·7750910	2·7489643	2·7232481
4	3·7170984	3·6730792	3·6298952	3·5875256	3·5459506
5	4·5797072	4·5150524	4·4518223	4·3899766	4·3294768
6	5·4171915	5·3285530	5·2421368	5·1578723	5·0756922
7	6·2302830	6·1145440	6·0020546	5·8927008	5·7863735
8	7·0196922	6·8739556	6·7327448	6·5958859	6·4632129
9	7·7861089	7·6076866	7·4353315	7·2687903	7·1078218
10	8·5302028	8·3166054	8·1108956	7·9127180	7·7217351
11	9·2526241	9·0015511	8·7604765	8·5289167	8·3064144
12	9·9540040	9·6633344	9·3850735	9·1185806	8·8632518
13	10·6349553	10·3027385	9·9856476	9·6828522	9·3935732
14	11·2960731	10·9205203	10·5631227	10·2228251	9·8986412
15	11·9379350	11·5174109	11·1183872	10·7395455	10·3796583
16	12·5611019	12·0941168	11·6522954	11·2340148	10·8377698
17	13·1661183	12·6513206	12·1656686	11·7071912	11·2740665
18	13·7535129	13·1896817	12·6592967	12·1599916	11·6895872
19	14·3237989	13·7098374	13·1339391	12·5932934	12·0853210
20	14·8774747	14·2124033	13·5903260	13·0079363	12·4622103
21	15·4150240	14·6979742	14·0291596	13·4047237	12·8211529
22	15·9369165	15·1671248	14·4511150	13·7844246	13·1630028
23	16·4436083	15·6204104	14·8568413	14·1477747	13·4885741
24	16·9355420	16·0583675	15·2469628	14·4954782	13·7986420
25	17·4131476	16·4815145	15·6220796	14·8282088	14·0939448
26	17·8768423	16·8903522	15·9827688	15·1466113	14·3751853
27	18·3270314	17·2853644	16·3295854	15·4513027	14·6430336
28	18·7641082	17·6670187	16·6630629	15·7428734	14·8981274
29	19·1884546	18·0357669	16·9837143	16·0218884	15·1410737
30	19·6004414	18·3920453	17·2920330	16·2888884	15·3724511
31	20·0004286	18·7362757	17·5884933	16·5443908	15·5928166
32	20·3887656	19·0688654	17·8735512	16·7888907	15·8026768
33	20·7657919	19·3902081	18·1476454	17·0228619	16·0025493
34	21·1318368	19·7006842	18·4111975	17·2467578	16·1929041
35	21·4872202	20·0006611	18·6646130	17·4610122	16·3741944
36	21·8322526	20·2904938	18·9082817	17·6660404	16·5468518
37	22·1672355	20·5705254	19·1425785	17·8622396	16·7112874
38	22·4924617	20·8410873	19·3678639	18·0499900	16·8678928
39	22·8082153	21·1024998	19·5844845	18·2296555	17·0170403
40	23·1147721	21·3550723	19·7927735	18·4015842	17·1590865

* *Present Worth of Annuities at Simple Interest.*

Theor. I. $\frac{r(t-1)+2}{2tr+2} \times tu = p$. II. $\frac{tr+1}{(r.(t-1)+2).t} \times 2p = u$.

NOTE. The above table is thus formed: £1 ÷ 1·05 = ·9523810, the present worth of the first year; this ÷ 1·05 = ·9070295, which, added to the first year's present worth, gives 1·8594105, the present worth of 2 years; then, ·9070295 ÷ 1·05, and the quotient added to 1·8594105 = 2·7232481, the present worth of 3 years, &c.

Case 1. When u, t, and r are given, to find p, *the present worth.*

RULE. $(u - \frac{u}{r^t}) \div (r-1) = p.$

Or, by the Table. Multiply the tabular present worth for the time given, by the given annuity, and the product will be the present worth required.

(29) What is the present worth of an annuity of £30, to continue 7 years, at £5 per cent per annum?*

(30) What is the present worth of a pension of £40 per annum, to continue 8 years, at £5 per cent per annum?

Ans. £258..10..6..3·264 *qrs.*

(31) What is the present worth of an annual salary of £35, to continue 7 years, at £4 per cent per annum?

Ans. £210..1..5·04*d.*

(32) What is the yearly rent of £50, to continue 5 years, worth in ready money, at £5 per cent per annum?

Ans. £216..9..5..2·08 *qrs.*

Case 2. When p, t, and r are given, to find u.

RULE. $\frac{pr^t(r-1)}{r^t-1} = u.$

(33) If an annuity be purchased for £173..11..10·08*d.* to

III. $\frac{(tu-p)\times 2}{(2p-(t-1)u).t} = r.$

IV. Put $\frac{1}{r} - \left(\frac{p}{u} + \frac{1}{2}\right) = m$; then $\sqrt{\left(\frac{2p}{ru} + m^2\right)} - m = t.$

For *Annuities in Reversion*, it is only necessary to observe the Rules for Reversionary Annuities at Compound Interest, and to calculate (according to the directions therein given) by the Theorems for Simple Interest.

* $30 - \frac{30}{1·05^7} = 30 - \frac{30}{1·4071} = 30 - 21·3204 = 8·6796$ and $8·6796 \div ·05 = £173·592 =$ £173..11..10·08*d.* *Ans.*

Or, by the Table, 5·7863735 × 30 = £173·591205. *Ans.*

be continued 7 years, at £5 per cent per annum, what is the annuity?*

(34) If £258..10..6..3·264 *qrs.* be paid down for a salary 8 years to come, at £5 per cent per annum, what is the salary? *Ans.* £40.

(35) If the present payment of £210..1..5·04*d.* be required for a pension for 7 years to come, at £4 per cent per annum, what is the pension? *Ans.* £35.

(36) If the present worth of an annuity 5 years to come, be £216..9..5..2·08 *qrs.* at £5 per cent per annum, what is the annuity? *Ans.* £50.

Case 3. When u, p, and r are given, to find t.

RULE. $\dfrac{u}{p+u-pr} = r^t$ which being continually divided by r till nothing remains, the *number of divisions* will be equal to t.

(37) How long may a lease of £30 yearly rent be had for £173..11..10·08*d.* allowing £5 per cent per annum to the purchaser?†

(38) If £258..10..6..3·264 *qrs.* is paid down for a lease of £40 per annum, at £5 per cent per annum, how long is the lease purchased for? *Ans.* 8 *years.*

(39) If a house is let upon lease for £35 per annum, and the lessor disposes of the lease for £210..1..5·04*d.* allowing after the rate of £4 per cent per annum; what term of the lease remains unexpired? *Ans.* 7 *years.*

(40) For what time is a lease of £50 per annum purchased, when present payment is made of £216..9..5..2·08 *qrs.* at £5 per cent per annum? *Ans.* 5 *years.*

ANNUITIES, &c. IN REVERSION.

To find the present worth of annuities in reversion.

RULE 1. Find the present worth of the annuity, *for the time of its continuance*, as if it were to commence immediately, by Case 1, page 151. Then find what principal will amount

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* 173·592 × 1·4071 × ·05 ÷ ·4071 = 12·213 ÷ ·4071 = £30. *Ans.* *Or, by the Table,* 173·592 ÷ 5·7863735 = £30. *Ans.*

† 173·592 + 30 — (173·592 × 1·05) = 203·592 — 182·2716 = 21·3204; and 30 ÷ 21·3204 = 1·4071; which being continually divided by 1·05, the number of divisions will be 7: therefore t = 7 years. *Ans.* *Or, by the Table:* 173·592 ÷ 30 = 5·7864: and referring to the column of 5 per cent, we find the number 5·786373 against 7 years.

to that sum in the given time *before the annuity commences* (by Case 2, *Compound Interest*, page 145); which will be the present worth.

RULE 2. Find the present worth of a similar annuity supposed *to commence immediately*, and continue during the *whole period*; and also the present worth of the same *till the time* when the reversionary annuity *actually commences*; and the *difference* of these two will be the present value required.

NOTE. When calculating by the Table, this is the most eligible method.

RULE 3. Find the *amount* of the annuity at the time of its cessation (by Case 1, page 148); and the *present worth* of *that amount* (being found by Case 2, *Compound Interest*, page 145) will be the value required.

(41) What is the present worth of a reversion of a lease of £40 per annum, to continue for 6 years, but not to commence till the end of 2 years, allowing £5 per cent per annum, to the purchaser?*

(42) What is the present worth of a reversion of a lease of £60 per annum, to continue 7 years, but not to commence till the end of 3 years, allowing £5 per cent per annum, to the purchaser? *Ans.* £299..18..2·16*d*.

(43) A house is let at £30 per annum, on a lease, of which 4 years are yet unexpired, and which the lessee is desirous of renewing at the same rental, to continue 7 years beyond the term of the present lease. What will the lessor expect as a *bonus* for such a renewal of the lease, considering the house to be worth double the present rent; and allowing interest for the money now advanced, at £5 per cent per annum?

Ans. £142..16..3..1·152 *qrs*.

To find the annuity in reversion, which a given sum will purchase.

RULE. Find the *amount* of the given sum for the time *before the annuity commences*; by Case 1, Compound Interest, page 145, which will be the value of the annuity at its *commencement*.

Call this value p, and then find the annuity as in Case 2, page 151.

(44) What annuity to be entered upon 2 years hence, and

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* $40 - \frac{40}{1{\cdot}05^6} = 40 - \frac{40}{1{\cdot}3400956} = 40 - 29{\cdot}84861 = 10{\cdot}15139$; and $10{\cdot}15139 \div {\cdot}05 = 203{\cdot}0278$: then $203{\cdot}0278 \div 1{\cdot}05^2 =$ £184·1522 = £184..3..0..2·112 *qrs*. *Ans.*

then to continue 6 years, may be purchased for £184..3..0..2 1·12 *qrs.* at £5 per cent per annum?*

(45) The present worth of a lease, taken in reversion for 7 years, but not to commence till the end of three years, is £299..18..2·16*d.* allowing £5 per cent per annum, to the purchaser. What is the yearly rent? *Ans.* £60.

(46) There is a lease that has yet 4 years to run, and the lessee has purchased the reversion of a renewed lease, at the same rental of £30 per annum, for the term of 7 years, commencing at the expiration of the present lease; for which he has paid down £142..16..3..1·152 *qrs.* What increase of rent is reckoned on the property, according to this contract, allowing £5 per cent per annum, for present payment? *Ans.* £30.

PERPETUAL ANNUITIES; OR FREEHOLD ESTATES.

Case 1. When u and r are given, to find p, the *present worth*, or *purchase money.*

RULE. $\dfrac{u}{r-1} = p.$†

(47) What is the worth of a freehold estate of £50 yearly rent, allowing £5 per cent per annum to the buyer?‡

(48) What is a real estate of £140 per annum, worth in present money, allowing £4 per cent per annum, to the purchaser? *Ans.* £3500.

(49) What must the purchaser give for a freehold estate of £437..10, yearly rent, so as to make £3½ per cent per annum, by the investment of his capital? *Ans.* £12500.

Case 2. When p and r are given, to find u.

RULE. $p \times \overline{r-1} = u.$

(50) If a freehold estate is bought for £1000, what must

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* $184{\cdot}1522 \times 1{\cdot}1025 = 203{\cdot}0278$; then

$$\frac{203{\cdot}0278 \times 1{\cdot}3400956 \times {\cdot}05}{{\cdot}3400956} =$$

$$\frac{203{\cdot}0278 \times 1 + (203{\cdot}0278 \times {\cdot}3400956)}{{\cdot}3400956} \times {\cdot}05 = \frac{203{\cdot}0278}{{\cdot}3400956}$$

$+\, 203{\cdot}0278 \times {\cdot}05 = 800{\cdot}0005 \times {\cdot}05 =$ £40. *Ans*

Or, by the Table: $184{\cdot}1522 \div (6{\cdot}4632129 - 1{\cdot}8594105) = 184{\cdot}1522 \div 4{\cdot}6038024 =$ £40. *Ans.*

† This rule is deduced from the *formula* in page 151: for in Annuities continuing for ever, t is *infinite*, and the subtractive quantity $u \div r^t = 0$; therefore the theorem assumes the above form.

‡ $50 \div {\cdot}05 =$ £1000. *Ans.*

be the yearly rent, to pay the purchaser £5 per cent per annum interest for his money?*

(51) If an estate be sold for £3500, what is the yearly rent, allowing to the purchaser £4 per cent per annum?

Ans. £140.

(52) If a freehold estate is bought for £12500, and will yield the purchaser £3½ per cent per annum, what is the yearly rent? *Ans.* £437..10.

Case 3. When p and u are given, to find r.

RULE. $\frac{u}{p} + 1 = r$.

(53) If an estate of £50 per annum is bought for £1000, what is the rate per cent per annum?†

(54) If a freehold estate of £140 per annum is sold for £3500, what interest will it pay to the purchaser?

Ans. £4 *per cent.*

(55) If an estate in perpetuity of £437..10 per annum is sold for £12500, what interest will it pay to the purchaser?

Ans. £3½ *per cent.*

FREEHOLD ESTATES IN REVERSION.

To find the present worth of a freehold estate in reversion.

RULE. Find the value of the estate, supposing it were to come into *immediate possession,* as in Case 1, page 154. Then suppose that value (p) to be a, and find what *principal* will amount to a, in the *time to come, previous to possession,* by Case 2, Compound Interest, page 145. Such principal will be the present value.

(56) What must be paid down for the purchase of a freehold of £50 per annum, to be entered upon 4 years hence, allowing the purchaser at the rate of £5 per cent per annum for his purchase money?‡

(57) What must be paid down for the reversion of a real estate of £200 per annum, so as to pay the purchaser £4 per cent per annum, for his capital; supposing 2 years to elapse before the estate comes into possession?

Ans. £4622..15..7..1·76 *qrs.*

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* $1000 \times \cdot 05 = £50$. *Ans.*

† $\frac{50}{1000} + 1 = 1\cdot 05 = £5$ *per cent. Ans.*

‡ $50 \div \cdot 05 = 1000$; then $1000 \div 1\cdot 2155 = £822\cdot 7067 = £822..$ 14..1..2·432 *qrs. Ans.*

(58) A freehold producing £280 annual rent is to be disposed of, with a reserve of the next 3 years' rent to the present proprietor. What is it worth in ready money, allowing £$3\frac{1}{2}$ per cent per annum to the purchaser?

Ans. £7215..10..9..3·36 *qrs.*

To find the yearly rent of an estate in reversion; having its present value given.

RULE. Find the amount of the given present value, in the *time before possession:* thus, $pr^t = a$. Then consider that amount to be the present value (p) of the perpetual annuity, and find the annuity thus: $p \times (r - 1) = u$.

(59) What must be the rent of a freehold property, to come into possession 4 years hence, for which £822..14..1..2·432 *qrs.* is paid down; allowing the purchaser £5 per cent per annum?*

(60) A freehold estate is sold for £4622..15..7..1·76 *qrs.* the vendor reserving to himself the first two years' rent. Required the annual value, to pay the purchaser £4 per cent per annum for his capital? *Ans.* £200.

(61) A freehold estate has been purchased for £7215..10..9..3·36 *qrs.* the possession of which is not to be given up till after the expiration of 3 years. What must be the annual rent, to pay the purchaser at the rate of £$3\frac{1}{2}$ per cent per annum? *Ans.* £280.

DISCOUNT; ON THE PRINCIPLES OF COMPOUND INTEREST.†

NOTE. The following table is constructed by the continual division of 1 by the *ratio* (r): thus $1 \div 1{\cdot}05 = {\cdot}9523810$, the first year's present worth; then ${\cdot}9523810 \div 1{\cdot}05 = {\cdot}9070295$, the second year's present worth; and ${\cdot}9070295 \div 1{\cdot}05 = {\cdot}8638376$, the third, &c.

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* $822{\cdot}70625 \times 1{\cdot}2155 = 1000$; then $1000 \times {\cdot}05 = £50$. *Ans.*

† This is merely a repetition of the various cases in Compound Interest. For instance, to find the present worth of any debt due some time hence, is precisely the same operation as finding what principal will amount to that sum in the given time; and this observation will equally apply to the identity of the other cases. The entire omission, therefore, of *Discount* (at *Compound Interest*) arranged under that specific head, would be no detriment to the learner. It is, however, retained here, for the sake of those who may think some repetition of the subject desirable. EDITOR.

A Table of the present Worth of £1, due any Number of Years hence, from 1 to 40.

yrs.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	$4\frac{1}{2}$ per Cent.	5 per Cent.
1	·9708738	·9661836	·9615385	·9569378	·9523810
2	·9425959	·9335107	·9245562	·9157299	·9070295
3	·9151417	·9019427	·8889963	·8762966	·8638376
4	·8884870	·8714422	·8548042	·8385613	·8227025
5	·8626088	·8419732	·8219271	·8024510	·7835262
6	·8374843	·8135006	·7903145	·7678957	·7462154
7	·8130915	·7859910	·7599178	·7348285	·7106813
8	·7894092	·7594116	·7306902	·7031851	·6768394
9	·7664167	·7337310	·7025867	·6729044	·6446089
10	·7440939	·7089188	·6755641	·6439277	·6139133
11	·7224213	·6849457	·6495809	·6161987	·5846793
12	·7013799	·6617833	·6245970	·5896639	·5568374
13	·6809513	·6394041	·6005741	·5642716	·5303214
14	·6611178	·6177818	·5774751	·5399729	·5050680
15	·6418619	·5968906	·5552645	·5167204	·4810171
16	·6231669	·5767059	·5339082	·4944693	·4581115
17	·6050164	·5572038	·5133732	·4731764	·4362967
18	·5873946	·5383611	·4936281	·4528004	·4155207
19	·5702860	·5201557	·4746424	·4333018	·3957340
20	·5536758	·5025659	·4563869	·4146429	·3768895
21	·5375493	·4855709	·4388336	·3967874	·3589424
22	·5218925	·4691506	·4219554	·3797009	·3418499
23	·5066918	·4532856	·4057263	·3633501	·3255713
24	·4919337	·4379571	·3901215	·3477035	·3100679
25	·4776056	·4231470	·3751168	·3327306	·2953028
26	·4636947	·4088377	·3606892	·3184025	·2812407
27	·4501891	·3950122	·3468166	·3046914	·2678483
28	·4370768	·3816543	·3334775	·2915707	·2550936
29	·4243464	·3687482	·3206514	·2790150	·2429463
30	·4119868	·3562784	·3083187	·2670000	·2313774
31	·3999872	·3442304	·2964603	·2555024	·2203595
32	·3883370	·3325897	·2850579	·2444999	·2098662
33	·3770263	·3213427	·2740942	·2339712	·1998725
34	·3660449	·3104761	·2635521	·2238959	·1903548
35	·3553834	·2999769	·2534155	·2142544	·1812903
36	·3450324	·2898327	·2436687	·2050282	·1726574
37	·3349829	·2800316	·2342968	·1961992	·1644356
38	·3252262	·2705619	·2252854	·1877504	·1566054
39	·3157536	·2614125	·2166206	·1796655	·1491480
40	·3065568	·2525725	·2082890	·1719287	·1420457

Case 1. *To find the present worth of any sum due after a certain period.*

RULE. The same as in Case 2, *Compound Interest;* considering *a* as the debt whose present value is required.

(1) If £344..14..9..1·92 *qrs.* be payable in 7 years time, what is the present worth, discount being made at £5 per cent per annum?*

(2) A debt of £409..9..00992*s.* payable 4 years hence, is agreed to be paid in present money: what sum must the creditor receive, discounting at £4 per cent per annum?

Ans. £350.

Case 2. *To find the debt whose present worth is given.*

RULE. See Case 1, *Compound Interest.*

(3) If £245 be received for a debt payable 7 years hence, allowing £5 per cent per annum to the debtor for present payment, what is the debt?†

(4) There is a sum of money due at the expiration of 4 years; but the creditor agrees to take £350 in ready money, allowing £4 per cent per annum, discount. What was the debt?

Ans. £409..9 00992*s.*

Case 3. *When the rest are given, to find the time.*

RULE. See Case 4, *Compound Interest.*

(5) A person receives £245 now for a debt of £344..14..9..1·92 *qrs.* discounting at £5 per cent per annum. In what time was the debt payable?‡

(6) There is a debt of £409..9·00992*s.* due a certain time hence; but £4 per cent per annum being allowed to the debtor for the present payment of £350, it is required to find in what time the sum was to be paid.

Ans. 4 *years.*

Case 4. *When the rest are given, to find the rate per cent.*

RULE. As in Case 3, *Compound Interest.*

(7) The present worth of £344..14..9..1·92 *qrs.* payable

---

* 344·7395 ÷ 1·4071 = £245. *Ans.*

Or, by the Table: ·7106813 × 344·7395 = £245. *Ans.*

† 245 × 1·4071 = £344·7395 = £344..14..9..1·92 *qrs. Ans.*

‡ 344·7395 ÷ 245 = 1·4071; the *continual divisions* of which by 1·05, will be 7 = the number of years. *Ans.*

7 years hence, is £245. At what rate per cent per annum is discount allowed?*

(8) There is a debt of £409..9·00992*s.* payable in 4 years; but it is agreed to take £350, present payment. Required the rate of discount. *Ans.* £4 *per cent.*

EQUATION OF PAYMENTS AT COMPOUND INTEREST.

Rule 1. Find the present worth of each payment respectively; and add them together for the *whole present worth:* then the time in which that present worth will amount to the sum of the debts will be the *true equated time* required.

2. Find the amount of each debt from the time of its becoming due till the time of the last payment, and add the respective amounts and the last payment into one sum. Then find *the time* in which the sum of the debts would amount to that sum of the amounts: subtract *this* from the time of the last payment, and the difference will be the *true equated time.*

(1) Required the true equated time for the payment of a debt of £400, of which £320 is now due, and the rest at the end of 5 years; reckoning compound interest at the rate of £5 per cent per annum? *Ans.* ·90714 *years.*

(2) If £100 will become due one year hence, and £104 three years hence, what is the true equated time for payment of the whole, allowing compound interest at £4 per cent per annum? *Ans.* 2 *years.*

(3) If a person will have to receive £200 at the end of 3 years, and £80 more at the end of 5 years, in what time ought he to receive the whole at one payment, allowing £5 per cent per annum, compound interest?

Ans. 3·5518 *years.*

DUODECIMALS

Are so named from the integer of each denomination containing *twelve* of the next inferior. They are in general use among artificers for computing the quantities of their materials and labour; both in *Superficial* and *Solid Measure.*†

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* $344{\cdot}7395 \div 245 = 1{\cdot}4071$; and $\sqrt[7]{1{\cdot}4071} = 1{\cdot}05$; which gives £5 per cent. *Ans.*

† For a clear and intelligible explanation of the *different Measures,* see the Tables, page 26, &c.

12 inches (′) make 1 foot.
12 seconds (″) ...... 1 inch, or prime.
12 thirds (‴) ...... 1 second, &c.

To multiply duodecimally.

RULE 1. Under the multiplicand write the corresponding terms of the multiplier.

2. Multiply by the feet in the multiplier, observing to *carry one* for every *twelve*, from each lower denomination to the next superior.

3. In the same manner multiply by the inches in the multiplier, setting the result from each term one place farther to the right.

4. Proceed in like manner with the remaining denominations, and the sum of the products will be the total product.

NOTE 1. Length and breadth multiplied together produce the *area* of a *superficies;* and this multiplied by the thickness, produces the *solid content* of a body.

2. t is generally more eligible to take *aliquot parts* out of the multiplicand for the inches, &c. in the multiplier.

		ft.	′	″		*ft.*	′	″		*ft.*	′	″	‴	⁗
(1)	Mult.	7	9		by	3	6*							
(2)	Mult.	8	5		by	4	7		*Ans.*	38	6	11		
(3)	Mult.	9	8		by	7	6		*a.*	72	6	0		
(4)	Mult.	8	1		by	3	5		*a.*	27	7	5		
(5)	Mult.	7	6		by	5	9		*a.*	43	1	6		
(6)	Mult.	4	7		by	3	10		*a.*	17	6	10		
(7)	Mult.	7	5	9	by	3	5	3	*a.*	25	8	6	2	3.
(8)	Mult.	10	4	5	by	7	8	6	*a.*	79	11	0	6	6.
(9)	Mult.	75	7		by	9	8		*a.*	730	7	8		
(10)	Mult.	97	8		by	8	9		*a.*	854	7	0		
(11)	Mult.	57	9		by	9	5		*a.*	543	9	9		
(12)	Mult.	75	9		by	17	7		*a.*	1331	11	3		
(13)	Mult.	87	5		by	35	8		*a.*	3117	10	4		
(14)	Mult.	179	3		by	38	10		*a.*	6960	10	6		
(15)	Mult.	259	2		by	48	11		*a.*	12677	6	10		
(16)	Mult.	257	9	†	by	39	11		*a.*	10288	6	3		
(17)	Mult.	311	4	7	by	36	7	5	*a.*	11402	2	4	11	11.
(18)	Mult.	321	7	3	by	9	3	6	*a.*	2988	2	10	4	6.

ft.	*in.*		*in.*	*Otherwise.*			*Proof by Decimals.*
* 7	9		6 = ½	7	9		7·75
3	6				3		3·5
23	3			23	3		3875
3	10	6″		3	10	6″	2325
27	1			27	1	6	27·125 *sq. feet.*

Glazing, *Mason's* flat work, and some parts of *Joiner's* work, are computed at so much per square foot.

Painters', *Plasterers'*, *Pavers'*, and descriptions of *Joiners'* work, are estimated by the square yard.

Roofs, Floors, Partitions, &c. by the square of 100 feet.

Bricklayers' work by the square rod, containing 272¼ feet.

(19) A certain house has 3 tiers of windows, 3 in a tier, the height of the first tier being 7 feet 10 inches, the second 6 feet 8 inches, and the third 5 feet 4 inches; and the breadth of each window is 3 feet 11 inches. What will the glazing cost at 14*d*. per square foot?*

(20) What is the price of 8 squares of glass, each measuring 4 feet 10 inches long, and 2 feet 11 inches broad, at 4⅛*d*. per square foot? *Ans*. £1..18..9.

(21) What is the value of 8 squares, each measuring 3 feet by 1 foot 6 inches, at 7¾*d*. per square foot? *Ans*. £1..3..3.

(22) What is the price of a marble slab, 5 feet 7 inches long, and 1 foot 10 inches broad, at 6*s*. per square foot? *Ans*. £3..1..5.

(23) What will be the expense of ceiling a room, the length of which is 74 feet 9 inches, and the width 11 feet 6 inches, at 3*s*. 10½*d*. per square yard? *Ans*. £18..10..1.

(24) What will the paving of a court-yard cost, at 4¾*d*. per square yard; the length being 58 feet 6 inches, and the breadth 54 feet 9 inches? *Ans*. £7..0..10.

(25) The circuit of a room is 97 feet 8 inches, and the height 9 feet 10 inches. What is the charge for painting it, at 2*s*. 8¾*d*. per square yard? *Ans*. £14..11..2.

(26) What is the expense of a piece of wainscot 8 feet 3 inches long, and 6 feet 6 inches broad, at 6*s*. 7½*d*. per square yard? *Ans*. £1..19..5.

---

	ft.	*in.*	
*	7	10	
	6	8	
	5	4	
	19	10	the whole height.
	3	11	
		3	
	11	9	the whole breadth.

		ft.	*in.*		
in.					
6 = ½		19	10		
			11		
in.		218	2		
3 = ½		9	11		
s.		4	11	6″	
1 = $\frac{1}{20}$		233	0	6	at 14*d*.
d. 2 = ⅙	£	11	13	0	the value at 1*s*.
		1	18	10	the value at 2*d*.
Value of 6″ =		0	0	0¼	
	£	13	11	10½	*Ans.*

(27) What will the paving of a court-yard cost, at 3*s.* 2*d.* per square yard, the length being 27 feet 10 inches, and the breadth 14 feet 9 inches? *Ans.* £7..4..5.

(28) A certain court yard is 42 feet 9 inches in front, and 68 feet 6 inches long; a causeway the whole length, and 5 feet 6 inches broad, is laid with Purbeck stone, at 3*s.* 6*d.* per square yard, and the rest is paved with pebbles, at 3*s.* per square yard. What is the expense? *Ans.* £49..17..0½.

(29) What will the plastering of a ceiling cost, at 10*d.* per square yard, supposing the length 21 feet 8 inches, and the breadth 14 feet 10 inches? *Ans.* £1..9..9.

(30) What will the wainscoting of a room cost, at 6*s.* per square yard, supposing the height of the room (including the cornice and moulding) is 12 feet 6 inches, and the compass 83 feet 8 inches: the three window shutters each 7 feet 8 inches by 3 feet 6 inches, and the door 7 feet by 3 feet 6 inches? The shutters and door being worked on both sides, are reckoned work and half-work. *Ans.* £36..12..2½.

(31) In a piece of partitioning 173 feet 10 inches long, and 10 feet 7 inches in height, how many squares?

Ans. 18 *squares,* 39 *feet,* 8′ 10″.

(32) A house of three stories, besides the ground floor, measuring 20 feet 8 inches by 16 feet 9 inches, is to be floored at £6..10 per square: there are 7 fire-places, two of which measure 6 feet by 4 feet 6 inches each, two others 6 feet by 5 feet 4 inches each, two others 5 feet 8 inches by 4 feet 8 inches each, and the seventh, 5 feet 2 inches by 4 feet; and the well-hole for the stairs is 10 feet 6 inches by 8 feet 9 inches. What will the whole amount to?

Ans. £53..13..3½.

(33) If a house measures within the walls 52 feet 8 inches in length, and 30 feet 6 inches in breadth, the roof being of a true pitch; what will it cost roofing at 10*s.* 6*d.* per square?

Ans. £12..12..11¼.

NOTE. A roof is said to be *of a true pitch,* when the rafters are ¾ of the breadth of the building. In this case, therefore, the breadth must be accounted equal to the breadth and half-breadth of the building.

(34) What will the tiling of a barn cost, at 25*s.* 6*d.* per square; the length being 43 feet 10 inches, and the breadth 27 feet 5 inches on the flat, the eave boards projecting 16 inches on each side? *Ans.* £24..9..5¾.

NOTE. Bricklayers compute their work at the rate of a brick and a half thick; therefore, if the thickness of a wall is more or less, it must be reduced to the standard thickness, by multiplying the area of the wall by the number of half bricks in the thickness, and dividing the product by 3.

*(35) If the area of a wall is 4085 feet, and the thickness two bricks and a half, how many rods does it contain of the standard thickness? *Ans.* 25 *rods,* 8 *feet.*

(36) If a garden wall is 254 feet in compass, 12 feet 7 inches high, and three bricks thick, how many rods does it contain? *Ans.* 23 *rods,* 136 *feet.*

(37) How many rods are there in a wall $62\frac{1}{2}$ feet long, 14 feet 8 inches high, and $2\frac{1}{2}$ bricks thick?

Ans. 5 *rods,* 167 *feet.*

(38) The end wall of a house is 28 feet 10 inches in length; the height of the roof from the ground is 55 feet 8 inches; and the gable (or triangular part at the top) rises 42 courses of bricks, reckoning 4 courses to a foot. The wall, to the height of 20 feet, is $2\frac{1}{2}$ bricks thick, 20 feet more, 2 bricks thick, and the remaining part, a brick and half thick; and the gable is 1 brick thick. What is the charge for the whole wall, at £5..16 per rod? *Ans.* £48..13..$5\frac{1}{2}$.

To multiply several figures by several, and obtain the product in one line only.

RULE. Multiply the units of the multiplicand by the units of the multiplier, set down the units of the product, and carry the tens; next multiply the tens in the multiplicand by the units of the multiplier, to which add the product of the units of the multiplicand multiplied by the tens in the multiplier, and the tens carried; then multiply the hundreds in the multiplicand by the units of the multiplier, adding the product of the tens in the multiplicand multiplied by the tens in the multiplier, and the units of the multiplicand by the hundreds in the multiplier; and so proceed till you have multiplied the multiplicand all through, by every figure in the multiplier.

Multiply 35234
by 52424
Product 1847107216

EXPLANATION.

First, $4 \times 4 = 16$, that is, 6 and carry 1. Secondly, $(3 \times 4) + (4 \times 2)$ and 1 that is carried $= 21$, set down 1 and carry 2. Thirdly, $(2 \times 4) + (3 \times 2) + (4 \times 4) + 2$ carried $= 32$; that is, 2 and carry 3. Fourthly, $(5 \times 4) + (2 \times 2) + (3 \times 4) + (4 \times 2) + 3$ carried $= 47$; set down 7 and carry 4. Fifthly, $(3 \times 4) + (5 \times 2) + (2 \times 4) + (3 \times 2) + (4 \times 5) + 4$ carried $= 60$; set down 0 and carry 6.

* In this and the three following examples, the rod is considered = 272 feet.

Sixthly, $(3 \times 2) + (5 \times 4) + (2 \times 2) + (3 \times 5) + 6$ carried $= 51$; set down 1 and carry 5. Seventhly, $(3 \times 4) + (5 \times 2) + (2 \times 5) + 5$ carried $= 37$; set down 7 and carry 3. Eighthly, $(3 \times 2) + (5 + 5) + 3$ carried $= 34$; set down 4 and carry 3. Lastly, $3 \times 5 + 3$ carried $= 18$; set down 18, and the work is finished.

MENSURATION OF SUPERFICIES.

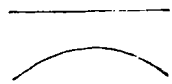
GEOMETRICAL DEFINITIONS.

GEOMETRY is the science which investigates the nature and properties of lines, angles, surfaces, and solid bodies.

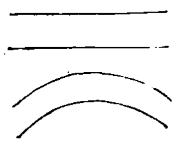
A *point* has no parts or magnitude.

A *line* has length only, without breadth or thickness.

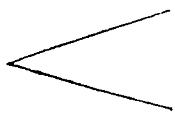
A line drawn wholly in the same direction, or the shortest distance between two points, is a *right* or *straight line.* That which continually changes its direction is a *curve.*



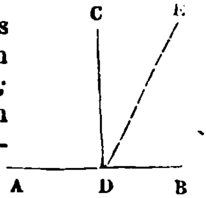
Parallel lines preserve the same distance from each other throughout; and therefore would never meet, though infinitely produced.



An *angle* is the degree of inclination of two lines, or the opening between them when they meet in a point; which is called the *angular point.*



When a line meeting another inclines not either way, but makes *equal angles* on each side, those are called *right angles;* and the lines are *perpendicular* to each other. Thus, the angle A D C = the angle B D C.*



An *oblique angle* is either *acute* or *obtuse.* An *acute angle* is *less* than a right angle, as B D E; and an *obtuse angle, greater* than a right angle, as A D E.

* When more than two lines meet, forming several angles at the same point, it is necessary to designate each angle by three letters, placing that which is at the angular point in the middle. Thus, the angle B D C is that formed by the lines B D and C D.

A *superficies*, or *surface*, is a space contained within lines; and has two dimensions, length and breadth.

A *solid* is a space or body bounded by surfaces, and has three dimensions, length, breadth, and thickness.

A *triangle* is a *superficies* bounded by three lines. A *quadrangle*, or *quadrilateral*, is bounded by four lines.

A *right-angled triangle* has one right angle; (Fig. page 118;) an *obtuse-angled triangle* has one obtuse angle; and an *acute-angled triangle* has all its angles acute.

An *equilateral triangle* has the three sides (and consequently the three angles) all equal to each other.

An *isosceles triangle* has two equal sides.

A *scalene triangle* has all the three sides unequal.

A *parallelogram* is a quadrangle having the opposite sides equal and parallel.[a] When the angles are right ones, it is called a *rectangle*.[b] And a rectangle having all its sides equal is a *square*.[c]

A *rhombus* has all its sides equal; but *oblique angles*.[d]

A *rhomboid* has oblique angles, and only its *opposite* sides equal.

All other quadrilaterals are *trapeziums:*[e] but a trapezium that has two sides parallel, is called a *trapezoid*.

The *base*[f] of a figure is the side on which it is supposed to stand; and a line drawn from the *vertex*, or opposite angle, perpendicular to the base, is the *altitude*,[f] or perpendicular height.

Right-lined plane figures of more than four sides, are called *polygons*. A polygon of five sides (or angles) is a *pentagon;* one of six, a *hexagon*, &c. *Vide Table, page* 168.

A *circle*[g] is a plain figure, contained under one curve line, called the *circumference;* which is in every part equidistant from the *centre*, or middle point within it. The circle contains more space than any other plane figure of equal compass.

A straight line passing through the centre, and meeting the circumference in two opposite points, is called the *diameter:*[h] and half the diameter, or the distance from the centre to the circumference, is the *radius*.[i]

An *arc* of a circle is any portion of the circumference.[k]

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[a] Figs. 1, 2, and 3. [b] Fig. 2. [c] Fig. 1. [d] Fig. 3. [e] Fig. 5.

[f] The line A B, Figs. 3 and 4, is the *base;* and C D, the *altitude*.

[g] Fig. 7. [h] The line A B, Fig. 7. [i] A C, or B C. [k] A D, or ADB, Fig 8.

A *chord* is a right line joining the extremes of an arc.[l]

The versed sine is part of the diameter cut off by the chord.[m]

A *segment* is a space contained between an arc and its chord.[n] A *semicircle* is a segment, the chord of which is the diameter.

A *sector* is bounded by an arc and two radii:[o] when the two radii are at right angles, it is a *quadrant*, or fourth part of the circle.

The circumference of every circle is supposed to be divided into 360 equal parts, called degrees; and each degree into 60 equal parts, called minutes, &c.

The measure of an angle is determined by the number of degrees in the arc of a circle contained between the two lines forming the angle, described round the angular point as a centre. Thus, the angle A C B (Fig. 8) is an angle of so many degrees as are contained in the arc A B. Hence a right angle contains 90 degrees.

An *ellipse* (or regular oval) is a plane figure bounded by a *curve* called the *circumference*, returning into itself, and described from two points, called the *foci*, or *focuses*, in the *transverse* (or longest) *diameter*. The shortest diameter, which intersects the *transverse* at right angles, is called the *conjugate*. The diameters are also called *axes*.[p]

MENSURATION.

Problem I. *To find the area of a Parallelogram; whether it be a Square, an oblong Rectangle, a Rhombus, or a Rhomboid.*

RULE. Multiply the length by the altitude or perpendicular breadth: the product will be the area.

Fig. 1.

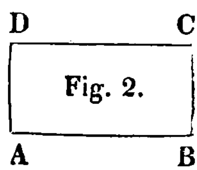
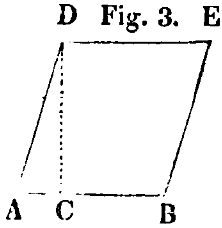


Fig. 2.

(1) The base of the largest Egyptian pyramid is a square, whose side is 693 feet. Upon how many acres of ground does it stand?

Fig. 3.



(2) Required the area of a rectangular board, whose length is 12½ feet, and breadth 9 inches.

(3) What quantity of land does a

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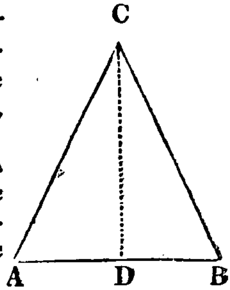
[l] A B, or A D, Fig. 8. [m] D E. [n] A E B D A. [o] Fig. 9. [p] Fig. 10.

rhombus contain, the base of which is 1490, and the perpendicular breadth 1280 links?

Problem 2. *To find the area of a Triangle.*

Fig. 4.



RULE. Multiply the base by the altitude, and half the product will be the area.

(1) Required the number of square yards in a triangle, whose base is 49 feet, and height 25¼ feet.

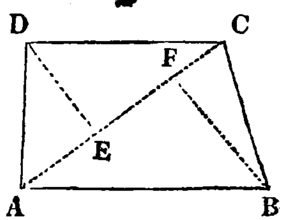
(2) What is the area of the gable of a house, the base or distance between the eaves being 22 feet 5 inches, and the perpendicular from the ridge to the middle of the base, 9 feet 4 inches?

RULE 2. When the three sides only are given.—From half the sum of the sides subtract each side severally; multiply the half sum and the three remainders continually together; and the square root of their product will be the area.

(3) The three sides of a triangular fish-pond are 140, 336, and 415 yards respectively. What is the value of the land which it occupies, at £225 per acre?

Problem 3. *To find the area of a Trapezium, or a Trapezoid.*

Fig. 5.



RULE. Divide the *trapezium* into two triangles by a *diagonal;* multiply the diagonal by the sum of the two perpendiculars falling upon it; and half the product will be the area.

That is, $\frac{DE + BF \times AC}{2}$ = the area.

For a *trapezoid.* Multiply the sum of the two parallel sides by the perpendicular distance between them; and half the product will be the area.

(1) How many square yards of paving are in the trapezium, whose diagonal is 65 feet, and the perpendiculars 28 and 33½ feet?

(2) Find the area of a trapezium whose south side is 2740 links, east side 3575, west side 4105, and north side 3755 links; and the diagonal, from the south-west to the north-east angle, 1835 links.

(3) Required the area of a trapezoid whose parallel sides are $20\frac{1}{2}$ feet and $12\frac{1}{4}$ feet respectively; the perpendicular distance being $10\frac{3}{4}$ feet?

(4) How many square feet are in a board, whose length is $12\frac{1}{2}$ feet, and the breadths of the two ends 15 inches and 11 inches respectively?

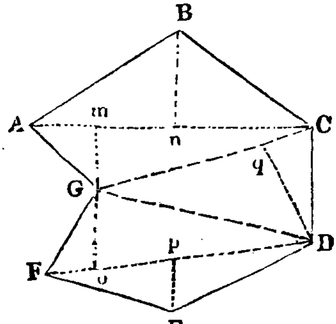
Problem 4. *To find the area of an Irregular Figure.*

RULE. Divide it, by drawing diagonals, into trapeziums and triangles. Find the area of each, and their sum will be the area of the whole.

1. Required the content of the irregular figure ABCDEFGA, in which are given the following diagonals and perpendiculars; namely,

AC = 5·5
FD = 5·2
GC = 4·4
Gm = 1·3
Bn = 1·8
Go = 1·2
Ep = 0·8
Dq = 2·3

Fig. 6.



Problem 5. *To find the area of a Regular Polygon.*

RULE 1. Multiply the perimeter (or sum of the sides) by the perpendicular drawn from the centre to one of the sides; and half the product will be the area.

RULE 2. Multiply the square of the side by the corresponding tabular area, or multiplier opposite to the name in the following table; and the product will be the area.

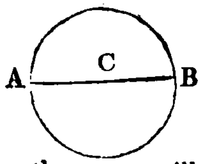
No. of sides.	*Names of Polygons.*	*Areas, or Multipliers.*
3	Trigon, or Equilateral Triangle ..	0·4330127
4	Tetragon, or Square	1·0000000
5	Pentagon	1·7204774
6	Hexagon	2·5980762
7	Heptagon	3 6339124
8	Octagon	4·8284271
9	Nonagon	6·1818242
10	Decagon	7·6942088
11	Undecagon	9·3656399
12	Duodecagon	11·1961524

(1) Required the area of a regular pentagon whose side is 25 feet.

(2) Required the area of an octagon whose side is 20 feet.

Problem 6. *To find the Diameter or Circumference of a Circle, the one from the other.**

Fig. 7.



RULE. $\left.\begin{array}{l}\text{As } 7 : 22\dagger \\ \text{or, as } 113 : 355\ddagger \\ \text{or, as } 1 : 3{\cdot}1416\,\S\end{array}\right\}$: : the diameter : the circumference; and reversing the terms will find the diameter.

(1) Required the circumference of a circle whose diameter is 12.¶

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* To find the proportion which the circumference bears to the diameter, and thence to find the area of a circle, is a problem that has engaged the anxious attention of mathematicians of all ages. It is now generally considered impossible to determine it exactly; but various approximations have been found, some of which have been carried to so great a degree of accuracy, that in a circle as immense in magnitude as the orbit of the planet Saturn, the diameter of which is about 158 millions of miles, we are enabled to express the circumference (the diameter being given) so nearly *approximating to the truth*, as not to deviate from it so much as the breadth of a single hair. The three approximations in the Rule are those in general use.

† This is the ratio assigned by Archimēdes, a celebrated philosopher of Syracuse, who flourished about two centuries before the Christian era. It answers the purpose sufficiently well, when particular accuracy is not required.

‡ This was discovered by Metius, a Dutchman, about two centuries since. It is a very good approximation, agreeing with the truth to the sixth figure.

§ This is an abridgment of the celebrated Van Ceulen's proportion, who was nearly contemporary with Metius. By a patient and most laborious investigation, he determined it truly to 36 places of figures. (3·141598, &c.) But it has been since extended to considerably more than 100 places.—This proportion is extremely convenient, from the circumstance of the first term being *unity*; which saves the labour of division, in finding the circumference of any other circle whose diameter is given. It is not quite so accurate as the preceding.

¶ $$\left.\begin{array}{l}\text{As } 7 : 22 : : 12 : \dfrac{22 \times 12}{7} = \dfrac{264}{7} = 37{\cdot}714285 \\ \text{or, as } 113 : 355 : : 12 : \dfrac{355 \times 12}{113} = 37{\cdot}699115 \\ \text{or, as } 1 : 3{\cdot}1416 : : 12 : 3{\cdot}1416 \times 12 = 37{\cdot}6992\end{array}\right\} \text{the circumference.}$$

(2) What is the circumference when the diameter is 45?

(3) What is the diameter of a column whose circumference is 9 feet 6 inches?

(4) If the circumference of a great circle of the earth (as the equator) were exactly 25000 miles, what would be the diameter?

Problem 7. *To find the area of a Circle.*

RULE 1. The area is equal to a fourth part of the product of the circumference into the diameter; or, the product of half the circumference into half the diameter.

Therefore, when the diameter is 1, the area $= \frac{1 \times 3{\cdot}1416}{4} = {\cdot}7854$; whence we have

RULE 2. Multiply the square of the diameter by ·7854; and the product will be the area.

RULE 3. Multiply the square of the circumference by ·07958 for the area.

(1) Required the area of the circle proposed in Example 1, Problem 6.*

(2) Find the area of the circle proposed in Example 2, Problem 6.

(3) What is the area of the end of a roller whose diameter is 2 feet 3 inches?

(4) Required the area when the circumference is $8\frac{1}{4}$ feet.

Problem 8. *To find the side of a square inscribed in a circle.*

RULE. Multiply the radius by 1·4142 (that is by $\sqrt{2}$), or multiply the diameter by ·7071.†

(1) Find the side of the square inscribed in the circle whose diameter is 12.

(2) What is the side of the square inscribed in a circle whose diameter is 6 feet 5 inches?

---

* $\left.\begin{array}{l}\frac{37{\cdot}6992 \times 12}{4} = 37{\cdot}6992 \times 3 = 113{\cdot}0976 \\ \text{or, } 12^2 \times {\cdot}7854 = 12 \times 12 \times {\cdot}7854 = 113{\cdot}0976\end{array}\right\}$ the area.

† The following Rules exhibit the principal relations between the circle and its equal square, or inscribed square.

$\left.\begin{array}{ll}\text{1. The diameter} & \times {\cdot}8862269 \\ \text{2. The circumfer.} & \times {\cdot}2820948\end{array}\right\} =$ the side of an equal square.

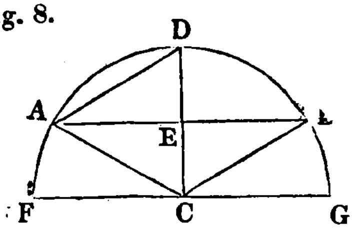
$\left.\begin{array}{ll}\text{3. The diameter} & \times {\cdot}7071068 \\ \text{4. The circumfer.} & \times {\cdot}2250791 \\ \text{5. The area .....} & \times {\cdot}6366197\end{array}\right\} =$ the side of the inscribed square.

Problem 9. *To find the length of a circular arc.*

RULE 1. From 8 times the chord of half the arc subtract the chord of the whole arc, and $\frac{1}{3}$ of the difference will be the length of the arc, *nearly*.

That is, $AD \times 8 - AB \div 3 =$ arc ADB.

Fig. 8.



NOTE. Half the chord of the whole arc, the chord of half the arc, and the versed sine, are sides of a right-angled triangle; any two of which being given, the third may be found as directed in page 117.

RULE 2. Multiply the number of degrees in the arc by the radius, and the product by ·01745, for the length of the arc.

(1) The chord of the whole arc is 30, and the versed sine 8: what is the length of the arc?

(2) What is the length of the arc when the chord of the half arc is 10·625, and its versed sine 5?

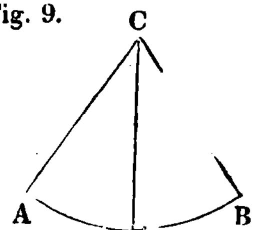
(3) Required the length of an arc of 12° 10′, the radius being 10 feet.

Problem 10. *To find the area of a Sector of a circle.*

RULE 1. Multiply the length of the arc by the radius, and half the product will be the area.

RULE 2. As 360° : the degrees in the arc : : the area of the circle : the area of the sector.

Fig. 9.



(1) Required the area of the sector, when the radius is 15, and the chord of the whole arc 18 feet.

(2) What is the area of a sector whose arc is 147° 29′, and the radius 25?

(3) Required the area of a sector whose radius is 20 feet, and the versed sine 1 foot 9 inches.*

---

6. The side of a square $\times$ 1·414214 = the diameter } of its circum-
7. The side of a square $\times$ 4·442883 = the circumf. } scribing circle.
8. The side of a square $\times$ 1·128379 = the diameter } of an equal cir-
9. The side of a square $\times$ 3·544908 = the circumf. } cle.

* By the properties of the circle, the versed sine $\times$ the remaining part of the diameter = the square of half the chord of the arc; whence all the requisites may be found.

(4) What is the area of the sector, when the chord of half its arc is 14 feet 2 inches, and the versed sine 6 feet 8 inches?*

Problem 11. *To find the area of a circular Segment.*

RULE 1. Find the area of the sector; and also the area of the triangle formed by the chord and the two radii of the sector: their *difference*, when the segment is *less* than a semicircle, or their *sum*, when it is *greater*, will be the area of the segment.

RULE 2. Divide the height of the segment by the diameter, and find the quotient in the column of heights in the following table. Multiply the corresponding area by the square of the diameter, for the area of the segment.†

Table of the Areas of Circular Segments.

Height.	*Area of Segment.*	*Height.*	*Area of Segment.*	*Height.*	*Area of Segment.*	*Height.*	*Area of Segment.*
·01	·00133	·14	·06683	·26	·16226	·39	·28359
·02	·00375	·15	·07387	·27	·17109	·40	·29337
·03	·00687	·16	·08111	·28	·18002	·41	·30319
·04	·01054	·17	·08854	·29	·18905	·42	·31304
·05	·01468	·18	·09613	·30	·19817	·43	·32293
·06	·01924	·19	·10390	·31	·20738	·44	·33284
·07	·02417	·20	[illegible]1182	·32	·21667	·45	·34278
·08	·02944	·21	·11990	·33	·22603	·46	·35274
·09	·03501	·22	·12811	·34	·23547	·47	·36272
·10	·04088	·23	·13647	·35	·24498	·48	·37270
·11	·04701	·24	·14494	·36	·25455	·49	·38270
·12	·05339	·25	·15355	·37	·26418	·50	·39270
·13	·06000			·38	·27386		

(1) What is the area of a segment, when the chord of the whole arc is 60, and the chord of half the arc $37\frac{1}{2}$?

* When the half chord (see AE, Fig. 7) of the arc is found by the properties of a right-angled triangle, then $AE^2 =$ the versed sine (DE) $\times$ the remaining part of the diameter; whence the diameter (and consequently the radius) will be known.

† When there is a remainder (or fraction) after the second quotient figure, in dividing the height by the diameter; having taken out the area answering to the two figures, add to it such *fractional part* of the *difference* between *that* and the *next succeeding* area, for the sake of greater accuracy.

(2) What is the area of a segment whose height is 18, and the diameter of the circle 48?

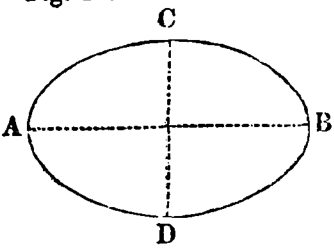
(3) Required the area of a circular segment whose height is 2, and chord 20?

(4) What is the area of the segment of a circle whose radius is 24, the chord of the whole arc 20, and the chord of half the arc 10·2?

(5) If the radius of a circle is 10 feet, what is the area of the segment whose chord is 12 feet?

Problem 12. *To find the circumference of an Ellipse, the transverse and conjugate diameters being given.*

Fig. 10.



Rule. Multiply the square root of half the sum of the squares of the two diameters by 3·1416, and the product will be the circumference nearly.*

(1) What is the circumference of an ellipse whose transverse diameter is 24, and conjugate 18?

(2) The two axes of an ellipse are 60 and 45 yards respectively: what is the circumference?

Problem 13. *To find the area of an Ellipse.*

Rule. Multiply the product of the axes by ·7854, for the area.

(1) Required the area of an ellipse whose axes are 35 and 25

(2) What will be the expense of trenching an elliptic garden, whose axes are 70 and 50 feet, at 3¼ per square yard?

(3) Required the area of the ellipse in Grosvenor Square, London; the transverse diameter being 8·40 chains, and the conjugate 6·12 chains.

Problem 14. *To find the area of an Elliptic Segment, the base being parallel to either axis.*

Rule. Divide the height of the segment by that axe of which it is a part, and find, in the *Table of Circular Segments*, a versed sine equal to the quotient.

---

* If the half sum of the two diameters be multiplied by 3·1416, the product will give the circumference *sufficiently near* for most practical purposes.

Multiply the corresponding *tabular area* and the two axes continually together, and the product will be the area required.

(1) What is the area of an elliptic segment cut off by a line (called a double ordinate) parallel at the conjugate diameter, at the distance of 36 yards from the centre; the axes being 120 and 40 yards respectively?

(2) Required the number of square yards in the segment of an ellipse, cut off by an ordinate parallel to the transverse diameter; the height being 5 feet, and the two axes 35 and 25 feet respectively.

A COLLECTION OF QUESTIONS.

(1) What is the value of 14 barrels of soap, at $4\frac{1}{2}$*d.* per *lb.* each barrel containing 254 *lb.*? *Ans.* £66..13..6.

(2) A and B joined in partnership; A put into the joint stock £320 for 5 months, and B £460 for 3 months: they gained £100. What is each man's share of the gain?
Ans. A's, £53..13..$9\frac{270}{298}$; and *B's*, £46..6..$2\frac{28}{298}$.

(3) How many yards of cloth, at 17*s.* 6*d.* per yard, can I have for 13 *cwt.* 2 *qrs.* of wool, at 14*d.* per *lb.*?
Ans. 100 *yards*, $3\frac{1}{5}$ *qrs.*

(4) If I buy 1000 ells of Flemish linen for £90, at what price must I sell it per English ell, to gain £10 by the whole? *Ans.* 3*s.* 4*d.* per ell.

(5) A has 648 yards of cloth, at 14*s.* per yard, ready money, but in barter will have 16*s.* B has wine at £42 per tun, ready money: what must he charge it per tun in barter, and what quantity must be given in exchange for the cloth?
Ans. £48 *per tun, and the quantity,* 10 *tuns,* 3 *hhds.* $12\frac{3}{5}$ *gals.*

(6) A jeweller sold jewels to the value of £1200, for which he has received in part 876 French pistoles, at 16*s.* 6*d.* each. How much more is due to him? *Ans.* £477..6.

(7) An oilman bought 417 *cwt.* 1 *qr.* 15 *lb.* gross weight of train oil, tare 20 *lb.* per *cwt.*: how many neat gallons were there, allowing $7\frac{1}{2}$ *lb.* to a gallon? *Ans.* 5120 *gallons.*

(8) If I buy cloth at 14*s.* 6*d.* per yard, and sell it at 16*s.* 9*d.* what is the gain per cent? *Ans.* £15..10..$4\frac{4}{29}$.

(9) Bought 27 bags of ginger, each weighing gross $84\frac{3}{4}$ *lb* tare $1\frac{3}{8}$ *lb.* per bag, tret as usual: what is the value at $8\frac{1}{2}$*d* per *lb.*? *Ans.* £76..13..$2\frac{1}{2}$.

(10) If $\frac{2}{3}$ *oz.* cost $\frac{7}{8}$*s.* what will $\frac{5}{6}$ *lb.* cost? *Ans.* 17*s.* 6*d.*

(11) If $\frac{5}{6}$ of a gallon cost $\frac{3}{8}$ of a £, what will $\frac{5}{9}$ of a tun cost? *Ans.* £105.

(12) A gentleman who spends, one day with another, £1..7..10$\frac{1}{2}$, lays up at the year's end £340. What is his annual income? *Ans.* £848..14..4$\frac{1}{2}$.

(13) What is the difference, in ounces, between 13 fothers of lead, and 39 boxes of tin, each box weighing 388 *lb?*
Ans. 212160 *ounces.*

(14) A captain, commanding a crew of 160 mariners, captured a prize worth £1360. The captain was allowed one-fifth, and the rest was equally divided among the sailors. What was each man's share?
Ans. The captain had £272; *and each sailor,* £6..16.

(15) At what rate per cent will £956 amount to £1314..10 in 7$\frac{1}{2}$ years, at simple interest? *Ans.* £5 *per cent.*

(16) A has 24 cows, worth £3..12 each, and B 7 horses worth £13 each. How much will make good the difference, in case they interchange their droves of cattle?
Ans. £4..12.

(17) A man left £120 to be given to three persons, A, B, and C; B to have twice as much as A, and C as much as A and B: what was the share of each?
Ans. A, £20; *B,* £40; *and C,* £60.

(18) £1000 is to be divided among three men, in such a manner, that if A has £3, B shall have £5, and C £8. How much will each man have?
Ans. A, £187..10; *B,* £312..10; *and C,* £500.

(19) A piece of wainscot is 8 feet 6$\frac{1}{2}$ inches long, and 2 feet 9$\frac{3}{4}$ inches broad. What is the superficial content?
Ans. 24 *feet* 0′ 3″ 4‴ 6⁗.

(20) A garrison of 360 men, who had originally six months provisions, having endured a siege of 5 months, without obtaining any relief or fresh supply, wish to know how many men must depart, that the provisions may suffice for the residue 5 months longer? *Ans.* 288 *men.*

(21) The less of two numbers is 187; the difference 34. The square of their product is required. *Ans.* 1707920929.

(22) A butcher sent his man with £216 to a fair to buy cattle; he bought oxen at £11, cows at 40*s.* colts at £1..5, and hogs at £1..15 each, and of each a like number. What was the number of each? *Ans.* 13 *of each sort, and* £8 *over.*

(23) What number added to 11$\frac{4}{7}$ will produce 36$\frac{397}{816}$?
Ans. 24$\frac{613}{816}$.

(24) What number multiplied by $\frac{3}{7}$ will produce $11\frac{8}{17}$?
Ans. $26\frac{46}{51}$.

(25) What is the value of 179 hogsheads of tobacco, each weighing 13 *cwt.* at £2..7..1 per *cwt.*? *Ans.* £5478..2..11.

(26) My factor informs me that he has bought goods on my account, of the value of £500..13..6. What will his commission come to at £$3\frac{1}{2}$ per cent? *Ans.* £17..10..5..$2\frac{17}{53}$ *qrs.*

(27) If $\frac{1}{3}$ of 6 were three, what would $\frac{1}{4}$ of 20 be?
Ans. $7\frac{1}{2}$.

(28) Reduce 3 *qrs.* 14 *lb.* to the decimal of a *cwt.*
Ans. ·875 *cwt.*

(29) How many *lb.* of sugar, at $4\frac{1}{2}$*d.* per *lb.* must be given in barter for 60 gross of inkle, at 8*s.* 8*d.* per gross?
Ans. $1386\frac{2}{3}$ *lb.*

(30) If I buy yarn for 9*d.* per *lb.* and sell it again for $13\frac{1}{2}$*d.* per *lb.* what is the gain per cent? *Ans.* £50.

(31) A tobacconist mixes 20 *lb.* of tobacco, at 9*d.* per *lb.* with 60 *lb.* at 12*d.* per *lb.*; 40 *lb.* at 18*d.* per *lb.*; and 12 *lb.* at 2*s.* per *lb.* What is a pound of the mixture worth?
Ans. 1*s.* $2\frac{1}{4}$*d.* $\frac{9}{11}$.

(32) What is the difference between twice eight and twenty, and twice twenty-eight; also between twice five and fifty, and twice fifty-five? *Ans.* 20 *and* 50.

(33) Whereas a noble and a mark just 15 yards did buy;
How many ells of the same cloth for £50 had I?
Ans. 600 *ells.*

(34) A broker bought for his principal, in the year 1720, £400 South Sea stock, at £650 per cent, and sold it again when it was worth but £130 per cent. What was the whole loss? *Ans.* £2080.

(35) C has candles at 6*s.* per dozen ready money, but in barter will have 6*s.* 6*d.* per dozen; D has cotton at 9*d.* per *lb.* ready money. What price must the cotton be charged in barter, and how much must be exchanged for 100 dozen of candles. *Ans. The cotton at* $9\frac{3}{4}$*d. per lb. and the quantity,* 7 *cwt.* 0 *qrs.* 16 *lb.*

(36) If a clerk's salary is £73 a year, what is that per day? *Ans.* 4*s.*

(37) B has an estate of £53 per annum, and pays 5*s.* 10*d.* to the subsidy. What must C pay, whose estate is worth £100 per annum? *Ans.* 11*s.* $0\frac{4}{53}$*d.*

(38) If I buy 100 yards of riband, at 3 yards for a shilling, and 100 yards more at 2 yards for a shilling, and sell the

whole at the rate of 5 yards for 2 shillings; whether do I gain or lose, and how much? *Ans. Lose* 3*s.* 4*d.*

(39) From what number must $\frac{3}{5}$ be deducted, that the remainder may be $\frac{1}{8}$? *Ans.* $\frac{29}{40}$.

(40) A farmer wishes to mix rye, at 4*s.* a bushel; barley, at 3*s.*; and oats, at 2*s.* How much must he take of each to sell the mixture at 2*s.* 6*d.* per bushel?

Ans. 6 *of rye,* 6 *of barley, and* 24 *of oats.*

(41) If $\frac{3}{8}$ of a ship is worth £3740, what is the value of the whole? *Ans.* £9973..6..8.

(42) Bought a cask of wine for £62..8, at 5*s.* 4*d.* per gallon. How many gallons were there? *Ans.* 234.

(43) A dissipated young fellow in a short time got through $\frac{1}{5}$ of his fortune; he then gave £2200 for a commission in the army: his profusion continued till he had no more than 880 guineas left; which was $\frac{3}{20}$ of his money after the commission was bought. What was his fortune at last? *Ans.* £10450.

(44) A sum of money is to be divided amongst four men, so that the first shall have $\frac{1}{3}$, the second $\frac{1}{4}$, the third $\frac{1}{6}$, and the fourth the remainder, which is £28. What is the sum?

Ans. £112.

(45) What is the amount of £1000 in $5\frac{1}{2}$ years, at £$4\frac{3}{4}$ per cent per annum, simple interest? *Ans.* £1261..5.

(46) Sold goods amounting to the value of £700, at two 4 months. What is the present worth, at £5 per cent per annum, simple interest? *Ans.* £682..19..$5\frac{1}{4}$ $\frac{177}{1891}$.

(47) A room 30 feet long, and 18 feet wide, is to be covered with painted cloth. How many yards of three-quarters wide will cover it? *Ans.* 80 *yards.*

(48) Betty told her brother George, that though her marriage portion took £19312 out of her family, it was but $\frac{3}{5}$ of two years' rent. What was his annual income?

Ans. £16093..6..8.

(49) A gentleman having 50*s.* to pay among his labourers for a day's work, gave to every boy 6*d.*; to every woman 8*d.*; and to every man 16*d.* There was an equal number of each description. What was that number? *Ans.* 20 *of each.*

(50) What is the solid content of a stone that measures 4 feet 6 inches long, 2 feet 9 inches broad, and 3 feet 4 inches deep? *Ans.* $41\frac{1}{4}$ *solid feet.*

(51) What does the pay of a ship's crew, consisting of 640 sailors, amount to for 32 months' service, each man's pay being 22*s.* 6*d.* per month? *Ans.* £23040.

(52) A traveller would change 500 French crowns, at 4*s.* 6*d.* per crown, into sterling money; but he must pay a halfpenny per crown for change. How much sterling money will he receive? *Ans.* £111..9..2.

(53) B and C traded together, and gained £100; B put in £640; C put in so much that he was entitled to £60 of the gain. What was C's stock? *Ans.* £960.

(54) From what principal sum did £20 interest arise in one year, at the rate of £5 per cent per annum? *Ans.* £400.

(55) How many French pistoles, at 17*s.* 6*d.* each, are equal to 672 Spanish guilders, at 2*s.* each? *Ans.* $76\frac{4}{5}$.

(56) Out of 7 cheeses, each weighing 1 *cwt.* 2 *qrs.* 5 *lb.* how many allowances for seamen may be cut, each weighing 5 *oz.* 7 *drams*? *Ans.* $3563\frac{35}{87}$.

(57) If 48 taken from 120 leaves 72, and 72 taken from 91 leaves 19, and 7 taken from thence leaves 12, what number is that, out of which, when you have taken 48, 72, 19, and 7, leaves 12? *Ans.* 158.

(58) A farmer, unskilled in numbers, ordered £500 to be divided among his 5 sons, thus: "Give A," says he, "$\frac{1}{3}$; B, $\frac{1}{4}$; C, $\frac{1}{5}$; D, $\frac{1}{6}$; and E, $\frac{1}{7}$ part." Divide this equitably among them, according to the father's intention.

Ans. *A*, £152..10..$1\frac{1}{4}\frac{105}{459}$; *B*, £114..7..$6\frac{3}{4}\frac{423}{459}$; *C*, £91..10..$0\frac{3}{4}\frac{63}{459}$; *D*, £76..5..$0\frac{1}{2}\frac{282}{459}$; *E*, £65..7..$2\frac{1}{4}\frac{45}{459}$.

(59) When first the marriage knot was tied
Between my wife and me,
My age did hers as far exceed,
As three times three does three;
But when ten years, and half ten years,
We man and wife had been,
Her age came then as near to mine,
As eight is to sixteen.

Quest. What was each of our ages when we married?
Ans. 45 *years the man,* 15 *the woman.*

SUPPLEMENTAL QUESTIONS.

(1) How many gallons of the imperial standard measure are respectively equal to a hogshead of wine, and a hogshead of ale, old measure; and what was the difference between the two hogsheads in cubic inches?

(2) What quantity of the old ale measure would correspond to 54 gallons of the imperial standard?

(3) How many gallons of the old wine measure are equal in quantity to 63 gallons, imperial measure?

(4) Reduce 15 quarters. 3 bushels, 1 peck, old measure, to its equivalent in the imperial standard measure.

(5) A lady who was asked the time of the day, said that it was between *three* and *four*: but being desired to name the *exact* time, she replied, "The minute hand is advanced half an hour precisely before the hour hand." Required the exact time.

(6) If 7 men can build a wall 40 yards long, 4 feet high, and 2 feet thick, in 32 days; how many men will build a wall 240 yards long, 6 feet high, and 3 feet thick, in 8 days?*

(7) The weight of a certain bar of iron, 2 feet long, 3 inches broad, and 1 inch thick, is 20 *lbs*. What is the weight of a bar of similar quality which is $7\frac{1}{4}$ feet long, $4\frac{1}{2}$ inches broad, and $3\frac{1}{2}$ inches thick?

(8) A person who had five-ninths of a mine, made his younger brother a present of half his share, and sold half the remainder to his cousin John, who soon after purchased $\frac{1}{3}$ of the younger brother's share; but now offers to dispose of half his interest in the mine for £150. Estimating at the same rate, what is the value of the whole mine, and of each brother's share?

(9) A, travelling from London to Manchester, and B, from Manchester to London, set out at the same time. They meet at the end of six days, A having travelled 3 miles a day

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* Questions of *Compound Proportion*, in which the terms are numerous, may be solved by Rule 4, for the Double Rule of Three; but the following method is more convenient.

RULE. Arrange the terms of the *first cause* and *effect* in one line, and the corresponding terms of the *second cause* and *effect* exactly under them; supplying the place of the term sought with an *asterisk*, and connecting the *contrary* causes and effects by cross lines. Multiply continually the terms of *each cause* and the *other effect*: divide the product arising from the *full* number of terms, by the product of those with which the *blank* term is connected, and the quotient will be the answer.

Solution of the above example.

$$\begin{array}{ccccccccc} \textit{men} & & \textit{days} & \textit{build} & \textit{y. long} & & \textit{ft. h.} & & \textit{ft. th.} \\ \text{If } 7 & : & 32 & \times & 40 & : & 4 & : & 2 \\ * & : & 8 & & 240 & : & 6 & : & 3 \\ & & & \textit{a wall.} & & & & & \end{array} \Bigg\} : \frac{7 \times \overset{\overset{1}{\cancel{4}}}{\cancel{32}} \times \overset{6}{\cancel{240}} \times \overset{3}{\cancel{6}} \times 3}{\underset{1}{\cancel{8}} \times \underset{1}{\cancel{40}} \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{2}}} = 378 \textit{ men. Ans.}$$

more than B. At what rate did each go, the distance being 186 miles?

(10) A coach which runs the whole distance in 31 hours, starts from London at the same time that another which does it in 21 hours, starts from Manchester. Required the number of hours elapsed, and the distance travelled by each when they meet.

(11) A load of corn was sold for £15, at a *loss* of 15 per cent. What should it have been sold for to *gain* as much per cent as the corn cost?

(12) Two men purchased a grinding stone, 42 inches in diameter, for a guinea; of which the first paid twelve shillings. They agree that the first shall use it till his share is worn down. What will be the diameter when the second receives it?

(13) If A and B together do a piece of work in $7\frac{1}{4}$ days, which A alone would accomplish in $12\frac{1}{2}$ days; in what time would B do it himself?

(14) A person lent £400, and agreed to receive in return a yearly payment of £50, for 13 years. Whether would he gain or lose thereby, reckoning Compound Interest at £5 per cent per annum?

(15) By selling a horse for £50, I gained one-fourth of what he cost me; but the whole cost (including the expense of his keep) was one-fourth more than the original purchase. How much did I give for him, what did I expend in keeping him, and what did I gain per cent?

(16) It has been found by experiment, that sound is conveyed through the air at the rate of 1142 feet in a second. How far distant is the cloud, when $7\frac{1}{4}$ seconds elapse between seeing the flash of lightning and hearing the thunder?

(17) What is the height of a tower that projects a shadow 75·75 yards long, at the same time that a perpendicular staff 3 feet high, gives a shade of 4·55 feet in length?

(18) A bankrupt owes £2580, and the value of his effects is £846, and the amount of recoverable debts £358..12, besides which he has an unexpired lease that has 13 years to run, valued at £12 a year more than the stipulated rent. If the lease be disposed of for present money, allowing Compound Interest at £5 per cent per annum, and if the working of the commission and other expenses amount to £472; what will his creditors have in the pound, provided they allow him £150 to recommence business?

(19) A youth aged 12 years, having had bequeathed to him

an annuity of £50 for 12 years, to commence when he comes of age; the executors think it will be more advantageous to exchange this for an annuity to commence immediately, and continue till he is 21; to enable them to give him some education and a trade. What will be such annuity, £100 being reserved at the conclusion to set him up in business?

(20) There is an island 73 miles in circumference; to travel round which three pedestrians all start at the same time: A travels 5 miles a day, B travels 8, and C 10 miles a day. In how many days will they all come together again, and how many circuits will each have made?

(21) What will a banker charge for discounting a bill of £52..10 on the 7th of April; the bill being due on the 19th of May?

(22) My agent in London having advised me, that he has purchased goods on my account to the amount of £756..10, at 6 months credit, or £7½ per cent discount for prompt payment; if I send a remittance of £400, to be paid down on account, after deducting out of it his charge for commission at £2¼ per cent; what will remain to be discharged at the end of 6 months?*

(23) If I insure a house for £250, at the annual charge of 1*s.* 6*d.* per cent, and the furniture, &c. for £150, at the rate of 4*s.* 6*d.* per cent; what shall I have to pay yearly to the Insurance office, including the duty paid to Government of ⅛, or 2*s.* 6*d.* per cent?

(24) If 12 oxen will eat 3⅓ acres of grass in 4 weeks, and 21 oxen will eat 10 acres in 9 weeks; how many oxen will eat 24 acres in 18 weeks, allowing the grass to grow uniformly? *Newton.*

(25) A bath is supplied with water by two cocks; from one of which it may be filled in 40 minutes, and from the other in 50 minutes: a discharging cock will empty it (when filled) in 25 minutes. If all the three be opened at the same time, in what time will the bath be filled, supposing the influx and efflux to be uniform?

(26) A person who had spent two-thirds of his money at one place, and half the remainder at another, found that he had £32..12 left. How much had he at first?

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* See Note to Example 11, Discount, page 73.

(27) The length, breadth, and height of a room are 9, 6, and 4 yards respectively. What is the longest right line that can be taken within that room?

(28) What must be the length of a cord with which a horse may be tethered to a certain point in a straight fence, so as to allow him the liberty of grazing to the extent of 1 rood; supposing that he can reach 2 yards beyond the tether?

(29) A person playing at cards, lost three nights successively. The first night he lost half his sovereigns, and half a sovereign besides; the second night he lost half the remainder, and half a sovereign more; and the third night, half the remainder, and half a sovereign more, which reduced his stock to twenty. How many sovereigns had he at first?

(30) The month of July, 1828, was remarkable, both in England, and several parts of the Continent, for excessive rains. At Derby, the quantity collected in the *pluviameter* (or rain-guage) between the hours of *nine*, A. M. of the 9th of that month, and *six* the following morning (an interval of 21 hours), was 3·69 inches: to the evening of the 15th, it amounted to $7\frac{1}{2}$ inches; and by the conclusion of the 29th, an interval of 21 days, of which 10 only were very rainy, the total depth of water collected was $11\frac{1}{3}$ inches. How many hogsheads of 54 and 63 imperial gallons respectively, fall on an acre of ground to amount to the depth of *one inch:* and how many hogsheads of each kind fell on the surface of an acre during each of the three several intervals above-mentioned?

(31) A person who occupies a piece of ground for which he pays a rent of £10 per annum, wishes to take it upon a lease for forty years, with the obligation of laying out upon it during the present year £600, in the erection of a building, which is to be left in good tenantable condition at the termination of the lease. The question is, how much will be a fair annual rent for the lessee to pay, during the term of continuance of such tenure, admitting the ground rent paid at present to be a fair one; and supposing the customary interest of money to be at the rate of £5 per cent per annum? Also, supposing interest to be at £4 per cent per annum?

(32) What will be the expense of covering and guttering a roof with lead, at 18*s.* per *cwt.*; the length of the roof being 43 feet, and the breadth or girt over it 32 feet; the guttering 57 feet long, and 2 feet wide: the lead for the former being 9·831 *lb.* and for the latter 7·373 *lb.* to the square foot?

A

COMPENDIUM OF

BOOK-KEEPING,

BY SINGLE ENTRY;

INTENDED FOR THE PURPOSE OF INITIATING YOUTH IN THE LEADING PRINCIPLES OF THAT IMPORTANT BRANCH OF SCIENCE.

BOOK-KEEPING is the art of recording pecuniary or commercial transactions in a regular and systematic manner.

The science of Book-keeping admits of innumerable varieties of method; but its general principles are invariable. These being well understood, the knowledge of any particular system, adapted to the peculiar concerns of any counting-house, will be easily acquired.

Single Entry, being the most simple and concise, is the method usually adopted in retail business.

The General Rule to be observed in every system of Book-keeping, is,

To make any person Debtor (Dr.) *for money or goods which* he receives *from me, and to make him* Creditor (Cr.) *for whatever* I receive *from him.*

The books usually kept in *Single Entry*, are the Day-Book, the Cash-Book, the Ledger, and the Bill-Book.

The Day-Book, when a person commences business, begins with an inventory of the existing state of his affairs: after which are entered, in the regular order of time, the daily transactions of Goods bought and sold.

The Cash-Book contains the particulars of all Money transactions. It is ruled in a folio form: on the left-hand page, *Cash is debited to all sums received*, and on the right, *Cash is credited by all sums paid.* The Balance (or quantity which the *Dr.* side exceeds the *Cr.*) shows the amount of *Cash in hand.* This should be ascertained weekly, and in some concerns daily, in order to prove if it corresponds with the *real Cash in hand.*

In the Ledger are collected the dispersed accounts of each person from the Day Book and Cash-Book, and entered in a concise manner in one folio; the sums in which he is *Dr.* being arranged on the left-hand, and those in which he is *Cr.* on the right-hand page of the folio: so that the *Balance* of his account (the difference between the *Dr.* and *Cr.* sides) may always be easily ascertained by inspection. The transferring of accounts from the Day-Book and Cash-Book to the Ledger, is called *posting.*

In many trades, it is found convenient to keep the account of Goods Sold at the former end, and those of Goods Bought at the latter end of the Ledger. But in concerns of magnitude, two Ledgers are more convenient; one for Goods Sold, and the other, called the "*Bought Ledger.*" for Goods Bought.

In the BILL-BOOK are copied the particulars of all *Bills of Exchange*, whether *Receivable* or *Payable* The former are those which come into the Tradesman's possession, and are drawn upon some other person; the latter are those which are drawn upon and accepted by him.—Printed Bill-Books may be had of any Bookseller.

Note.—In the following transactions, Bills Receivable are considered as Cash; but many Accountants do not enter them as such, till Cash has been actually received for them.

MEMORANDUMS

OF THE TRANSACTIONS STATED IN THE FOLLOWING BOOKS.

Jan. 5. Received from Allen, Wild, and Co. of Leeds, *on credit*, 2 pieces of super blue cloth, each 36 *yds.* at 25*s.* 6*d.* *per yard*;—and 2 pieces of narrow brown, 84 *yds.* at 4*s.* 9*d.*

8. Sold Bernard Mason 2 *st.* raw sugar, at 9½*d.* *per lb.*—3¼ *lb.* green tea, at 8*s.* 6*d.*—and 3¾ *yds.* blue cloth, at 28*s.*

9. Bought of Samuel Fletcher 1 *cwt.* 1 *qr.* 5 *lb.* Kent hops, at £5..7 *per cwt.*—and 1½ *cwt.* of Worcester, £5..11..6;—*six months' credit*, or 5 *per cent discount for cash.*—Paid him *Bill*, No. 1, £24..3..9, and received from him a *cheque* on Smith and Co. Bankers, for the *difference*, including *discount*.

10. Bought of Simmonds and Co. Liverpool, 2 *cwt.* yellow soap, at 76*s* 12 *doz.* dip candles, at 8*s.* 6*d.*—and 4 *doz.* mould do. at 11*s.* 3*d.*

14. Sold William Tomlinson 7 *yds.* narrow cloth, at 5*s.* 3*d.* and 15 *yds.* calico, at 8½*d.*

19. Sold Hazard and Jones 1½ *st.* yellow soap, at 9*d.* per *lb.*—½ *st.* mottled, at 9½*d.*—9 *lb.* candles, at 9*d.*—and 3 *lb.* mould do. at 12½*d.*
Paid *Cash* for *Bill*, No. 1, W. Holmes, £45..10.

23. Received from J. Sanderson, goods as *per invoice*, £7..3..6.

28. Sold Hazard and Jones 17½ *lb.* loaf sugar, at 1*s.* 1*d.*—12 *lb.* raw do. at 10*d.*—1¼ *lb.* Congou tea, at 7*s.* 6*d.*—and ½ *lb.* Hyson, at 12*s.*

31. Paid the *balance* due to J. Herdson, *cash* £37..5..6, *abatement* 4*d.*
Sold him on credit 10 *lb.* hops, at 13*d.*—and half a ream of cap paper, at 7*d.* *per quire.*

Feb. 1. Sold W. Tomlinson 2 *st.* yellow soap, at 9*d.* *per lb.*—6 *lb.* mould candles, at 1*s.* 1*d.*—and 16 *lb.* lump sugar, at 12½*d.* Received of him *cash* for account due, including the present goods, £4..13, *abatement* 3½*d.*
Paid Bernard Mason the *balance* due, *cash* £832; *abatement* 5*s.* 2½*d.*

4. Received of James Taylor half year's interest on £70, due this day, £1..15.

5. Sold W. Tomlinson 1 piece super blue cloth, 36 *yds.* at 27*s.* 6*d.*—for *Bill at one month.*

8. W. Tomlinson paid me a *Bill* on Jones and Co. London, due May 10, £60, which should have been *at one month.*—Charged him *discount*, 10*s.*—Paid, agreeably to his order, the *difference* to J. Sims, in *cash*, £10.

10. Bought of J. Sanderson, cheese 25 *cwt.* 3 *qrs.* 17 *lb.* at £3..2..6 *per cwt.*—Paid his whole account in *cash*, £88, *abatement* 1*s.* 8¾*d.*
Accepted Allen, Wild, and Co.'s *Bill at two months*, drawn Jan. 3, £80.

12. Received of Hazard and Jones's assignee, *cash* £2..7..2¼, for *composition* on £3..15..6, at 12*s.* 6*d.* in the pound.

DAY-BOOK. (page 1)

Folio of Ledger.*	Inventory, January 1st, 1830.	£	s.	d.
	I have in *Ready Money*	1500	0	0
	Bills Receivable, No. 1, on S. Johnson, due 29th instant	24	3	9
	cwt. qr. lb. Tea, 3 chests, gross *wt.* 3 1 17 tare 0 2 7 Duty included on 2 3 10 at 6*s.* 2*d.* per *lb.*	98	1	0
	cwt. qr. lb. Raw Sugar, 2 hhds. weighing 27 3 18 neat, at £3..14..8 per *cwt.*	104	4	0
1	*James Taylor* owes me on bond, dated Aug. 4th, 1826, with interest at £5 per cent per annum	70	0	0
1		1796	8	9
	I owe as follows:			
1	*John Herdson*, a balance of accounts	37	5	10
1	*Bernard Mason*, for purchase of my house by auction, to be paid 1st Feb. next .. £800 Duty on do. at £5 per cent. 40	840	0	0
2	*Bills payable*, viz. No. 1, W. Holmes's bill, to H. Williams or order, accepted by me, due 19th instant	45	10	0
1		922	15	10
	5.			
	Allen, Wild, and Co. Leeds *Cr.*			
	†By 3 pieces superfine blue cloth, each 36 *yds.* at 25*s.* 6*d.* per *yd.*	137	14	0
	2 pieces narrow brown, 84 *yds.* at 4*s.* 9*d.*	19	19	0
	Wrappers	0	5	6
2		157	18	6
	8.			
	Bernard Mason *Dr.*			
	‡To 2 *st.* raw sugar, at 9½ per *lb.*	1	2	2
	3¼ *lb.* green tea, at 8*s.* 6*d.*	1	7	7½
	3¾ *yds.* blue cloth, at 28*s.*	5	5	0
1		7	14	9½

* This column contains the figures of reference to the folio of the Ledger in which any account is posted. They should be written in red ink.

† The preposition *By* is always put before any article for which a person is *credited*.

‡ The preposition *To* is generally placed before articles for which a person is *debited*: but some Book-keepers omit it.

		£	s.	d.
	1830, Jan. 9.			
	Samuel Fletcher Cr.			
	cwt. qr. lb. £ *s. d.*			
	By Kent hops ... 1 1 5 at 5 7 0	6	18	6
	Worcester do. 1 2 0 at 5 11 6	8	7	3
	Six months' credit, or £5 per cent discount			
2	for present payment	15	5	9
	10.			
	Simmonds and Co. Liverpool Cr.			
	By yellow soap, 2 *cwt.* at 76*s.*	7	12	0
	12 *doz.* candles ... at 8*s.* 6*d.*	5	2	0
	4 *doz.* mould do. . at 11*s.* 3*d.*	2	5	0
2		14	19	0
	14.			
	William Tomlinson Dr.			
	To narrow cloth, 7 *yds.* at 5*s.* 6*d.*	1	18	6
	calico, ... 15 *yds.* at 8½*d.*	0	10	7½
2		2	9	1½
	19.			
	Hazard and Jones Dr.			
	To 1½ *st.* yellow soap, at 9*d.* per *lb.*	0	15	9
	½ *st.* mottled do. at 9½*d.*	0	5	6½
	9 *lb.* candles, ... at 9*d.*	0	6	9
	3 *lb.* moulds, ... at 1*s.* 0½*d.*	0	3	1½
3		1	11	2
	23.			
	James Sanderson Cr.			
3	By Goods as per Invoice*	7	3	6
	28.			
	Hazard and Jones Dr.			
	To 17½ *lb.* loaf sugar, . at 1*s.* 1*d.*	0	18	11½
	12 *lb.* raw do. ... at 10*d.*	0	10	0
	1¼ *lb.* Congou tea, at 7*s.* 6*d.*	0	9	4½
	½ *lb.* Hyson, ... at 12*s.*	0	6	0
3		2	4	4
	31.			
	John Herdson Dr.			
	To hops, 10 *lb.* at 1*s.* 1*d.*	0	10	10
	½ ream cap paper, at 7*d.* per quire	0	5	10
1		0	16	8
	Feb. 1.			
	William Tomlinson Dr.			
	To 2 *st.* yellow soap, .. at 9*d.* per *lb.*	1	1	0
	6 *lb.* mould candles, at 1*s.* 1*d.*	0	6	6
	16 *lb.* lump sugar, .. at 1*s.* 0½*d.*	0	16	8
2		2	4	2

* Many tradesmen keep an *Invoice Book*, into which all invoices of goods purchased are regularly transcribed. Some paste the invoices themselves within a blank book. The account of Aiken, Wild,

		£	s.	d.
	1830, Feb. 4.			
	James Taylor *Dr.*			
	To half a year's interest on £70, at £5 per			
1	cent per annum	1	15	0
	5.			
	William Tomlinson *Dr.*			
	To 1 *piece* superfine blue cloth, 36 *yds.* at			
2	27*s.* 6*d.*	49	10	0
	For bill at one month.			
	10.			
	James Sanderson *Cr.*			
	cwt. *qr.* *lb.* £ *s.* *d.*			
3	By cheese,* 25 3 17 at 3 2 6 per *cwt.*	80	18	2¾
	10.			
	Allen, Wild, and Co. Leeds *Dr.*			
	To my acceptance of their Bill at 2 mon.			
2	drawn 3d Jan. *B. P. Book*, No. 2.	80	0	0
	12.			
	Oats' Purveyance, in Partnership with			
	J. Herdson † *Dr.*			
	To Cash, for Oats purchased by me ‡	26	11	0
	Do. do. by J. Herdson	449	0	3
	Do. for warehouse room, &c.	1	13	8
3		477	4	11
	Cr.			
	By Cash received for Oats sold		9	2
	Do. do. by J. H.	55	2	8
3		562	11	10
	£ *s.* *d.*			
	To Profit, ½ (42..13..5½) being my share			
3	J. H. ½ (42..13..5½) being his share	85	6	11
	12.			
	John Herdson *Dr.*			
	To Cash advanced to him on Oats' concern	433	17	0
	Oats sold and received for by him	55	2	8
1		488	19	8
	Cr.			
	By Oats purchased by him	449	0	3
	his share of profit	42	13	5½
1		491	13	8½

and Co. (D. B. p. 1.) is supposed to be a copy from their invoice. Some make the *Invoice-Book* serve also for the "*Bought Ledger*." This last method, when the nature of the trade will admit, is well worth adopting.

* This is supposed to be bought by the long *cwt.* (120 *lb.*) agreeably to the general practice.

† Vide Example of a Partnership Concern, p. 189.

‡ In this concern, the entries of Cash are not made in the Cash-Book; this being only a general statement of transactions supposed to have taken place before the opening of these Books.

1830	CASH *Dr.*		£	s.	d.
Jan. 1	To Stock	1	1500	0	0
	Bills Receivable, No. 1, on S. Johnson	1	24	3	9
9	S. Fletcher, cheque on Smith and Co.	2	9	13	3
			1533	17	0
	To Balance		1426	17	9
Feb. 1	W. Tomlinson (abatement 3½*d.*)	2	4	13	0
4	J. Taylor, ½ year's int. on £70	1	1	15	0
8	W. Tomlinson, Bill on Jones and Co. London, No. 2, due May 10	2	60	0	0
	Should have been at one month: debit him with discount .. 10*s.*	2	–	–	–
	His acct. £49 10*s.* Diff. £10. see *Cr.* side		–	–	–
12	To Hazard and Jones's Assignee, composition on £3..15..6, at 12*s.* 6*d.* in the £. Loss, £1..8..3¾.	3	2	7	2¼
			1495	12	11¼

1830	CONTRA *Cr.*		£	s.	d.
Jan. 9	By S. Fletcher, paid him Bill No. 1	2	24	3	9
	His acct. £15..5..9 less 15*s* 3*d* discount .... £14..10..6		–	–	–
	Diff. see *Dr.* side ... 9..13..3		–	–	–
19	By Bills payable, No. 1, W. Holmes's	2	45	10	0
31	John Herdson, balance of accts. (abatement 4*d.*)	1	37	5	6
	Balance		1426	17	9
			1533	17	0
Feb. 1	By Bernard Mason (total £832..5..2½, abatement 5*s.* 2½*d.*) ...	1	832	0	0
8	W. Tomlinson, paid to J. Sims, by his order	2	10	0	0
10	James Sanderson (£88..1..8¾, abatement 1*s.* 8¾*d.*)	3	88	0	0
	Balance	3	565	12	11¼
			1495	12	11¼

INDEX TO THE LEDGER.*

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M	Mason, Bernard.... 1	Z	

* The Ledger has an Alphabetical Index, showing, at one view, in what folio any person's account may be found.

EXAMPLE OF A PARTNERSHIP CONCERN.

John Herdson and I have been engaged in a joint concern as purveyors of oats for the army. I have purchased oats for the joint stock to the amount of £26..11. J. H. has paid for oats purchased by him, £449. 0..3. I have received for oats that have been disposed of, £507..9..2; and my partner has received from the same source, £55..2..8. I have advanced to him at different times, £433..17; and have paid for warehouse room and other sundry expenses, £1..13..8.—From these general heads, collected from the particulars recorded in the Granary-Book, it is required to state the transactions.

It may, perhaps, be interesting to the learner to be informed, that the above was a real occurrence; for an accurate statement of which the writer was some time since applied to by the parties concerned.

	Dr.		£	s.	d.		Cr.		£	s.	d.
1830	*Stock* Dr.					1830	*Contra* Cr.				
Jan. 1	To Sundries, amount of my debts	1	922	15	10	Jan. 1	By Sundries, amount of property	1	1796	8	9
Feb. 12	*Balance account*	fo. 3	460	18	0¾						
	Note.—Stock is a term used to represent the owner of the Books. This account shows what he is worth at the commencement of business; and, when a general balance is taken, will enable him to discover the value of his property at that time, and the gain or loss attending trade. It cannot be carried on by Single Entry. But						**on drawing out the general Balance, the quantity and *value* of the *goods on hand* must be ascertained by an actual examination, called the *taking of stock.* The objects of the Stock Account may then be accomplished, by adding that value to the *Cr.* side.**				
1830	*James Taylor* Dr.					1830	*Contra* Cr.				
Jan. 1	To Money on bond	1	70	0	0	Feb. 4	By Cash for interest	c.b. 1	1	15	0
Feb. 4	half a year's interest	3	1	15	0		*Balance*	fo. 3	70	0	0
			71	15	0				71	15	0
1830	*John Herdson* Dr.					1830	*Contra.* Cr.				
Jan. 31	To Cash £37..5..6; abatem. 4*d.*	c.b. 1	37	5	10	Jan. 1	By a Balance of accounts	1	37	5	10
	To Sundries	* 2	0	16	8	Feb. 12	By Sundries, on Oats' concern	* 3	491	13	8½
Feb 12	Do. on Oats' concern	3	488	19	8						
	Balance	fo. 3	1	17	4½						
			491	13	8½						
1830	*Bernard Mason* Dr.					1830	*Contra* Cr.				
Jan 8	To Sundries	1	7	14	9½	Jan. 1	By purchase of house, and duty, due Feb. 1	1	840	0	0
Feb 1	Cash, £832; abat. 5*s.* 2½*d.*	c.b. 1	832	5	2½						
			840	0	0						

* When an account is balanced, a double line is drawn through the money columns, both on the *Dr.* and *Cr.* sides of the Ledger; as in J. Herdson's account, above.

LEDGER. (folio 2)

1830	*Bills Payable* Dr.		£	s.	d.	1830	*Contra* Cr.		£	s.	d.
Jan. 19	To Cash	c.b.1	45	10	0	Jan. 1	By Holmes's Bill, No. 1	1	45	10	0
Feb. 12	*Balance*	fo. 3	80	0	0	Feb. 10	Allen, Wild, and Co.'s Bill, No. 2	b.b.1	80	0	0

1830	*Allen, Wild, and Co. Leeds* Dr.		£	s.	d.	1830	*Contra* Cr.		£	s.	d.
Feb. 10	To Bills Payable	3	80	0	0	Jan. 5	By Cloth	1	157	18	6
12	*Balance*	fo. 3	77	18	6						
			157	18	6						

1830	*Samuel Fletcher* Dr.		£	s.	d.	1830	*Contra* Cr.		£	s.	d.
Jan. 9	To Bill, and discount on hops	c.b.1	24	19	0	Jan. 9	By Hops	2	15	5	9
							Cheque on Smith and Co.	c.b.1	9	13	3
									24	19	0

1830	*Simmonds & Co. Liverpool* Dr.		£	s.	d.	1830	*Contra* Cr.		£	s.	d.
Feb. 12	To *Balance*	fo. 3	14	19	0	Jan. 10	By Goods	2	14	19	0

1830	*William Tomlinson* Dr.		£	s.	d.	1830	*Contra* Cr.		£	s.	d.
Jan. 14	To Goods	2	2	9	1½	Feb. 1	By Cash	c.b.1	4	13	0
Feb. 1	Do.	2	2	4	2		Abatement	c.b.1	0	0	3½
			4	13	3½				4	13	3½
5	To Cloth	3	49	10	0	8	By Bill	c.b.1	60	0	0
8	Cash, £10. Discount, 10s.	c.b.1	10	10	0						
			60	0	0						

Date	Dr.	Fo.	£	s.	d.	Date	Cr.	Fo.	£	s.	d.
1830	*Hazard and Jones* Dr.					1830	*Contra* Cr.				
Jan. 19	To Soap and Candles	2	1	11	2	Feb. 1	By Cash for composition	c.b.1	2	7	2¼
28	Sugar and Tea	2	2	4	4		Remainder lost	c.b.1	1	8	3¾
			3	15	6				3	15	6
1830	*James Sanderson* Dr.					1830	*Contra* Cr.				
Feb. 10	To Cash, £88; abat. 1*s.* 8¾*d.*	c.b.1	88	1	8¾	Jan. 23	By Goods	2	7	3	6
						Feb. 10	Cheese	3	80	18	2¾
									88	1	8¾
1830	*Oats' Purveyance*, in Co. with *J. Herdson* Dr.					1830	*Contra* Cr.				
Feb. 12	To Sundries	3	477	4	11	Feb. 12	By Sundries	3	562	11	10
	Profit, ½ to Self... £42..13..5½ } ½ to J. H. . £42..13..5½ }	3	85	6	11						
			562	11	10						
	GENERAL						BALANCE.				
1830	*Balance* Dr.					1830	*Contra* Cr.				
Feb. 12	To Cash in hand	c.b.1	565	12	11¼	Feb. 12	By J. Herdson, I owe	fo. 1	1	17	4½
	James Taylor owes me	fo. 1	70	0	0		Bills Payable	2	80	0	0
							Allen, Wild, and Co.	2	77	18	6
							Simmonds and Co.	2	14	19	0
							Stock account debited	1	460	18	0¾
			635	12	11¼				635	12	11¼

FINIS.