# THE BRITISH AMERICAN
# COMMERCIAL ARITHMETIC,

FOR THE USE OF

## SCHOOLS, COLLEGES, AND COUNTING-HOUSES,

EMBRACING

AN EXTENSIVE COURSE BOTH IN THEORY AND PRACTICE.

BY

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# PREFACE.

We have for some time looked upon it as a kind of reproach that Canada has never produced a treatise on Arithmetic adequate to the wants of a commercial community such as this country has become. It is not enough that the school-boy should be provided with a course suited to his years; there must be supplied to him something higher as he advances in years and progress, and nears the period when he is to enter on real business life.

We have felt this keenly in our experience in conducting the British American Commercial Colleges. We have hitherto been obliged to have recourse to United States' publications, which, without any disparagement to their intrinsic merit, we are forced to say, are not suited, in many important respects, to the wants of Canada. The great increase in our sphere of operations, and the grave responsibilities devolving on us thereby, have made us feel in duty bound to supply to our students a book such as they require. We have therefore undertaken the task of supplying the deficiency. As we proceeded with the work we found it necessary to extend our original programme considerably, and, therefore, also the limits of the book, so as to make it useful, not only to our own Colleges, but to the community at large.

In carrying out our plan we have endeavoured to unfold the theory of Arithmetic as a SCIENCE in as concise a manner as seemed consistent with clearness, and at the same time to show its applications as an ART. We have striven to make the business part so copious and practical as to afford the young student ample information and discipline in all the principles and usages of commercial intercourse. For the same reason, we have introduced some articles on Commercial Law, written by J. D. Edgar, Esq., Barrister-at-Law, a prominent part of our aim being to produce a work which shall be found useful, not only in the class-room and the learner's study, but also on the merchant's table and the accountant's desk. We have taken especial care not to enunciate any rule without explaining the *reason*, for, without a knowledge of the principle, the operator is a mere calculating machine that can work but a certain round, and is almost sure to be at fault when novel cases arise. In giving those explanations we have not followed any predecessors, but have been guided entirely by our own experience in teaching. The great mass of the exercises likewise are entirely new, though we have not scrupled to make selections from some of the most approved works on the subject; but in doing so, we have confined ourselves to such questions as are to be found in almost all popular works, and which, therefore, are to be looked upon as the common property of science.

We have, as much as possible, avoided Algebraic forms and notation, as being unsuited to a large proportion of those for whom the book is intended, and to many altogether unintelligible. We have been encouraged to follow out this course from the reflection that those who understand Algebraic modes will have all the less difficulty in understanding the common Arithmetical ones. Even in the Mathematical parts, we have endeavoured to popularize the subject as much as possible.

We were compelled to follow a certain logical order in arranging the subjects treated of, but the teacher and learner will often find it necessary to depart from that order. (See Suggestions to Teachers.)

As we consider that rules and definitions should always be expressed in the smallest possible number of words consistent with perspicuity and accuracy, we have taken great pains to carry out this principle in every case.

We have appended copious exercises to each rule, especially to the most important, as Fractions, Proportion, Analysis, and Interest. Besides these, we have introduced extensive collections of mixed exercises throughout the body of the work, besides a large number at the end. The utility of such miscellaneous questions will be admitted by all, but the *reason* why they are of such importance seems strangely overlooked or misunderstood, even by writers on the subject. They are spoken of as mere *review* exercises. Their great value depends on something still more important. A class is working questions on a certain rule, and each member of the class has just heard the rule enunciated, and readily applies it. So far, one important object is attained, viz., freedom of operation. But something more is necessary. The learner must be taught to discern *what rule is to be applied* to the solution of each question proposed. The pupil, under careful teaching, may be able to understand fully every rule, and never confound any one with any other, and yet be doubtful what rule to apply to an individual case. The miscellaneous problems, therefore, are intended, not so much as exercises on the *operations* of the different rules, as on the *mode of applying* those rules; or, in other words, to practise the pupil in perceiving of what rule any proposed question is a particular case. To this we attach great importance, not only as regards readiness in real business, but also as a mental exercise to the young student.

We are far from supposing, much less asserting, that the work is complete, especially as the whole has been prepared in less than the short space of six months. We present it, however, to the public, in the confident expectation that it will meet, to a great extent at least, the necessities of the times. With this view, also, we have given the great mass of the examples and exercises, involving money, in dollars and cents, with, however, a number in pounds, shillings and pence, sufficient for the purpose of illustration. We have followed this course because we do not see any use in perpetuating the cumbrous system of the old currency, and we even hope to see the day when the decimal system will be universally adopted in British America. For this reason, too, in teaching Book-keeping, we uniformly employ the decimal notation.

We beg, also, respectfully to call the attention of the Government to the desirableness of taking steps to introduce the decimal system in weights and measures as well as in money. Its great simplicity must commend it to every intelligent mind. Some difficulty would, no doubt, for a time attend the change, but this would soon pass away, especially as the country is already familiarized with the notation by the use of the decimal coinage. We feel confident that a Parliamentary bill for this object, co-ordinate with the same movement now going on in Britain, would be most acceptable to the great majority of the people of Canada, if the matter were only taken up by some persons of energy and influence.

The rule for finding the Greatest Common Measure, though not new, is given in a new, and, we think, a concise and convenient form.

The rule for finding the Cube Root is a modification of that recommended by Dr. Hind, and will be found very ready and short.

On the subjects of Logarithms and Mensuration, we have only given the general principles, and a few of the most important cases. To do justice to these subjects would require separate books. In treating of Common Fractions we have placed Multiplication and

Division before Addition and Subtraction, for two reasons:—**FIRST.** In Common Fractions, Multiplication and Division present much less difficulty than Addition and Subtraction; and, SECONDLY, as in Whole Numbers, Addition is the rule that regulates all others; so in Fractions, which originate from Division, we see, in like manner, that all other operations result from Division, and, in connection with it, Multiplication. We hope this will be accounted reason sufficient for following so unwonted a course. By the ordinary routine the pupil is, in effect, called upon to apply rules and principles that he has not learned.

Several subjects, commonly treated of in works on Arithmetic, have been omitted, in order to leave space for more important matter bearing on commercial pursuits. Duodecimals, for example, have been omitted, as that mode of calculation is now virtually superseded by the use of Decimals.

Barter, too, has been passed by, as questions of that class can easily be solved by the rule of Proportion, which has been fully explained.

The subject of Analysis has been gone into at considerable length, and we expect that the new manner in which the explanations and solutions are presented, and the extensive collection of exercises appended, will contribute to make this a valuable part of the treatise.

We think that the view we have given of Decimal Fractions is the only true one, and is calculated to give to the student clear notions of the nature of the notation, and to show the great convenience and utility of Decimals. We have ignored the distinction sometimes made between *Decimals* and *Decimal Fractions*, as being "a distinction without a difference." *Decimal* is merely a short way of writing *Decimal Fraction*. Thus: .7 is merely a convenient mode of writing $\frac{7}{10}$. These differ in form only, but otherwise are as perfectly identical as $\frac{3}{4}$ and $\frac{6}{8}$.

The contracted methods of Multiplication and Division will be found, after some practice, extremely useful and expeditious in Decimals expressed by long lines of figures.

We have, after some hesitation, introduced an article on Logarithms, and also Tables, as the logarithmic mode of computation saves, in many cases, an immense amount of time and labour, and will be found extremely useful in surveying and mechanical calculations, especially when angular and linear units have to be compared. As the book has expanded to much greater dimensions than we anticipated, we have judged it better not to insert a table of Squares and Cubes, as we had intended. For the same reason, we found it impossible to introduce tables of Logarithmic Sines, Cosines, &c.

We have entrusted a great part of the composition of the work to the Teacher of Mathematics and English in our Toronto College, T. A. Bryce, M.A., Glasgow University, Scotland. As our own time was so much occupied in teaching, we were anxious to procure the assistance of a gentleman who was at once a sound Mathematician, an accomplished English scholar and an accurate writer, and at the same time an experienced practical teacher, believing that all these qualifications were needed for the composition of such a treatise. These requisites we found in Mr. Bryce. In the explanation of principles and framing of rules we think he has been peculiarly happy, and we take pleasure in acknowledging the valuable aid he has rendered us in the preparing of the book.

# SUGGESTIONS TO TEACHERS AND STUDENTS.

WE would first refer to our remark in the Preface, that we did not expect the teacher to follow our logical arrangement, and even advised that he should not. We know by experience that the same course does not suit all students, any more than the same medical treatment suits all patients. The course requires to be varied according to age, ability and acquirements. The greatest difficulties generally present themselves at the earliest stages. What more serious difficulty, for example, has a child to encounter than the learning of the alphabet? Though this is, perhaps, the extreme case, yet others will be found to be in proportion. For beginners, therefore, we recommend the following course:

Let the elementary rules be carefully explained and illustrated by *simple examples*, and the pupil shown how to work *easy exercises.* This done, let the whole be reviewed, and exercises of a more dificult kind be proposed. The Decimal Coinage should then be taken up. In explaining this part of the subject the teacher ought to notice carefully that the operations in this case differ in no way from those already gone through, in reference to Abstract Numbers, except in the preserving of the position of the mark that separates the cents from the dollars, usually called the decimal point. The next step should be the whole subject of Denominate Numbers, and, in illustration and application, the rule of Practice. After a thorough review of all the ground now gone over, Simple Proportion may be entered upon, using only such questions as do not involve Fractions. Then, after a course of Fractions has been gone through, Proportion should be reviewed, and questions which involve Fractions proposed. After this, it will generally be found desirable to study Percentage, with all its applications, the most important of which is Interest. The order in which the rest of the course shall be taken is comparatively unimportant, as the student has now realized a capital on which he can draw for any purpose.

We would, in the strongest manner possible, impress on the minds of teachers and students the great utility of frequent reviews, and especially of constant exercise in the addition of money columns.

We have endeavoured, as far as possible, to make the exercises under each rule of progressive difficulty. We have also made it an object to give each exercise the semblance of a real question, for all persons, and especially the young, take greater interest in exercises that assume the form of reality than in such as are merely abstract; and, besides, this is a preparatory exercise to the application of the rules afterwards.

In the foregoing suggestions we have had in view the case of a child beginning from the very elements, but the judicious teacher will readily modify and apply these principles according to circumstances. We shall only briefly add the following cautions and hints:

At every stage the greatest care should be taken that the learner understands the meaning of each rule, and the terms and conditions of each question, before he attempts to solve it.

The teacher should never attempt to explain two things at the same time, and he should be satisfied that the first idea is fully grasped and stored before he approaches the second.

Unruffled temper and untiring patience are essential on the part of the teacher, in order that he may be able to discover the source of every difficulty that presents itself to the pupil's mind, and remove it by careful, and, when necessary, repeated explanation.

The teacher should not always be talking or working on the black-board; he should require the pupils to speak a good deal in answer to questions, and also to work much on their slates.

The pupils should never be made to work or listen till they are fatigued, or till their attention flags.

Finally, we would suggest to every teacher to keep constantly before his mind both of the two great works he has to accomplish—*First*, the development of the mental powers of his pupil; and, *secondly*, imparting to him such knowledge as he will require to use when he enters upon life, either as a professional man or a merchant. Some seem to consider these two objects incompatible, as if taking up time in mental training left insufficient time for the imparting of actual knowledge. We, however, consider this a palpable error, for the more the mind is cultivated, the more readily and rapidly will it take in any species of knowledge, and the more surely too will it retain what it has mastered. Mental culture is at once the foundation and the means; the other is the superstructure raised on that foundation, and by that means; or it may be compared to a great capital judiciously embarked in trade, and often turned, and therefore yielding good profits. It frequently happens, however, from the peculiar circumstances of individuals and families, and even communities, that young men require to be hurried into business, so as to be able to support themselves; but even in such cases the desired object will be much more speedily and securely attained by such a course than by what is usually and not inappropriately called "*cramming.*" We have striven to give the character here recommended to this book, especially in the explanatory portions.

We solicit the attention of the student, as well as the teacher, to these suggestions, and in particular of the self-taught student, whose wants also we have kept in view throughout.

MUSGROVE & WRIGHT,

# TABLE OF CONTENTS.

# ARITHMETIC.

ARTICLE 1.—ARITHMETIC treats of numbers in theory and practice. In relation to theory it is a science, and in relation to practice it is an art.

All computations are made by fixing on a certain quantity, called a unit or one, and repeating that unit any required number of times. Various units are selected, according to the nature and extent of the quantity or space to be measured. For example, in measuring length or distance, if the extent is small, such as the length of a pane of glass, we select a small unit, called an inch, and repeating that unit any required number of times, say twelve, we say the pane is twelve inches long,—if a more extended space is to be measured, it is convenient to adopt a larger unit,—thus, if we wish to measure the length of a desk, we should probably select a unit called a foot, equal to twelve of the preceding units,—if we wish to measure the length of a room, we should select a still larger unit, called a yard, equal to three of the last,—again, if we wish to measure the length of a field, we should adopt a unit equal to five and a-half of the last, and called a perch or rod,—if we wish to note the distance between Toronto and Montreal, we have recourse to a still larger unit, called a mile, and equal to three hundred and twenty of the last,—finally, when astronomers are estimating the distance of any planet, say the earth, from the sun, they generally use a unit equal to a million of the last-mentioned, and they say that the earth is ninety-five millions of miles from the sun, but they simply note the distance as ninety-five; and in the same manner they mark the distance of Venus as sixty-nine, meaning in both cases that each unit is a million of miles. A similar illustration may be applied to every kind of measurement.

The symbols or characters now almost universally used to denote quantity or magnitude, are the Arabic figures, or digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. These, by various combinations, can be made to represent any quantity or magnitude whatsoever. The first nine are called significant figures, because they always denote some real quantity,—the last, called nought (often improperly ought), or cipher, or zero, simply indicates the absence of any significant figure.

## NUMERATION.

2.—NUMERATION is the mode of marking and reading off any line of figures that has been written down, so as to ascertain its value readily and express that value in words. For this purpose every such line is divided into sets or lots of three figures each, counting from right to left, and each set is called a period,—thus, 888888888 forms three periods by marking the figures in threes from right to left by a character of the same form as the comma in composition,—thus, 888,888,888. The first period is called the period of units, the second the period of thousands, the third the period of millions, and so on,—billions, trillions, quadrillions, &c., &c., to any required extent, which seldom exceeds millions.

The first figure of each period denotes units* of that period, the second tens, and the third hundreds of that period. Thus, in the example given above, the first figure denotes eight units in the period of units, or eight ones, or, as it is usually read, simply eight; so, also, the fourth denotes eight units in the period of thousands; or eight times one thousand, or eight thousands; the seventh figure again denotes eight units in the period of millions, or eight times one million, or eight millions; again, the second, fifth, and eighth figures denote tens in the period of units, thousands and millions, respectively; lastly, the third, sixth and ninth figures denote hundreds in the periods of units, thousands and millions, respectively. Such a line, then, as 888,888,888 is read eight hundred and eigety-eight millions, eight hundred and eighty-eight thousands, eight hundred and eighty-eight.

Every period but the last must have three figures. Thus, in the line 43,279,865 the first and second periods have three digits each, units, tens and hundreds, but the third has only two, units and tens, but no hundreds, and therefore is read forty-three millions, two hundred and seventy-nine thousands, eight hundred and sixty-five.

### RULE FOR NUMERATION.

Beginning at the right, count off periods of three digits each till not more than three are left; then read off each period from left to

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* It is somewhat awkward that the term units is used for two purposes, viz.: as the name of the first period and also as the name of the first figure of each period. Though we cannot well change what usage has so long established, yet the teacher may obviate the difficulty by varying the expression occasionally, if not habitually, saying, E. G., units in the *unity* period, or the *place* of units in the units period.

right by naming as many hundreds, tens and units as each contains, and adding at the end of each period its proper name. The name of the unity period is usually omitted. When a cipher occurs no mention is made of that place in the period, but the cipher is counted as a digit; thus, in the line 360,708,091 each cipher is counted a digit, but the reading is three hundred and sixty millions, seven hundred and eight thousands and ninety-one.

### EXERCISES

Divide into periods and read the following lines:

1.—586729341 2.—976852734 3.—2178427385
4.—92879357485 5.—4638709120 6.—1111111111111111
7.—2822828228288 8.—10904870 9.—1010101010101

## NOTATION.

3 —NOTATION is the mode of expressing any quantity or magnitude by the combination of conventional symbols or characters. Thus, by the Roman notation, the letter I. stands for *one*, II. for *two*, X. for *ten*, &c.; thus, XII. stands for *one* ten and *two* units. By the Arabic notation, any digit standing alone, as 5 in the margin, denotes simply five units, but if another digit (5) be placed to the right of it, then the new 5 denotes units and the other 5 becomes tens, so that appending a second digit makes the first one ten times its original value; again, if another digit (5) be subjoined, it takes the place of units, and the 5 next to it becomes tens and the third becomes hundreds, so that each of them has ten times the value in the third line that it had in the second; so also, if another digit (5) be added, each of the three to the left of it will have ten times the value that it had in the third line, and so on. Universally, every digit placed to the right makes every one to the left ten times its previous value.

5
55
555
5555

The use of the tenth of the Arabic characters, the cipher (0) will be made more clear by the rule of notation than by numeration. If I am counting my cash and find that I have *eight* ten-dollar bills, and *eight* one-dollar bills, it is plain from Art. 2 that if I write 8 alone this must represent the one-dollar bills, and to represent the ten-dollar bills along with the one-dollar bills I must write 88, for the figure to the left being ten times that to the right, will stand correctly for the ten-dollar bills, just as that to the right, being in the units' place, stands for the one-dollar bills.—But if I have no one-dollar bills and write 8, this would stand for only one-dollar bills, and hence the necessity for introducing a non-significant character and writing 80, for though the cipher represents no quantity, yet by being put in the place of units it throws the 8 to be in the place of tens, and therefore the 8 now stands fitly for the *eight* ten-dollar bills, and is written $80.—Again, if I find that I have *two* one-hundred-dollar bills, *six* one-dollar bills, but *no* ten-dollar bills, and I write only 26, this would be plainly incorrect, for the 2 would stand for ten-dollar bills only, but by inserting a zero mark between the figures I throw the 2 into the place of hundreds, and $206 represents correctly that I have *two* one-hundred dollar bills, and *six* one-dollar bills, but *no* ten-dollar bills. The superiority of this simple system over the cumbrous Roman one will be manifest from its simplicity and brevity by writing *eighty-eight* according to both systems—thus: LXXXVIII. and 88.

### RULE FOR NOTATION.

Write the significant figures of the first period named in their proper places, filling up any places *not named* with ciphers, just as if you were writing the units period with nothing to follow; then, to indicate that something *is* to follow, place a comma to the right, and do the same for every period down to units, inclusive. For example, teacher says: "Write down one hundred and six millions;" pupil writes 106 and pauses; teacher adds, "ninety thousand;" pupil fills up thus: 106,090, and pauses; teacher concludes: "and eighteen;" pupil completes 106,090,018. If the teacher should say sixteen millions and the pupil write 016, the cipher would be manifestly superfluous, as it has no effect on figures placed to the right of it, but only on those placed to the left.

### EXERCISES.

Write in figures and read the following quantities:

1. Ten millions, seven thousand and eleven.
2. Ninety billions, seven thousand and ten.
3. Eighteen millions, sixty thousand and nine hundred.
4. Forty thousand and nine hundred.

5. Eighty-seven millions and one.
6. Ninety thousand, seven hundred and eight.
7. Eleven millions, eight hundred thousand and twenty-four.
8. Six hundred and seven thousand and ninety-seven.
9. Eight hundred and seventy billions, sixty thousand and eighteen.
10. Eleven billions, eleven millions, eleven thousand and eleven.

## AXIOMS.

**4.**—Axioms used in the sequel:

I. Things that are equal to the same thing, or to equals, are equal.

II. If equals be added to equals, the wholes are equal.

*Corollary.*—If equals be multiplied by the same, the products are equal.

III. If equals be taken from equals, the remainders are equal.

*Cor.*—If equals be divided by the same, the quotients are equal.

IV. The whole is greater than its part

*Cor.*—The whole is equal to all its parts taken together.

V. Magnitudes which coincide, or occupy the same or equal spaces, are equal.

N. B.—This axiom is modified by, but still is the principle of all business transactions, purchases, sales, barters, exchanges, &c., &c., where the articles traded in are not *equals*, but *equivalents*.

## ADDITION.

**5.**—Addition is the mode of combining two or more numbers into one. The operation depends on axiom II. The result is called the sum. Thus: $8+$9+$6=$23. The sign plus (+) indicates addition.

$287654
758287
612873
494768
836195

To illustrate the operation, let it be required to find the sum of the five numbers of dollars noted in the margin. First, the numbers are placed so that those of the same *name* are in vertical columns, *i. e.*, units under units, tens under tens, &c. Next, we find that the sum of the units' column is (Ax. IV., *Cor.*) 27, *i. e.*, *two* tens and *seven* units. Next, we find that the sum of the tens' column is 35, but, as it is the *tens'*

|  |
|---:|
| 27 |
| 350 |
| 2400 |
| 27000 |
| 260000 |
| 2700000 |
| $2989777 |

column, we write (Art. 3) 350; in the same manner we find the sum of the hundreds' column to be 2400; the sums of the others will be seen by inspection. Having thus obtained the sum of each column, each being *summed* as if *units*, but placed in succession towards *the left* (by Arts. 2 and 5), we now take the sum of the partial results, which (Axiom IV. *Cor.*) is the sum of the whole, viz.: $2,969,777. In practice the operation is much abbreviated in the following manner:—When the units' column has been added, and we find the sum to be 27, *i. e.*, 7 units and 2 tens, we write down the 7 units under the units' column, and add up (Art. 3) the 2 tens with the tens' column, and we find the sum is 35 *tens*, *i. e.*, 5 *tens* and 3 *hundreds*, and we place the 5 *tens* under the *tens'* column, and add up the 3 *hundreds* with the *hundreds'* column, and so on. The transferring of the tens, obtained by adding the *units'* column to the *tens'* column, and the *hundreds* obtained by adding the *tens'* column to the *hundreds'* column, &c., &c., is called carrying. In all such operations the learner should carefully bear in mind the principle explained in Art. 3., that every figure to the left is ten times the value that it would have if one place farther to the right.*

|  |
|---:|
| $287654 |
| 758287 |
| 612873 |
| 494768 |
| 836195 |
| $2989777 |

EXERCISES.

Find the sums of the following quantities:

| (1) | (2) | (3) | (4) |
|---:|---:|---:|---:|
| 895763 |  | 99876 |  |
| 49176 | 987654231 | 63879 | 89765324 |
| 283527 | 123456789 | 54387 | 42356798 |
| 659845 | 908760504 | 789 | 56798423 |
| 7984 | 890705063 | 137568 | 23567989 |
| 31659 | 759086391 | 278652 | 79842356 |
| 968438 | 670998767 | 85945 | 65324897 |
| 2896392 | 4340661745 | 721096 | 357655787 |

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* We would strongly recommend every one who wishes to become an expert accountant, to avoid the common practice of drawling up a column of figures in the manner that may be sufficiently illustrated by the adding of the units' column of the above example. Never say 5 and 8 are 13; 13 and 3 are 16: 16 and 7 are 23; 23 and 4 are 27; but run up your column thus: 5, 13, 16, 23,

| (5) | (6) | (7) | (8) |
|---|---|---|---|
| | | | 738 |
| | | | 659 |
| | | | 471 |
| 78563 | | | 897 |
| 47986 | 12345 | | 658 |
| 5798 | 67890 | 918273 | 856 |
| 19843 | 98765 | 651928 | 789 |
| 56479 | 43219 | 374859 | 978 |
| 28795 | 87654 | 263748 | 654 |
| 897 | 32169 | 597485 | 999 |
| 1984 | 78912 | 986879 | 888 |
| 68195 | 65439 | 98765 | 777 |
| 3879 | 98765 | 9876 | 666 |
| 698 | 43288 | 987 | 555 |
| 5879 | 77877 | 456879 | 897 |
| 17985 | 98989 | 345678 | 978 |
| 336981 | 805312 | 4705357 | 12460 |

| (9) | (10) | (11) | (12) |
|---|---|---|---|
| 189 | | | 1298 |
| 976 | 98 | 47 | 764 |
| 85 | 89 | 96 | 5837 |
| 73 | 76 | 83 | 6495 |
| 338 | 67 | 59 | 789 |
| 793 | 281 | 74 | 638 |
| 49 | 592 | 82 | 546 |
| 75 | 678 | 97 | 98 |
| 218 | 58 | 68 | 475 |
| 365 | 67 | 75 | 394 |
| 113 | 98 | 49 | 89 |
| 279 | 149 | 76 | 157 |
| 67 | 67 | 54 | 638 |
| 76 | 54 | 78 | 594 |
| 84 | 72 | 69 | 789 |
| 1379 | 298 | 37 | 114 |
| 5159 | 2744 | 1044 | 19715 |

27, for that is the mode to secure both rapidity and accuracy. The same remark will apply equally to multiplication, and therefore to every arithmetical operation. To enforce this advice let us add a simple example to caution the student before he approaches multiplication. In multiplying 497 by 6, avoid the tediousness of saying 6 times 7 is 42—2 and carry 4—6 times 9 is 54, and 4 is 58—8 and carry 5—6 times 4 is 24, and 5 is 29 ; but practice the eye, aided by the memory, to take in at a glance 6 times 7 is 42, &c.—The quick operator uses the eye, and not the tongue.

There is no method of proving the correctness of any addition with positive certainty, but a very convenient mode of checking is to add each column both upwards and downwards. Another mode is, to add by parts and take the sum of those. This is a very secure method in the case of long columns, but not so ready as the former. If the same result is found by each method, the sum may be accounted correct.

## SUBTRACTION.

**6.** SUBTRACTION is the converse of addition, *i. e.*, it is the mode of finding the difference between two numbers, or, in other words, the excess of one number above another. The number to be subtracted is called the subtrahend, and that from which it is to be taken the minuend, and the result is called the remainder, difference or excess. The sign used for subtraction is a line (—) called *minus*, or less. Let it be required to find the difference between $578643957 and $235412712. Having placed them in vertical columns, as in addition, it is obvious that 2 units taken from 7 units will leave 5 units, and that 1 ten taken from 5 tens will leave 4 tens, and so on. But if it is required to find the excess of $513674208 above $347895319, we find that each figure of the subtrahend, except the last, counting from right to left, is greater than the corresponding one of the minuend, and therefore, to find the correct difference, we have recourse to a simple artifice, which is deduced from the principle of the notation, and may be illustrated in the following manner:—Taking the question in the margin, we are first required to subtract 7 units from 3 units. Now, though the algebraic notation furnishes the means of noting the difference directly, the ordinary arithmetical form does not, but still it furnishes the means of doing it indirectly. By Art. 3 each figure to the left is ten times the value of the next to its right, therefore we take one of the 3 tens and call it ten units, and add it to the 3 units, and thus we have 13 units, which let us enclose in a parenthesis or bracket, thus: (13), to indicate that the whole quantity, 13, is to occupy the units' place; when *one* of the *three* tens has been thus transferred to the units'

$$\begin{array}{r}578643957\\235412712\\\hline 343231245\end{array}$$

$$\begin{array}{r}333{,}333\\177{,}777\\\hline 155{,}556\end{array}$$

$$\begin{array}{cccccc}2&(12)&(12)&(12)&(12)&(13)\\1&7&7&7&7&7\\\hline 1&5&5&5&5&6\end{array}$$

place, only *two* tens remain in the place of tens, and we are now required to take 7 tens from 2 tens; to do this we have recourse to the same artifice, by calling one of the hundreds *tens*, which gives 10 tens and 2 tens, and so on to the end, the last 3 necessarily becoming 2. We can now subtract 7 from 13, &c., &c. This mode of resolution depends on the corollary to Axiom IV. The parts into which the whole is virtually resolved are shown in the margin. This artifice is popularly called borrowing. In practice the resolution can be effected mentally as we proceed, and as each figure from which we *borrow* is diminished by unity, it is usual to count it as it stands, and to compensate for this to increase the one below it by one, for, as in the example, 7 from 12 is the same as 8 from 13, and 2 from 3 is the same as 1 from 2. We are now prepared to answer the proposed question, as annexed, and we say 9 from 8, we cannot, and there are no tens to borrow from, we therefore take one of the hundreds and call it 10 *tens*, and one of the tens and call it 10 units, which with 8 units makes 18 units, and we take 9 from 18 and 9 remain. We have now only 9 tens left, but we reckon them as ten, and to compensate for the surplus ten, we reckon the 1 below as 2, and say 2 from 10 and 8 remain. We proceed thus to the end, and find the whole remainder to be $165778889.

$$\begin{array}{r}200000\\120000\\12000\\1200\\120\\13\\\hline 333333\end{array}$$

$$\begin{array}{r}\$513674208\\\$347895319\\\hline \$165778889\end{array}$$

EXERCISES. REMAINDERS.

1.—From 847639021 take 476584359=371054662.
2. " 1010305061 " 670685093=339619968.
3. " 59638743 " 18796854= 40841889.
4. " 7813257 " 3745679= 4067578.
5. " 111111111 " 98657293= 12453818.

In Subtraction, as in Addition, we have no method of proof that arrives at positive certainty, but either of the two following methods may be generally relied upon.

1.—Add the remainder and subtrahend, and if the sum is equal to the minuend, it is to be presumed that the work is correct.

2.—Subtract the remainder from the minuend, and if this second remainder is the same as the subtrahend, the work may be accounted correct.

## MULTIPLICATION.

7.—MULTIPLICATION may be simply defined by saying that it is a short method of performing addition, when all the quantities to be added are the same or equal. Thus : $6+6+6+6+6+6+6+6$, means that eight sixes are to be added together, or that six is to be repeated as often as there are units in eight, and we say that 8 times 6 is 48, and write it thus: $8\times6=48$. So also $8+8+8+8+8+8$ gives 48. So that $6.8=8.6=48$, and thus we can construct a multiplication table. The number to be repeated is called the multiplicand, and the one that shows how often it is to be repeated is called the multiplier, and the result is called the product, or what is produced, and hence the multiplier and multiplicand are also called the factors or makers, or producers, and the operation may be called *finding* a product when the factors are given. Hence also the mode of carrying is the same in multiplication as in addition.

### MULTIPLICATION TABLE.

| Twice | 3 times | 4 times | 5 times | 6 times | 7 times |
|---|---|---|---|---|---|
| 1 is 2 | 1 is 3 | 1 is 4 | 1 is 5 | 1 is 6 | 1 is 7 |
| 2 — 4 | 2 — 6 | 2 — 8 | 2 — 10 | 2 — 12 | 2 — 14 |
| 3 — 6 | 3 — 9 | 3 — 12 | 3 — 15 | 3 — 18 | 3 — 21 |
| 4 — 8 | 4 — 12 | 4 — 16 | 4 — 20 | 4 — 24 | 4 — 28 |
| 5 — 10 | 5 — 15 | 5 — 20 | 5 — 25 | 5 — 30 | 5 — 35 |
| 6 — 12 | 6 — 18 | 6 — 24 | 6 — 30 | 6 — 36 | 6 — 42 |
| 7 — 14 | 7 — 21 | 7 — 28 | 7 — 35 | 7 — 42 | 7 — 49 |
| 8 — 16 | 8 — 24 | 8 — 32 | 8 — 40 | 8 — 48 | 8 — 56 |
| 9 — 18 | 9 — 27 | 9 — 36 | 9 — 45 | 9 — 54 | 9 — 63 |
| 10 — 20 | 10 — 30 | 10 — 40 | 10 — 50 | 10 — 60 | 10 — 70 |
| 11 — 22 | 11 — 33 | 11 — 44 | 11 — 55 | 11 — 66 | 11 — 77 |
| 12 — 24 | 12 — 36 | 12 — 48 | 12 — 60 | 12 — 72 | 12 — 84 |

| 8 times | 9 times | 10 times | 11 times | 12 times |
|---|---|---|---|---|
| 1 is 8 | 1 is 9 | 1 is 10 | 1 is 11 | 1 is 12 |
| 2 — 16 | 2 — 18 | 2 — 20 | 2 — 22 | 2 — 24 |
| 3 — 24 | 3 — 27 | 3 — 30 | 3 — 33 | 3 — 36 |
| 4 — 32 | 4 — 36 | 4 — 40 | 4 — 44 | 4 — 48 |
| 5 — 40 | 5 — 45 | 5 — 50 | 5 — 55 | 5 — 60 |
| 6 — 48 | 6 — 54 | 6 — 60 | 6 — 66 | 6 — 72 |
| 7 — 56 | 7 — 63 | 7 — 70 | 7 — 77 | 7 — 84 |
| 8 — 64 | 8 — 72 | 8 — 80 | 8 — 88 | 8 — 96 |
| 9 — 72 | 9 — 81 | 9 — 90 | 9 — 99 | 9 —108 |
| 10 — 80 | 10 — 90 | 10 —100 | 10 —110 | 10 —120 |
| 11 — 88 | 11 — 99 | 11 —110 | 11 —121 | 11 —132 |
| 12 — 96 | 12 —108 | 12 —120 | 12 —132 | 12 —144 |

Regarding the following part of this table, see suggestions to Teachers :

| 13 times | 14 times | 15 times | 16 times | 17 times | 18 times | 19 times |
|---|---|---|---|---|---|---|
| 2 is 26 | 2 is 28 | 2 is 30 | 2 is 32 | 2 is 34 | 2 is 36 | 2 is 38 |
| 3 — 39 | 3 - 42 | 3 — 45 | 3 — 48 | 3 — 51 | 3 - 54 | 3 — 57 |
| 4 — 52 | 4 — 56 | 4 — 60 | 4 — 64 | 4 — 68 | 4 — 72 | 4 — 76 |
| 5 — 65 | 5 — 70 | 5 — 75 | 5 — 80 | 5 — 85 | 5 — 90 | 5 — 95 |
| 6 — 78 | 6 — 84 | 6 — 90 | 6 — 96 | 6 — 102 | 6 — 108 | 6 — 114 |
| 7 — 91 | 7 — 98 | 7 — 105 | 7 — 112 | 7 — 119 | 7 — 126 | 7 — 133 |
| 8 — 104 | 8 — 112 | 8 — 120 | 8 — 128 | 8 — 136 | 8 — 144 | 8 — 152 |
| 9 — 117 | 9 — 126 | 9 — 135 | 9 — 144 | 9 — 153 | 9 — 162 | 9 — 171 |

We have in the above table corrected the gross grammatical blunder so common of saying eight times two ARE sixteen.

When more than two factors are given, the operation is called continued multiplication, as $6\times3\times2\times5=180$.

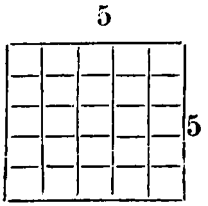
When the factors consist of more figures than one, the most convenient mode of operation is that shown by the annexed example, where the multiplicand is first repeated 8 times, then 60 times, or which is the same thing 6 times when the first figure of the second line is placed under the second figure of the first line, *i. e.* (art. 2,) in the place of tens, and then the partial products are added, which (Ax. IV. Cor.) gives the full product. Hence we deduce the

$$\begin{array}{r}
345186 \\
268 \\
\hline
2761488 \\
2070916\phantom{0} \\
690372\phantom{00} \\
\hline
92507848
\end{array}$$

### RULE FOR MULTIPLICATION.

Place the multiplier under the multiplicand, units under units, tens under tens, &c., &c.,—commencing at the right, multiply each figure of the multiplicand by each figure of the multiplier in succession, placing the results in parallel lines, and units, tens, &c., in vertical columns,—add all the lines, and the sum of all the partial products will (Ax. IV. Cor.) be the whole product required.

As far as the learner has committed a multiplication table to memory, say to 12 times 12, the work can be done by a single operation.



When any number is multiplied by itself, the product is called the square or second power of that number, and the product of three equal factors is called a cube or third power, the product of four equal factors the fourth power, &c., &c. The terms *square* and *cube* are derived from superficial and solid measurement. The annexed square has each of its sides divided into 5 equal parts, and it will be found on inspection that the whole figure contains

25 (=5×5) small squares, all equal in area, and having all their sides equal.—Hence because 5×5 represents the whole area, 25 is called the square of 5, or the second power of 5, because it is the product of the two equal factors 5 and 5. A cube is a solid body, the length, breadth and thickness of which are all equal, and hence, if these dimensions be each represented by 5, the whole solid will be represented by 5×5×5=125, which is therefore called the cube or third power of 5. The terms square and cube are often used without any reference to superficial and solid measure. For example, in lineal measure an expression for distance in a straight line is often called the square and cube of a certain number, thus: 81 is called the square, and 729 the cube of 9, although these are only used to show that the distance is not 9 in either case, but in the one 9×9, and in the other 9×9×9. In such cases the terms second and third power are therefore to be preferred, and since no solid can have more than three dimensions, we have no term corresponding to square and cube for the product of four or more equal factors, and therefore we are obliged to use the words fourth power, fifth power, &c., &c.

### CONTRACTIONS AND PROOF.

There are many cases in which multiplication may be performed by contracted methods, but the utility of these, for the purposes of accuracy, is, at least, doubtful. The most secure method in the great majority of cases, is to follow the general rule. Multiplication by 10, 100, &c., is effected at once by adding a cipher for ten, two for 100, &c., &c. The following is, next to the above, the most safe and useful contraction that can be adopted. It is exhibited in the subjoined examples, but purposely without explanation, as an exercise for the learner's reflection:

| ORDINARY METHOD. | CONTRACTED METHOD. | ORDINARY METHOD. | CONTRACTED METHOD. |
|---|---|---|---|
| 35697×17 | 35697×17 | 35697×71 | 35697×71 |
| 17 | 249879 | 71 | 249879 |
| 249879 | 606849 | 35697 | 2534487 |
| 35697 | | 249879 | |
| 606849 | | 2534487 | |

The only practically useful proof of the correctness of the product, is the one subjoined, but even it, though it seldom fails, does not secure positive certainty:

Add together all the figures of each factor separately, rejecting 9 from all sums that contain it, and multiply the remainders together, rejecting every 9 from the result,—add the figures of the product in the same manner, and if the two remainders are equal, the work *may* be accounted as correct, but if they are not equal, the work *must* be wrong. The reason of this proof depends on the property of the number 9, that if any number be divided by 9, the remainder will be the same as if the sum of its digits were divided by 9.—Thus: 7422153÷9=824683+6, and the sum of the digits is 24, and 24÷9=2+6, *i. e.* 9 is contained in 24 twice with a remainder 6. Every 9 is rejected because 9 is contained in itself once evenly, and therefore cannot affect the remainder. Let it now be required to multiply 122 by 24. Now, 122=9×13+5, and 24=9×2+6, and if we multiply together the two factors thus resolved, we get 9×13×9×2+9×2×5+9×13×6+6×5, and since 9 is a factor of all but the last, the last only will give a remainder when divided by 9, and therefore the whole product will give the same remainder when divided by 9, as 6×5÷9, which gives the remiainder 3, for 6×5=30 and 30÷9 gives 3 with a remainder 3. To test this by trial, we find 122÷9=13 with a remainder 5, and 24÷9=2 with a remainder 6, and the product of these remainders is 6×5=30, and 30÷9=3 with a remainder 3. Again, 122×24=2928, and 2928÷9=325 with a remainder 3, as in the case of the factors.

### EXERCISES.

1. 7896×5=39480.
2. 581967×8=4655736.
3. 938746×4=3754984.
4. 193784×7=1356488.
5. 391876×9=3526884.
6. 987456×6=5924736.
7. 496783×52=25832716.
8. 719864×43=30954152.
9. 375967×64=24061888.
10. 27859×29=807911.
11. 679854×83=56427882.
12. 759684×187=142060908.
13. 5372×1634=8777848

14. Find the second power of 389 ? Ans. 151321.

15. Find the third power of 538 ? Ans. 155720872.

16. Find the fourth power of 144 ? Ans. 429981696.

17. Find the cube of 99 ? Ans. 970299.

18. 5796 seamen have to be paid 169 dollars each ; what is the amount of the treasury order for that purpose ? Ans. $979,524.

19. A block of buildings is 87 feet long; 38 feet deep, and 29 feet high ; how many cubic yards does it contain ? Ans. $3550\frac{8}{9}$ cubic yards:

20. If 29 oil wells yield 19 gallons an hour each; how much will they all yield in a year? Ans. 201115 gals.

21. If the rate on each of 1597 houses be $19; what is the whole assessment? Ans. $30343.

22. If 1297 persons have paid up 9 shares each in a railway company, and each share is $15; what is the working capital of the company? Ans. $172095

## DIVISION.

**8.**—Division is the converse operation to multiplication. It is the mode of finding a required factor when a product and another factor are given. It bears the same relation to subtraction that multiplication does to addition, as will be seen below. By Ax. IV. Cor. we may resolve any complex quantity into its component parts; so division is resolving a certain quantity called the dividend into the number of parts indicated by another quantity called the divisor, (divider,) and the result is called the quotient (how often.) Let it be required to find how often 8 is contained in 279,856. We can resolve 279,856 as in the margin; then dividing the lines separately by 8, we obtain the partial quotients, the sum of which is the whole quotient. But this resolution may be done mentally as we proceed. We first see that 8 is not contained in 2, therefore we take 27, and find that 8 is contained in it 3 times, with a remainder 3; next combining this 3 with the next figure 9, we get 39, in which 8 is contained 4 times, with a remainder 7; combining this 7 with the next figure 8, we have 78, in which 8 is contained 9 times, with a remainder 6; combining this with the 5 following, we obtain 65, and 8 is contained in it 8 times, with a remainder 1, which combined with the 6 makes 16, and 8 is contained twice in 16. The correctness of the result may be tested by multiplying the quotient by the divisor. When the divisor consists of more than one figure, the learner must have recourse to a trial quotient, but after some practice he will have little difficulty in finding each figure by inspection.

| | | |
|---|---:|---:|
| 8 | 240000 | 30000 |
| | 32000 | 4000 |
| | 7200 | 900 |
| | 640 | 80 |
| | 16 | 2 |
| 8 | 279856 | 34982 |

Let it be required to find how often 298 is contained in 431766.—The numbers being arranged in the convenient order indicated in the margin, we mark off to the right of the dividend blank spaces for the trial and true quotients. We readily see that 2 is contained *twice* in 4, but cannot so easily see whether the whole divisor 298 is contained twice in the same number of figures of the dividend, (viz. 431,) we therefore make trial, and place the 2 in the trial quotient, and multiply the divisor by 2 to find how much we shall have to subtract from 431. We find 298×2=596, larger than 431, and therefore we reject 2 and try 1. Now 298×1=298, less than 431, so we subtract and find a remainder of 133, and as this proves correct, we place the 1 obtained in the true quotient. We find our next partial divdend by writing 7, the next figure of the dividend after the remainder 133. Our experience of the first case suggests to us that though 2 is contained 6 times in 13, yet on multiplying something will have to be carried from the 98 which we expect will make the result too large, and therefore we at once try 5, but we find that 298×5=1490, which is larger than 1337, and so we try 4, and find 298×4=1192, which being less than 1337, we subtract and find a remainder of 145; and having placed the 4 in the true quotient, we bring down the next figure of the dividend, giving a partial dividend 1456. By inspection, as before, we see that 6 would be too large, owing to the carrying from 98, we try 5 and find 298×5=1490, which is larger than 1456; we try 4, and find 298×4=1192, which is less than 1456, so we subtract and find a remainder of 264. Having placed this 4 after the other 4 in the true quotient, we bring down 6, the last figure of the dividend, we try 9, and find 298×9=2682, which is greater than our last partial dividend, 2646; we try 8, and find 298×8=2384, and this being less than 2646, we subtract it from

```
298)431766(2.1.5.4.5.4.9.8 trials.
    298    1448 true quotient.
    ----
    1337
    1192
    ----
     1456
     1192
     ----
      2646
      2384
      ----
       262
      ----
       298
```

that number, and find a final remainder of 262, and close the question by entering 8 in the true quotient. The mode adopted to indicate that the remainder 262 still remains to be divided, which cannot be actually done, as it is less than the divisor, is to write the 298 below the 262, and draw a line between them, thus $\frac{262}{298}$, as also is seen in the margin. The resolution into partial dividends is also shown in the margin, where it will be seen that the partial dividends, including the remainder, make up the whole original dividend. So also the partial quotients are exhibited, making up the whole true quotient. That the trial quotient is not a single number, like the true quotient, but merely a succession of detached numbers, used as separate trials, is indicated by placing a full point between each pair. When we have multiplied the divisor by any figure in the trial quotient, and subtracted the product from the partial dividend, should the remainder be greater than the divisor, we perceive that the trial figure is too small, and we must try a larger.

| | | |
|---|---|---|
| 298 | 298000 | =1000 |
| | 119200 | = 400 |
| | 11920 | = 40 |
| | 2384 | = 8 |
| Remainder | 262 | |
| | | 1448 |
| Dividend | 431766 | |

From these illustrations we can deduce a

### RULE FOR DIVISION.

(1.) Place the given numbers in the same horizontal line, putting the divisor to the left of the dividend, with a vertical line between them, draw another vertical line to the right of the dividend, and enter the quotient, figure by figure as obtained, to the right of that line. (2.) Find by the principles of multiplication, how often the divisor is contained in the same number of figures of the dividend; place the number thus obtained in the quotient, and multiply the divisor by it, and subtract the product from the corresponding partial dividend. (3.) To the remainder annex the next figure of the dividend, and proceed as before, and so on till all the figures of the dividend are exhausted. (4.) Should there be a remainder, write it and the divisor after the quotient, thus: $\frac{\text{remainder,}}{\text{divisor,}}$

The divisor is often written to the right of the dividend, and the quotient written below it, a horizontal line separating the two.

EXAMPLE OF FORM 1

476)8593504(18053$\frac{276}{476}$.
476
3833
3808
2550
2380
1704
1428
276

EXAMPLE OF FORM 2

1860904 | 87
174 | 21389$\frac{61}{87}$
120
87
339
261
780
696
844
783
61

### EXERCISES.

1. 1554768÷216=7198.
2. 31884470÷779=40930.
3. 57380625÷7575=7575.
4. I2810098÷732=17500$\frac{98}{732}$.
5. 9313702859÷4687319=1987$\frac{9}{4687319}$.
6. 449148410476÷73885246=6079$\frac{32}{73885246}$.
7. 109588282929÷1386=7902468$\frac{279}{1386}$.
8. 35676210832÷79094451=764095$\frac{1673987}{79094451}$.
9. 536818834÷907=591862.
10. 170064915561÷759=2240644479.
11. 554270297961÷7584=73084163$\frac{5769}{7584}$.
12. 60435674634529÷764095=79094451$\frac{97684}{764095}$.

13. How many bags, each containing 87 pounds, will 24,853,464 pounds of flour fill? Ans. 285,672.

14. 857 houses pay annually a tax of $41136; what is the average on each per quarter? Ans. $12.

15. $9297175 of prize money are to be divided among 97,865 sailors; what is the share of each? Ans. $95.

16. 120,815,231 pounds of cotton are made up in 233,879 bales; how many pounds in each bale? Ans 89.

### DIVISION.

1. 49687532÷2=24843766.
2. 57986327÷3=19328775$\frac{2}{3}$.

3. $87965328 \div 4 = 21991332$.
4. $7963821 \div 5 = 1592764\frac{1}{5}$.
5. $6875324 \div 6 = 1145887\frac{1}{3}$.
6. $3987654 \div 7 = 569664\frac{6}{7}$.
7. $19876532 \div 8 = 2484566\frac{1}{2}$.
8. $2976854 \div 9 = 330761\frac{5}{9}$.
9. $4967532 \div 10 = 496753\frac{1}{5}$.
10. $46879352 \div 11 = 4261759\frac{3}{11}$.
11. $18765314 \div 12 = 1563777\frac{1}{6}$.
12. $78654246 \div 18 = 4369680\frac{1}{3}$.
13. $75088 \div 52 = 1444$.
14. $1674918 \div 189 = 8862$.
15. $31884470 \div 779 = 40930$.
16. $57380628 \div 7575 = 7575\frac{3}{7575}$.
17. $554270292198 \div 7584 = 73084163\frac{1}{1266}$.
18. $88789980979 \div 9584 = 9264397\frac{131}{9584}$.
19. $102030429729 \div 123456 = 826452\frac{95073}{12345}$.
20. $267817946000 \div 36500 = 10077204$.

21. 497 men fell 163798 trees; how many does each fell on an average? Ans. 329.

22. If 148 houses pay a tax of $7844; what is the rate on each on an average? Ans. $53.

23. If $4151436030 are levied from 4455 townships; what is the portion of each on an average? Ans $93186.

24. How many lots of 6754 each are contained in 396809151372? Ans. 58763718.

25. What quotient will be obtained by dividing 961504803 twice by 987? Ans. 987.

## 9.—TABLES of MONEY, WEIGHTS & MEASURES.

| BRITISH OR STERLING MONEY. | | DECIMAL COINAGE. | |
|---|---|---|---|
| 4 farthings, or 2 half pennies, are........ | 1 penny (d.) | 10 mills (M) are........ | 1 cent (ct.) |
| 12 pence............ | 1 shilling (s.) | 10 cents.......... .... | 1 dime (d.) |
| 20 shillings........... | 1 pound (£) | 10 dimes, or 100 cents... | 1 dollar ($) |

### AVOIRDUPOIS WEIGHT.

TABLE.

| | | | |
|---|---|---|---|
| 16 drams make........................... | 1 ounce, | marked | oz. |
| 16 ounces.................................. | 1 pound, | " | lb. |
| 25 pounds.................................. | 1 quarter, | " | qr. |
| 4 quarters.............. ............ | 1 hundredweight, | " | cwt. |
| 20 cwt..................................... | 1 ton, | " | t. |

NOTE.—This weight is used in weighing heavy articles, as meat, groceries, vegetables, grain, etc.

## TROY WEIGHT.

TABLE.

| | | | |
|---|---|---|---|
| 24 grains (grs.) make | 1 pennyweight, | marked | dwt. |
| 20 pennyweights | 1 ounce, | “ | oz. |
| 12 ounces | 1 pound, | “ | lb. |

NOTE.—Troy weight is used in weighing the precious metals and stones.

## APOTHECARIES' WEIGHT.

TABLE.

| | | | |
|---|---|---|---|
| 20 grains (grs.) make | 1 scruple, | marked | scr. |
| 3 scruples | 1 dram, | “ | dr. |
| 8 drams | 1 ounce, | “ | oz. |
| 12 ounces | 1 pound, | “ | lb. |

NOTE.—Apothecaries and Physicians mix their medicines by this weight, but they buy and sell by Avoirdupois.

## PRODUCE WEIGHT-TABLE FOR CANADA.

| GRAIN. | | | | | SEEDS. | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Wheat | 60 | pounds | to the | bushel. | Clover | 60 | pounds | to the | bushel. |
| Oats | 34 | “ | “ | “ | Flax | 50 | “ | “ | “ |
| Corn | 56 | “ | “ | “ | Timothy | 48 | “ | “ | “ |
| Corn in cob | 80 | “ | “ | “ | Hemp | 54 | “ | “ | “ |
| Barley | 48 | “ | “ | “ | Blue grass | 14 | “ | “ | “ |
| Rye | 56 | “ | “ | “ | Red Top | 8 | “ | “ | “ |
| Buckwheat | 48 | “ | “ | “ | Hungarian grass | 48 | “ | “ | “ |
| Peas | 60 | “ | “ | “ | Millet | 48 | “ | “ | “ |
| Beans | 60 | “ | “ | “ | Rape | 50 | “ | “ | “ |
| Tares | 60 | “ | “ | “ | | | | | |

| VEGETABLES. | | | | | VEGETABLES. | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Potatoes | 60 | pounds | to the | bushel. | Castor Beans | 40 | pounds | to the | bushel. |
| Parsnips | 60 | “ | “ | “ | Malt | 36 | “ | “ | “ |
| Carrots | 90 | “ | “ | “ | Dried Peaches | 33 | “ | “ | “ |
| Turnips | 90 | “ | “ | “ | Dried Apples | 22 | “ | “ | “ |
| Beets | 60 | “ | “ | “ | Salt | 56 | “ | “ | “ |
| Onions | 60 | “ | “ | “ | Bran | 20 | “ | “ | “ |

## LINEAR (OR LONG) AND SQUARE MEASURE.

| LINEAR. | | SQUARE. | |
|---|---|---|---|
| 12 inches (in.) make | 1 foot (ft.) | 144 inches make | 1 foot (ft.) |
| 3 feet | 1 yard (yd.) | 9 feet | 1 yard (yd.) |
| 5½ yards | 1 rod or perch. | 30¼ yards | 1 rod (rd.) |
| 40 rods | 1 furlong (fur.) | 40 rods | 1 rood (r.) |
| 8 furlongs | 1 mile (m.) | 4 roods | 1 acre (a.) |

## LAND MEASURE.

| LENGTH | | AREA. | |
|---|---|---|---|
| $7\frac{92}{100}$ inches make | 1 link. | 10,000 square links make | 1 sq. chain |
| 25 links | 1 rod. | 10 square chains | 1 acre. |
| 4 rods or 100 links | 1 chain. | | |
| 80 chains | 1 mile. | | |

In solid measure, *i. e.*, the measurement of solids, 1728 (the third power or cube of 12,) inches make 1 cubic foot, and 27 cubic feet (*i. e.* 3×3×3,) make 1 cubic yard. In measuring timber, 40 cubic feet of round timber make what is called a ton, and the same name is given to 50 feet of hewn timber. A cord of firewood is 8 feet long, 4 feet wide, and 4 feet high, and therefore its solid content is 8×4×4=128 feet.

Dry goods are measured by the yard, and fractions of a yard, the fractions used being one-quarter, one-eighth, and one-sixteenth.

## MEASURES OF CAPACITY.

| DRY. | |
|---|---|
| 2 pints make...... | 1 quart (qt.) |
| 4 quarts .... .... | 1 gallon (gal.) |
| 2 gallons.......... | 1 peck (pk.) |
| 4 pecks .......... | 1 bushel (bu.) |
| 36 bushels ......... | 1 chaldron (ch.) |

The last is seldom used.

| LIQUID. | |
|---|---|
| 4 gills make..... | 1 pint (pt.) |
| 2 pints.......... | 1 quart (qt.) |
| 4 quarts ........ | 1 gallon (gal.) |
| 63 gallons........ | 1 hogshead (hhd.) |
| 2 hogsheads ..... | 1 pipe (pi.) |
| 2 pipes .......... | 1 tun (tun.) |

| MEASURE OF TIME. | |
|---|---|
| 60 seconds make......... | 1 minute. |
| 60 minutes.............. | 1 hour. |
| 24 hours................ | 1 day. |
| 365¼ days .............. | 1 year. |

| ANGULAR OR CIRCULAR MEASURE. | |
|---|---|
| 60 seconds make. | 1 minute ($1'$.) |
| 60 minutes ...... | 1 degree ($1^{\circ}$.) |
| 360 degrees ...... | 1 complete circle. |

There are other units applied to certain articles, e. g., 12 articles, one dozen ; 20 articles, one score ; 144 articles, one gross ; 24 sheets of paper, one quire ; 20 quires, one ream.—14lbs., one stone. This last weight is varied in many places, 15lbs. and 16lbs., according to the nature of the article sold, e. g.,—potatoes, as an allowance for earth adhering.

## THE CALENDAR MONTHS OF THE YEAR.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| January ............ | has | 31 | days. | July.................. | has | 31 | days |
| February ........... | " | 28 | " | August ............... | " | 31 | " |
| March .............. | " | 31 | " | September............ | " | 30 | " |
| April............... | " | 30 | " | October.............. | " | 31 | " |
| May ................ | " | 31 | " | November............. | " | 30 | " |
| June ............... | " | 30 | " | December............. | " | 31 | " |

Every fourth year is called Leap-year, in which February has 29 days.—If the last two figures denoting the year can be divided evenly by 4, it is Leap-year.

## DECIMAL COINAGE.

**10.** The principle of the decimal coinage is generally understood to depend on the rules of decimal fractions ; but as it is merely a separate and co-ordinate result of the common system of notation, we may explain it here, independently of the theory of decimal fractions.

We have already explained, that according to the Arabic notation, each digit has one-tenth the value that it would have if situated one place farther to the left. Thus, in the number 88, the digit to the right expresses 8 units, while that to the left expresses 8 tens. Now we cannot have any integer less than unity, but we may have to make calculations respecting quantities less than the unit under consideration, *e. g.*, in calculating by dollars, we may have to take cents into account, and as the cent is a sub-division of the unit, a dollar, some new character must be introduced to indicate this transition from the integral unit to a part of it. This is done very simply by interposing a mark like the period or full point (.) in printing.—This is usually called the decimal point, though it sometimes gets the vague and awkward name of the separatrix. This simple but admirable contrivance is ascribed to one Stevinus or Stevens, of the Netherlands, who gave his suggestion to the public about the year 1585. Its excellence consists in its being simply an extension of the common notation. The original system marks only the repetition of the unit of measure,—this applies the same principle to the sub-division of the unit into parts. To explain this, we have only to carry out the illustration already given regarding integers. We saw that the extreme right hand figure, 8 in our example, stood for 8 units, and was *one-tenth* of the preceding one; just in the same manner another figure, 8, placed to the right of the units' figure, will express *one-tenth* of those units, and the decimal point is used to mark this descending from integers to parts of the integral unit, and is written thus: 8.8, and means eight units, and eighth-tenths of that unit. If another 8 be added, thus: 8.88, it will express eight-tenths of the preceding unit, *i. e.*, eight-tenths of one-tenth, which is the same as eight one-hundredths of unity, and thus we have the descending scale by tenths towards the right of the decimal point, in the same manner as we had the ascending scale by tens towards the left. As a farther illustration, we may begin at the extreme right, as in 888.888, and we find throughout that each figure to the left is ten times that immediately to its right.

The decimal coinage adopts a certain unit called a dollar—the dollar is then sub-divided into ten equal parts, and each part is called a dime, the dime, in like manner, is divided into ten equal parts, and each part is called a cent; and the cent is divided into ten equal parts, and each part is called a mill. The mill enters into many calculations, though no coin of its value has ever been issued. It is from this sub-division by ten, that the name *accimai*, derived from the latin *decem*, ten, is applied to this coinage. In the example 8.888, the first 8 means 8 dollars; the second, 8-tenths of a dollar, or 8 dimes, or 80 cents; the third, 8-tenths of a dime, or 8 one-hundredths of a dollar, or 8 cents; and the fourth 8-tenths of a cent, or 8 one-hundredths of a dime, or 8 one-thousandths of a dollar, or a mill, (from the latin *mille*, a thousand.) In naming any sum, it is not usual to mention either dimes or mills, but only dollars and cents. Thus: $\overset{\$}{12}\cdot\overset{d}{8}\overset{c}{7}\overset{m.}{5}$ is written $12.875, or $$12.87\frac{1}{2}$ and is read twelve dollars eighty-seven and-a-half cents, which is perfectly correct, as 8 dimes make 80 cents, and 5 mills make half a cent. We noted, in treating of simple division, that when the terms in which a question is expressed require us to divide a less number by a greater, or in the case of remainders, the division is indicated by writing the dividend above the divisor, and separating them by a line,—thus: 7÷8 is written $\frac{7}{8}$. So to indicate that 1 dollar is divided into 100 cents, we write $$\frac{1}{100}$, which means the one-hundredth part of a dollar, and therefore dollars and cents are sometimes written, especially in bills and drafts, in this manner, $$12.\frac{25}{100}$, but the form $12.25 is generally preferable. To show the reason of the form $1.05, for one dollar and five cents, we have only to notice that the form $1.5, would mean one dollar and five dimes, or fifty cents; whereas $1.05 means one dollar, no dimes, and five cents.

From the foregoing explanations, it is plain that the rules for the addition, subtraction, multiplication and division of abstract numbers, or applicate numbers of only one denomination, apply also to dollars and cents, because they increase from right to left, and decrease from left to right, according to the same law, that is, in the former case by tens, and in the latter by tenths.

It would be of great benefit to the whole commercial community, and perhaps still greater to the farmer, if the decimal scale were adopted in weights and measures, as well as in money, as it would materially simplify and expedite all calculations. Every one must feel and admit the very great ease and rapidity with which every operation is effected, accounts made up, and books kept in dollars and cents, in comparison with the sub-division into pounds, shillings and pence, and the difference would be at least as great regarding weights and measures. It would also very much accelerate the learner's progress, for it would save him the heavy labour of committing to memory the formidable host of tables, through which he has now to cut his way—the whole processes of reduction would be compressed into "nut-shell" dimensions, and the memory would not be over-taxed in after years to keep up the recollection of the tables conned in youth. Besides, by the plan we have suggested, the pupil could pass at once from the elementary rules to the higher ones, such as proportion and interest, and could either get into business in a much shorter time than is possible at present, or devote his time to higher and more important studies.

## EXERCISES.

*Addition of dollars and cents.*

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| | | $85.50 | $116.20 |
| | $13.19 | 49 63 | 291.45 |
| $125.75 | 14.16 | 92.18 | 89.75 |
| 98.50 | 85.92 | 37.09 | 365.84 |
| 25.15 | 64.15 | 8.92 | 91.50 |
| 76.05 | 37.25 | 76.45 | 76.15 |
| 91.11½ | 91.20 | 25.75 | 485.00 |
| 43.87½ | 18.75 | 64.16 | 157.92 |
| 84.20 | 29.10 | 18.60 | 263.75 |
| 67.62½ | 47.85 | 59.11 | 188.25 |
| 39.80 | 55.55 | 148.17 | 39.48 |
| 17.37½ | 72.63 | 265.90 | 136.13 |
| 669.44 | 529.75 | 931.46 | 2301 42 |

| (5.) | (6.) | (7.) | (8.) |
|---|---|---|---|
| $11.27 | | | $55.63 |
| 45.15 | $44.50 | $296.75 | 17.75 |
| 54.72 | 67.23 | 176.84 | 84.18 |
| 31.30 | 89.75 | 518.50 | 29.88 |
| 49.50 | 27.63 | 369.63 | 45.13 |
| 16.75 | 95.13 | 627.45 | 38.81 |
| 84.28 | 38.88 | 258.13 | 67.25 |
| 14.85 | 17.45 | 591.18 | 96.20 |
| 9.44 | 56.64 | 179.25 | 77.63 |
| 28.09 | 73.85 | 567.42 | 8.75 |
| 345.35 | 511.06 | 3585.15 | 521.21 |

(9.)

*Sold to* J. JONES,

| | |
|---|---|
| 20 yards cloth | $75.25 |
| 14 mats | 21.56 |
| 16 hats | 33.50 |
| 5 pairs of blankets | 28.75 |
| 15 yards sealskin | 40.25 |
| 15 yards of serge | 9.63 |
| 28 yards fine cloth | 112.88 |
| | 321.82 |

(10.)

| |
|---|
| $157.29 |
| 268.73 |
| 985.45 |
| 197.06 |
| 385.18 |
| 876.75 |
| 795.85 |
| 567.13 |
| 659.63 |
| 4893.07 |

(11.)

*Sold to* S. FULTON, Aurora,

| | |
|---|---|
| 12 pairs of worsted stockings | $13.50 |
| 18 " " flannel drawers | 22.75 |
| 24 " " kid gloves | 8.63 |
| 56 school books | 49.72 |
| 29 yards of satin | 83.23 |
| 96 school copy books | 1.84 |
| 180 yards of ribbon | 29.76 |
| 84 yards of ticking | 22.68 |
| 122 yards of sheeting | 23.18 |
| | 255.29 |

12. The shares in an oil-well speculation are $5 each; A. takes 15 shares; B. 25; C. 20; D. 1; E. 11; F. 37; G. 16; H. 18; I. 8; K. 21; L. 14; and 14 other persons take 10 shares each; what is the capital of the company, and how many shares are there?

Ans. $1,630 and 326 shares.

13. If 17 vessels bring to the port of Boston cargoes of the following values; what does the whole amount to? $2365.75, $1793.87, $3815.25, $2718.63, $4186.50, $3179.13, $1623.88, $4311.75, $1987.38, $2975.75, and the other 7 average $2689.13.

Ans. $47781.80.

*Subtraction of dollars and cents.*

| (1.) | (2.) | (3.) |
|---|---|---|
| $567819.83 | $83756.17 | $17423.37½ |
| 278956.89 | 76489.71 | 9654.63½ |
| 288862.94 | 7266.46 | 7768.74 |

4. What is the difference between 2769 dollars and 50 cents, and 987 dollars 87½ cents? Ans. $1781.62½.

5. The debit side of a ledger is $1770.80, and the credit side $876.50; what is the balance? Ans. $894.30.

6. The credit side of a cash book is $8795.88, and the debit side is $10358.18; what is the balance? Ans. $1562.30.

7. A firm owes $227968.25, and the estate is worth $98764.75; what is the state of the affairs of the firm? Ans.—The firm is unable to pay $129,203.50 over and above the assets.

8. A ship and cargo were worth $27509.50,—the ship was lost, and only $6784.60 worth of the cargo saved; what was the loss? Ans. $20724.90.

9. A cotton mill was totally destroyed by fire; the mill and its contents were worth $78616; it was insured in one office for $11760; in another, for $9845; in another, for $10800; and, in a fourth, for $12685; did the owner lose, and if so, how much? Ans.—He lost $33526.

10. I have sold for cash, during the last month, $2786.88 worth of goods, I have received payment of S. Fulton's account, $255.29 received proceeds of J. Jones' note, $302.64; received interest on sundry debentures, $278.50; sold my shares in the G. T. R. for $785.75; received in cash, interest from Royal Canadian Bank, $187.25; sold block of buildings in King street for $1719.00, and shares in Rossin house for $718.50. Paid S. Smellie's account, $261.88; for sundry insurances, $879.60; rent of office, $150.00; for consignment to Liverpool, $2786.50, and charges on same, $175.63; what is the balance of receipts above expenditure? Ans. $2780.20.

*Multiplication of dollars and cents.* *

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| $365.75 | $1873.47 | $865.63 | $24786.38 |
| 87 | 69 | 93 | 45 |
| 256025 | 1686123 | 259689 | 12393190 |
| 292600 | 1124082 | 779067 | 9914552 |
| 3182025 | 12926943 | 8050359 | 111538710 |

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* We must here caution the tyro against such modes of expression as this,—"multiply $85 by $12." Such an expression is simply absurd, for to say $12 times $85, might as well mean 1200 times $85, or 12000 times $85,

Such questions as the following may be worked in three different ways:

| | | | | | |
|---|---|---|---|---|---|
| (5.) | $487.63½ | or | $487.63½ | or | $487.63½ add in 14. |
| | 28 | | 28 | | 28 |
| | 390104 | | 390104 | | 390118 |
| | 97526 | | 97526 | | 97526 |
| | | ½ of 28= | 14 | | |
| | 1365364 | | | | 1365378 |
| ½ of 28= | 14 | | 1365378 | | |
| | 1365378 | | | | |

It is often convenient to make the number expressing the dollars and cents the multiplier, especially when they form the shorter line. Thus to multiply 63 cents by 3587, we make .63 the multiplier, and 3587 the multiplicand, and so in examples 7, 8 and 9.

| (6.) | (7.) | (8.) | (9.) |
|---|---|---|---|
| 3587 | 2876 | 5796 | 14986 |
| .63 | 8.63 | 4.87½ | 9.12½ |
| 10761 | 8628 | 40572 | 29972 |
| 21522 | 17256 | 46368 | 14986 |
| | 23008 | 23184 | 134874 |
| 225981 | | 2898 | 7493 |
| | 2481988 | | |
| | | 2825550 | 13674725 |

10. If 987 houses pay a tax of $3.37½; what does the whole amount to? Ans. $3331.12½.

---

which would all give widely different results. We may indeed have to multiply a denominate number representing $85, by another denominate number representing $12, as often happens in questions involving proportion, *e. g.*, in interest; but so soon as we use the number 12, or any denominate number as a multiplier it ceases to be denominate, and becomes abstract, and no longer represents any denomination, but merely the number of times the other is to be repeated. We object even to the putting of such questions as "catch questions," for the learner is but too apt to look at the question just as it stands, without ever thinking of the principle on which it is intended to try him. The absurdity of the expression may be shown by the different lights in which the long discussed question, to multiply 2s. 6d. by 2s. 6d, may be viewed. (1.) As 2s. 6d. is ⅛ of a pound, the question may be taken as meaning that 2s 6d is to be divided into 8 equal parts, and 1 of them taken, which would be 3¾d. (2.) As 2s. 6d. is 2½ shillings, the question might be taken as meaning that 2s. 6d. was to be

Division of dollars and cents.

1. \$28642.14÷29=\$987.66.
2. \$37133.34÷87=\$426.82.
3. \$60509.68÷76=\$796.18.
4. \$43009.75÷98=\$438.87½.
5. \$1943243.55÷983=\$1976.85.
6. \$31421.25÷63=\$498.75.
7. \$28479.75÷78=\$365.12½.
8. \$2595.37½÷769=\$3.37½.

9. \$2927.30 a year; how much per day? Ans. \$8.02.

10. \$3953.19 a year; how much for every working day? Ans. \$12.63.

11. 269 persons have to pay a tax of \$1312.72; what is the average tax on each? Ans. \$4.88.

12. A collection of \$544.04 is made by 1876 persons; how much did each give on an average? Ans. 29 cents.

To reduce currency money to the denominations of the decimal coinage. Since 100 cents make 1 dollar, and 4 dollars make 1 pound, 400 cents make 1 pound currency, and therefore to find the number of cents in any given number of pounds, we must multiply the pounds by 400. Again, since 20 cents make 1 shilling or 12 pence, to find the number of cents in any given number of shillings, we must multiply the shillings by 20. Lastly, 5 cents, are equal to 3 pence, and 12 farthings are also equal to 3 pence, and (Ax. I.) things that are equal to the same thing, are equal to one another; therefore, 5 cents are equal to 12 farthings, and 1 farthing is the $\frac{1}{12}$ of 5 cents, or $\frac{5}{12}$ of 1 cent. Hence to find the number of cents in any number of pence and farthings, we multiply the number of farthings in the given pence and farthings by 5, and divide the product by 12. Having obtained the three results, we add them all together. Thus to change £48 18s. 9¾d. to dollars and cents, we multiply 48 by 400, 18 by 20, and take $\frac{5}{12}$ of 9¾, or 39 farthings, and add the three together, which gives us 19576¼ cents, or \$195.76¼.

$$\begin{array}{r}
48\times400=19200\\
18\times\ 20=\ \ 360\\
9\tfrac{3}{4}=39\text{f.}\times\tfrac{5}{12}=\ \ 16\tfrac{1}{4}\\
\hline
19576\tfrac{1}{4}
\end{array}$$

---

repeated 2½ times, which would make 6s. 3d. (3.) The interpretation might be, that as 2s. 6d. is 30 pence, that the other 2s. 6d. is to be repeated 30 times, which would give £3 15s. 0d. (4.) The phrase may also be interpreted as meaning that 30d. was to be repeated 30 times, which would also give £3 15s. 0d. The last two interpretations are the same in two different forms, and give the same result. This is the only view in which the expression has any sense, and proves our statement, that whenever a denominate number is used as a multiplier, it ceases to be denominate, and becomes abstract. The same principle will apply to division.

### EXERCISES.

(1.)

$$\begin{array}{lrr}
\pounds 79 & \times 400= & 31600 \\
16 & \times\ 20= & 320 \\
6\frac{1}{2}\text{d} & \times\ \frac{5}{12}= & 10\frac{5}{6} \\
\hline
 & & \$319.30\frac{5}{6} \\
\hline
\end{array}$$

(2.)

$$\begin{array}{lrr}
\pounds 117 & \times 400= & 46800 \\
17 & \times\ 20= & 340 \\
8\frac{3}{4}\text{d} & \times\ \frac{5}{12}= & 14\frac{7}{12} \\
\hline
 & & \$471.54\frac{7}{12} \\
\hline
\end{array}$$

3. $\pounds 87.14.10\frac{3}{4}=\$350.97\frac{1}{12}$.
4. $\pounds 29.19.9=\$119.95$.
5. $\pounds 67.13.4\frac{3}{4}=\$270.67\frac{11}{12}$.
6. $\pounds 279.15.10\frac{1}{2}=\$1119.17\frac{1}{2}$.
7. $\pounds 118.11.4\frac{1}{2}=\$474.27\frac{1}{2}$.
8. $\pounds 79.8.4=\$317.66\frac{2}{3}$.
9. $\pounds 37.18.8=\$151.73\frac{1}{3}$.
10. $\pounds 57.8.11\frac{3}{4}=\$229.79\frac{7}{12}$.
11. $\pounds 49.7.6=\$197.50$.
12. $\pounds 137.16.8=\$551.33\frac{1}{3}$.
13. $\pounds 236.19.2\frac{1}{2}=\$947.84\frac{1}{6}$.
14. $\pounds 19.16.8=\$79.33\frac{1}{3}$.
15. $\pounds 98.1.1\frac{1}{2}=\$392.22\frac{1}{2}$.
16. $\pounds 87.11.8=\$350.33\frac{1}{3}$.
17. $\pounds 457.12.6=\$1830.50$.
18. $\pounds 219.4.7\frac{3}{4}=\$876.92\frac{11}{12}$.
19. $\pounds 49.9.4\frac{3}{4}=\$197.87\frac{11}{12}$.
20. $\pounds 287.18.10\frac{1}{2}=\$1151.77\frac{1}{2}$.

To change dollars and cents to Halifax currency, we must reverse the above operation. Thus, to reduce \$195.76¼ to £. s. d.—First, reduce the dollars and cents to cents, then divide by 400, which gives 48, the even number of pounds, with a remainder of 376¼ cents; then divide this remainder by 20, which gives 18, the number of shillings, with a remainder of 16¼ cents, as in the converse operation, we multiplied by 5, and divided by 12, so now we multiply by 12, and divide by 5; thus, 16¼×12=195, and 195÷5=39, the number of farthings, and this being reduced to pence and farthings, gives 9¾, so that \$195.76¼=£48.18.9¾.

```
400)19576¼(48
     400
    ------
     3576¼
     3200
    ------
  20)376¼(18
     20
    ------
     176
     160
    ------
      16¼
      12
    ------
    5)195 (39
```

Or the work may be shortened by the following method. As \$4 make £1, the number of £'s in \$195.76¼, will be the same as the number of times that 4 is contained in the 195 dollars, which gives £48, and \$3 remain-

```
$195—76¼
 ————
4)195
 ————
 £48—300
     ————
   20)376¼
      s18—16¼
            3
          ————
        5 48¾
          ————
           9¾d.
```

ing. Now, these three dollars are equivalent to 300 cents, which added to the remaining 76¼ cents, gives 376¼ cents; this divided by 20, will give the shillings, because 20 cents are equal to one shilling, and it is self-evident that the number of shillings in 376¼ cents, will be the same as the number of times 20 is contained in that number, which gives 18 shillings, and 16¼ cents remaining. Lastly, as 5 cents are equal to 3 pence, one cent will be equal to ⅕ of 3 pence, which is ⅗ of a penny; therefore, if one cent is equal to ⅗ of a penny, the remaining 16¼ cents will be equal to 16¼ times ⅗ of a penny, which is 9¾d.; hence we have $195.76¼ equal to £48.18.9¾.

### EXERCISES.

| | | |
|---|---|---|
| 1. | Reduce $119.95 to Halifax currency. | Ans. £29.19.9. |
| 2. | Reduce $270.67$\frac{11}{12}$ " " | Ans. £67.13.4¾. |
| 3. | Reduce $474.27½ " " | Ans. £118.11.4½ |
| 4. | Reduce $197.50 " " | Ans. £49.7.6. |
| 5. | Reduce $1119.17½ " " | Ans. £279.15.10½. |
| 6. | Reduce $551.33⅓ " " | Ans. £137.16.8. |
| 7. | Reduce $1830.50 " " | Ans. £457.12.6: |
| 8. | Reduce $1151.77½ " " | Ans. £287.18.10½. |

### MIXED EXERCISES.

| | | |
|---|---|---|
| 1. | Reduce £436.7.8½ to dollars and cents. | Ans. $1745.54⅙. |
| 2. | Reduce $547.87 to Halifax currency. | Ans. £136.19.4⅕. |
| 3. | Reduce £783.13.5¼ to dollars and cents. | Ans. $3134.68¾. |
| 4. | Reduce $576.85 to Halifax currency. | Ans. 144.4.3. |
| 5. | Reduce £606 19.8¾ to dollars and cents. | Ans. $2427.94$\frac{7}{12}$. |
| 6. | Reduce $375.99 to Halifax currency. | Ans. £93.19.11⅗. |
| 7. | Reduce 3s. 8¼d. to dollars and cents. | Ans. 73¾ cents. |
| 8. | Reduce 17 cents to Halifax currency. | Ans. 10⅕ pence. |
| 9. | Reduce 10¾ pence to dollars and cents. | Ans. 17$\frac{11}{12}$ cents |
| 10. | Reduce 23 cents to old Canadian currency. | Ans. 13⅘ pence |

## THE FARMERS' RULE FOR REDUCING CENTS TO PENCE, AND PENCE TO CENTS.

### QUESTION.

Said farmer A. to grocer B.
There's something here that puzzles me ;
I sold some butter here to-day,
I sold by cents, by pence they pay ;
How shall I change the cents to pence,
And know the trick from this day hence ?

### ANSWER.

Five cents are three pence you must know,
As twenty cents to twelve pence go ;
Three times the cents, the fifth of that
Is just the thing you would be at ;
And if you buy from grocers here,
That other case is just as clear,
Five times the pence, the third of it
Will make you safe and always fit.

◆

## REDUCTION.

**11.**—REDUCTION is the mode of expressing any given quantity in terms of a higher or lower denomination, *e. g.*, expressing any given number of dollars as cents, and *vice versa*, any number of cents as dollars.

When a higher denomination is changed to a lower, (as dollars to cents,) the process is called reduction *de*scending, and when a lower is changed to a higher, (cents to dollars,) it is called reduction *as*cending.

### RULE.

To express any given quantity in terms of a lower denomination, multiply it by the number of units which it contains of the next lower denomination, and add in the given units of that denomination, and so on to the lowest denomination given. Thus, to express 8 dollars and 25 cents as cents, multiply 8 by 100, giving 800, and add the 25 cents, giving 825 cents. So also, as in the margin, the pounds are multiplied by 20, for 20s.=£1, and the 11 shillings added in, giving 511 shillings, then these shillings are multiplied by 12, and the 4 pence added in, giving 6136 pence, and this finally is multiplied by 4, and the two farthings added in, giving 24546 farthings. So also \$98×100=9800 cents. To express a lower in terms of a higher denomination, divide the lower by the number that denotes how many units of the lower are contained in one unit of the higher. Thus

£25.11.4½
20
———
511
12
———
6136
4
———
24546

to reduce 24546 farthings to £. s. d.—since 4 farthings make 1 penny, we divide by 4, and get 6136 pence, with a remainder of 2 farthings, or 1 half-penny. Again, since 12 pence make one shilling, we divide 6136 by 12, and get 511 shillings, with a remainder of 4 pence. Lastly we divide 511 by 20, and get 25 pounds, with a remainder of 11 shillings, so that 24546 farthings make £25 11s. 4½d. So also, since 100 cents make one dollar, to reduce 12579 cents to dollars, divide by 100, and 12579÷100=$125.79. We thus see that cents can be changed to dollars and cents, by simply cutting off two figures from the right. So also dollars can be changed to cents by adding two ciphers, or dollars and cents can be changed to cents by removing the decimal point two places towards the right.

### EXERCISES.

1. How many dollars are there in 47986 cents? Ans. $479.86.
2. How many cents are there in 187 dollars? Ans. 18700.
3. How many pence are there in £87.12.8? Ans. 21032.
4. How many pence are there in £113.18.4? Ans. 27340.
5. How many farthings are there in £79.15.10½? Ans. 76602.
6. How many half pence in £97.17.6? Ans. 46980.
7. How many pounds, &c., are there in 7983 pence?
Ans. £33.5.3.
8. How many pounds, &c., are there in 156793 farthings?
Ans. £163.6.6¼.
9. How many pounds are there in 2 tons 16 cwt. 2 qrs. and 21 lbs.? Ans. 5671.
10. How many pounds are there in 18 cwt. and 22 lbs.?
Ans. 1822.
11. Reduce 14796 lbs. to tons, &c.?
Ans. 7 tons, 7 cwt., 3 qrs., 21 lbs.
12. Reduce 7643 quarters to tons, &c.?
Ans. 95 tons, 10 cwt., 3 qrs.
13. How many drams are there in 18 lbs., 13 oz. and 15 drs.?
Ans. 4831.
14. How many pounds are there in 2785 drams?
Ans. 10 lbs., 14 oz., 1 dr.
15. How many grains are there in 17 lbs., 11 oz., 18 dwt. and 22 grains? Ans. 103654.
16. How many lbs. in 46891 grs.?
Ans. 8 lbs., 1 oz., 13 dwt., 19 grs.

17. Reduce 98 miles, 5 furlongs and 30 rods to rods ?
Ans. 31590 rods.

18. How many inches from Toronto to Hamilton, (38 miles) ?
Ans. 2407680.

19. How many miles are there in 527168 feet ?
Ans. 99 miles, 6 fur., 29 pr., 2 yds., 3 ft., 6 in.

20. Reduce 57 acres, 3 roods and 24 rods to rods ?
Ans. 9264 rods.

21. How many square yards are there in 17 acres, 2 roods and 18 rods ? Ans. 85244½ yards.

22. Find the number of acres, &c., in 479685971 square inches ?
Ans. a. 76.1.35.19.2.119.

23. How many acres do 176984 square yards make ?
Ans. a. 36.2.10.21½.

24. How many square links are there in 37 acres ?
Ans. 3,700,000 links.

25. How many acres, &c., in 479,863,201 square links ?
Ans. 4798 a., 6 ch., 3201.

26. 7,864,391 cubic inches ; how many cubic yards ?
Ans. yds. 168.15.263.

27. 9 cubic yards, 7 cubic feet, 821 cubic inches ; how many cubic inches ? Ans. 432821 cubic inches.

28. How many gills does a tun contain ? Ans. 8064 gills.

29. How many gallons, &c., do 479865 gills make ?
Ans. gals. 14995.3.0.1.

30. How many pints are there in 28 bu., 3 pecks and 1 gal. ?—
Ans. 1848 pints.

31. 27 yards, 3 qrs., 3 nails ; how many nails ? Ans. 447 nails.

32. 286 nails ; how many yards, &c. ?
Ans. 17 yards, 3 qrs., 2 nls.

33. 36° 40′ 25″ ; how many seconds ? Ans. 132025″.

34. How many degrees, &c., in 49786″ ? Ans. 13°.49′.46″.

35. The population of Toronto is 45,288 ; what would a poll tax of 5 cents each amount to ? Ans. $2264.40.

36. How long would it take a railway train to move a distance equal to that of the earth from the sun, (95 millions of miles,) at a speed of 52 miles an hour ? Ans. 208 years, 201 days, $19\frac{1}{13}$ hours.

37. The area of Upper Canada is 94,720,000 acres ; how many square feet ? Ans. 4,126,003,200,000 square feet.

38. Sound moves about 1130 feet in a second of time; how long would it be in moving from the earth to the sun?

Ans. 14years, 27 days, 15 hours, 50 min., $5\frac{350}{1130}$ sec.

39. How many seconds of this century had elapsed at the end of 1864, counting the day at 24 hours? Ans. 2,019,686,400″.

40. The great bell of Moscow weighs 127,836 lbs.; how many tons, &c., does it weigh, the quarter being 28 lbs.?

Ans. 57t. 1c. 1q. 16lbs.

41. How many days from the 11th July, 1861, to the 1st of April, 1864? Ans. 995 days.

42. A congregation of 569 persons made a collection of £40.6.1; how many pence did each give on an average? Ans. 17d.

43. The British mint can strike off 20,000 coins in an hour; what is the value of all the pennies coined in one day of 12 hours' work? Ans. £1,000.

44. 417 tons of fish were caught at Newfoundland in one season, and sold by the stone of 14 lbs., at an average price of 42 cents a stone; what did they bring? Ans. $25020.

45. How many feet from pole to pole, the earth's diameter being 7945 miles? Ans. 41949600 feet.

## DENOMINATE NUMBERS.

**12.**—WHEN numbers are spoken of in general, without reference to any particular articles, such as money or merchandise, they are called abstract, but when they are applied to such articles they are sometimes called *applicate*, as being *applied* to some particular articles to express their quantity; sometimes they are called *concrete*, (growing together,) as attached to some particular substances, and sometimes they are called *denominate*, as denoting quantities that consist of different denominations, as dollars and cents,—pounds, ounces, &c. The elementary rules of addition, subtraction, multiplication and division, are performed on denominate numbers, exactly in the same way as on abstract numbers, with this single difference, that when a lower denomination is added, and gives a sum equal to one or more units of the next higher denomination, we carry that unit, or those units, to the next higher denomination. Thus: if the sum were 24 inches, we should call that two feet. In abstract and decimal numbers we always reduce, or carry, by tens.

### EXAMPLE

Here we find the sum of the pence to be 28, and as 12 pence make 1 shilling, the number of shillings in 28 pence will be the same as the number of times that 12 is contained in 28, which is twice, with a remainder of 4; therefore we write the 4 pence under the pence column, and add up the 2 shillings with the shillings' column, and obtain 54 shillings, and as 20 shillings make 1 pound, the number of pounds in 54 shillings will be the same as the number of times that 20 is contained in 54, which is twice, with a remainder of 14, and therefore we write the 14 shillings under the shillings' column, and add up the 2 pounds with the units' column of pounds, and now since the remaining columns are all of the same denomination, we proceed as in simple addition, and find the whole to be £236.14.4. The same illustration will apply to the subtraction, multiplication and division of all kinds of denominate numbers.

£49. 7. 6
83.15.10
67.12. 8
35.18. 4
236.14. 4

In the exercises on the addition of denominate numbers, one question in abstract numbers is given to contrast with the denominate.

### EXERCISES.

| (1.) | (2.) | (3.) | (4.) |
|---|---|---|---|
| | | £76.18. 4 | $1967.87½ |
| | $857.63 | 17.11. 4½ | 2075.75 |
| 7865437 | 189.50 | 99.19. 9 | 3194.62½ |
| 198675 | 684.87½ | 11.11.11 | 7658.50 |
| 8476154 | 498.75 | 67.15.10½ | 8976.37½ |
| 1869538 | 867.12½ | 79.19. 9 | 2873.12½ |
| 4187643 | 365.37½ | 28.12. 1 | 1769.25 |
| 5768299 | 917.25 | 63. 8. 4½ | 2481.92 |
| 28365746 | 4380.50½ | 445.17. 5½ | 30997.42 |

| (5.) | (6.) | (7.) | (8.) |
|---|---|---|---|
| lbs. oz. drs. | t. cwt. qrs. lbs. | lbs. oz. dwt. grs. | lbs. oz. drs. ser. grs. |
| 13.14.10 | 26.17.3.21 | 3.11.16.21 | 5.11.7.2.19 |
| 15.11.10 | 18.11.0.19 | 5. 8. 7.11 | 4.10.4.1. 7 |
| 11. 4. 9 | 25.15.1.16 | 7. 9.18.23 | 3.11.6.2.14 |
| 8.12.13 | 13.17.2.20 | 11.10.15.17 | 1. 9.3.1.12 |
| 15. 7. 8 | 39. 4.1.23 | 12. 7. 9. 8 | 2. 4.5.0.10 |
| 10.13.11 | 28.16.3.14 | 16.10.11.22 | 6. 7.2.2. 9 |
| 8. 9. 6 | | 18. 8.19.18 | 2. 8.1.1.13 |
| 4.15.15 | | | |
| 89.10. 2 | 153. 3.2.13 | 77. 8. 0. 0 | 28. 4.0.1. 4 |

| (9.) | (10.) | (11.) | (12.) |
|---|---|---|---|
| m. fur. rods. yds. | yds. ft. in. l. | ac. roods, rd. | rods. yds. ft. in. |
| 176.7.39.5 | 18.2.11.11 | 29.3.39 | 39.30.8.143 |
| 85.4.20.1 | 14.2. 7. 9 | 57.2.18 | 18.11.4. 68 |
| 79.6.29.3 | 8.1.10. 7 | 118.0.26 | 24. 4.7.118 |
| 42.3. 8.2 | 11.0. 7. 6 | 75.3.11 | 11.21.2. 96 |
| 67.1.11.2 | 7.2. 8. 5 | 51.1. 8 | 15.27.0.124 |
| 118.3.10.3 | 16.2. 9.10 | 94.1.19 | 27. 6.3. 87 |
| 81.2.31.1 | 8.1. 7. 6 | 63.2.21 | 19.25.2. 38 |
| 79.0.21.2 |  | 78.1.15 |  |
| 18.3.33.3 |  | 19.3.33 |  |
| 749.2. 6.0 | 87.0. 3. 6 | 589.0.30 | 157. 6.3. 98 |

| (13.) | (14.) | (15.) | (16.) |
|---|---|---|---|
| a. ch. links. | ch. b. p. g. qt. pt. | tu. pi. hhd. gal. qt. pt. gl. | yds. qrs. nls. |
| 79.9.9999 | 5.35.3.1.3.1 | 6.1.1.1.3.1.3 | 36.3.2 |
| 117.4.3650 | 7.18.2.0.1.1 | 4.0.1.1.2.0.2 | 19.1.3 |
| 47.5. 941 | 8. 7.1.1.0.1 | 5.1.0.0.1.1.1 | 87.2.1 |
| 56.2.1182 | 3.26.0.0.1.0 | 1.1.0.1.0 | 63.0.2 |
| 27.7.2813 | 4.18.0.1.0.1 |  | 74.2.2 |
| 36.1. 771 |  |  | 93.3.3 |
| 84.8.1160 |  |  |  |
| 449.8. 516 | 29.34.0.0.3.0 | 16.1.1.5.0.0.2 | 375.2.1 |

| (17.) | (18.) | (19.) | (20.) |
|---|---|---|---|
| cwt. qrs. lbs. |  | yrs. days. hrs. min. sec. | cwt. qrs. lbs. |
| 87.3.11 | 359°.59′.59″ | 33.364.23.59.59 | 18.1.18 |
| 49.1.18 | 153 .40 .45 | 28.113.11.48.48 | 22.3.11 |
| 28.3.15 | 270 . 0 . 0 | 17. 97.12. 0. 0 | 9.2.18 |
| 36.1. 8 | 179 .45 .30 | 1.307.23.48.49 | 12.1.15 |
| 88.1.16 | 81 .30 .10 | 12.114. 0. 0. 0 | 8.3.24 |
| 57.3.14 | 89 .59 .59 |  | 31.2. 0 |
| 348.3.7 | 1134 .56 .23 | 93.267.23.37.36 | 103.3.11 |

## LEDGER ACCOUNTS.

The debit and credit sides of four folios of a ledger are as below, what are the balances?

| (21.) Dr. | (21.) Cr. | (22.) Dr. | (22.) Cr. |
|---|---|---|---|
| $1214.75 | $2763.80 | $198.75 | $118.50 |
| 863,09 | 471.38 | 47.63 | 9.05 |
| 291.45 | 365.50 | 18.11 | 16.25 |
| 318.25 | 297.11 | 97.38 | 37.08 |
| 1789.87 | 584.88 | 85.88 | 19.13 |
| 947.63 | 963.15 | 76.20 | 47.75 |
| 2000.00 | 1257.75 | 4.50 | 65.92 |
| 798.38 | 189.60 | 181.60 | 32.40 |
| 2018.50 | 98.13 | 19.25 | 76.50 |
| 164.30 | 756.25 | 76.38 | 7.75 |
| 277.15 | 87.50 | 219.50 | 197.25 |
| 1165.20 | 163.63 | 48.75 | 15.75 |
| 367.40 | 1291.00 | 93.15 | 8.38 |
| 984.70 | 784.25 | 25.50 | 93.15 |
| 273.60 | 79.75 | 81.05 | 67.45 |
| 584.10 | 81.18 | 28.30 | 5.45 |
| 1200.00 | 318.50 | 69.08 | 18.09 |
| 68.75 | 1819.20 | 157.11 | 4.12 |
| 79.15 | 58.50 | 278.00 | 57.60 |
| 56.18 | 176.25 | 59.50 | 28.88 |
| 2860.14 |  | 11.25 |  |
|  |  | 941.12 |  |

| (23.) Dr. | (23.) Cr. | (24.) Dr. | (24.) Cr. |
|---|---|---|---|
| $81.19 | $80.10 | $177.88 | $156.92 |
| 17.11 | 15.65 | 291.16 | 285.15 |
| 45.38 | 39.88 | 356.13 | 356.12 |
| 19.63 | 10.13 | 189.38 | 178.25 |
| 187.13 | 176.15 | 471.63 | 469.10 |
| 87.63 | 89.92 | 785.88 | 698.80 |
| 87.88 | 77.81 | 911.50 | 930.75 |
| 111.11 | 99.88 | 583.15 | 496.20 |
| 134.56 | 16.97 | 432.61 | 547.60 |
| 179.51 | 87.63 | 355.55 | 478.99 |
| 340.25 | 75.75 | 638.27 | 546.54 |
| 224.12 | 56.51 | 436.15 | 372.25 |
| 156.12 | 37.23 | 325.36 | 252.12 |
| $ | $ | $ | $ |

| (25.) Dr. | (25.) Cr. | (26.) Dr. | (26.) Cr. |
|---|---|---|---|
| $176.93 | $1237.75 | $1087.63 | 4786.87 |
| 27.85 | 2763.18 | 457.88 | 183.05 |
| 79.37 | 194.25 | 190.37 | 97.75 |
| 98.11 | 39.37 | 87.12 | 149.15 |
| 35.40 | 8.25 | 94.25 | 13.25 |
| 83.50 | 11.87 | 47.20 | 41.18 |
| 1127.25 | 29.05 | 39.15 | 8.50 |
| 48.18 | 63.20 | 8.75 | 9.75 |
| 250.00 | 71.80 | 367.40 | 11.12 |
| 779.63 | 13.10 | 18.93 | 183.62 |
| 154.20 | 45.50 | 67.45 | 79.10 |
| 59.75 | 25.20 | 21.63 | 814.00 |
| 68.87 | 43.15 | 298.50 | 95.50 |
| 18.75 | 7.50 | 78.60 | 218.00 |
| 28.63 | 50.00 | 189.00 | 59.87 |
| 71.38 | 87.75 | 47.15 | 18.05 |
| 293.63 | 5.00 | 68.10 | 77.40 |
| 185.10 | 31.60 | 54.30 | 38.87 |
| 9.05 | 13.40 | 12.12 | 15.62 |
| 64.20 | 90.75 | 89.75 | 9.87 |
| 38.75 | 15.15 | 118.00 | 14.12 |
| 45.45 | 67.63 | 69.50 | 89.50 |
| 215.87 | 58.50 | 48.75 | 4.20 |
| 7.75 | 67.05 | 36.12 | 67.37 |
| 93.92 | 49.35 | 91.20 | 81.09 |
| 81.88 | 21.25 | 87.63 | 7.05 |
| 68.25 | 35.15 | 90.00 | 57.20 |
| 99.99 | 20.13 | 100.75 | 114.25 |
| 18.12 | 92.87 | 49.15 | 297.00 |
| 27.13 | 35.28 | 87.63 | 78.75 |
| 168.00 | 81.18 | 43.25 | 564.87 |
| 75.75 | 10.80 | 81.37 | 961.34 |
| 738.38 | 51.25 | 92.65 | 268.34 |
| 18.24 | 67.54 | 37.49 | 567.84 |
| 136.25 | 91.12 | 46.87 | 987.69 |
| 126.72 | 18.35 | 91.13 | 356.78 |
| 834.15 | 42.54 | 54.12 | 978.65 |
| 128.71 | 16.21 | 64.54 | 546.37 |
| 136.18 | 25.51 | 57.62 | 786.42 |
| 178.16 | 53.99 | 38.94 | 428.97 |
| 284.77 | 62.87 | 61.87 | 642.85 |
| 326.54 | 91.54 | 93.89 | 529.64 |
| 412.13 | 32.21 | 89.78 | 428.04 |
| 391.15 | 54.12 | 21.46 | 106.70 |
| 267.18 | 77.99 | 64.98 | 500.00 |
| 125.13 | 42.51 | 73.75 | 250.09 |

## SUBTRACTION.

| | | |
|---|---|---|
| (1.) | (2.) | (3.) |
| $147985.87½ | £1573.11. 4½ | $810731.37½ |
| 86997.75 | 976.15.10½ | 341876.62½ |

4. I have taken this month in trade £1796.18.11, and paid £673.10.10 for fall goods, and expended for private purposes, £36. 8.1, and lodged the rest in the Ontario Bank; how many dollars have I banked ? Ans. $4348.

5. I bought 47 tons, 17 cwt., 1 qr., 18 lbs. of grain, and have sold 29 tons, 18 cwt., 3 qrs., 22 lbs. of it; how much have I in store ? Ans. 17 tons, 18 cwt., 1 qr. 21 lbs.

6. If the distance from Toronto to Quebec is 503 miles, 1 fur., 20 rods; and the distance from Montreal to Quebec is 180 miles, 2 fur., 35 rods: what is the distance from Toronto to Montreal?— Ans. m .322.6.25.

7. A farmer possessed 1279 acres, 2 roods, 21 rods, and by his will left 789 acres, 3 roods, 36 rods to his eldest son, and the rest to the second; how much had the younger ?
Ans. 489 acres, 2 roods, 25 rods.

8. The latitude of London (England,) is 51°.30′.49″ N., and that of Gibraltar 36°.6′.30″ N.; how many degrees is Gibraltar south of London ? Ans. 15°.24′.19″.

9. The earth performs a revolution round the sun in about 365 days, 5 hours, 48 minutes and 48 seconds, and the planet Jupiter in about 4332 days, 14 hours, 26 minutes and 55 seconds; how much longer does it take Jupiter to perform one revolution than the earth ?
Ans. 3967 days, 8 h., 38 min., 7 sec.

10. I bought 54 lbs., 10 oz. of tobacco, and 11 oz. of it were lost by drying; and I sold 36 lbs., 12 oz. of it to A.; and 11 lbs., 9 oz. to B.; and used 3 lbs., 14 oz. myself; how much have I remaining, and how much did I get for what I sold, at 6 cents an ounce, and how much did my own consumption and drying come to at the cost price, which was 5 cents an ounce ?
Ans. (1.) 1 lb., 12 oz. (2.) $46.38. (3.) $3.65.

## MULTIPLICATION.

1. $1796×47=$84412.
2. £2.19.2¼×144=£426.3.0.
3. $168.87½×64=$10808.
4. £1.2.9×225=£255.18.9

5. Find the duty on 97 consignments of merchandise at $86.62½ each? Ans. $8402.62½.

It is often convenient to multiply denominate numbers by the *factors* of the multiplier. Thus: to multiply by 84 is the same as to multiply by 7 and 12. Thus, in the annexed examples, since 12×7=84, 18 tons, 12 cwt., 2 qrs., 11 lbs.×84, is the same as 18 tons, 12 cwt., 2 qrs., 11 lbs.×12×7, &c.

(6.)

tons. cwt. qrs. lbs.
18.12.2.11×84
12
223.11.1. 7
7
1564.19.0.24

(7.)

ac. roods. rds.
27.2.29×72
8
221.1.32
9
1993.0. 8

(8.)

£3.15.6×150
5
18.17.6
5
94. 7.6
6
566. 5.0

(9.)

cwt. qrs. lbs.
23.3.22×49
7
167.3. 4
7
1174.2. 3

(10.)

lbs. oz. drs.
49.11.12×63
7
348. 2. 4
9
3133. 4 .4

Questions, such as No. 10, may also be worked by multiplying each denomination separately by the whole given multiplier at once, reducing to the next higher denomination, and adding this to the next result.—Thus: £3.15.6, multiplied by 150, will give (1.) 900 pence=75 shillings,—(2.) 15 shillings multiplied by 150, will give 2250 shillings, which, added to the 75 shillings already found, will give 2325 shillings, which reduced will give 116 pounds, 5 shillings, and (3.) the 3 pounds multiplied by 150, will give 450 pounds, which added to what has already been found, will give a final result of £566.5.0, as already obtained by the method of factors.

£3.15.6
150
900
75.0
2250.0
2325.0
£116.5.0
450.0.0
£566.5.0

| | | |
|---|---|---|
| cwt. qrs. lbs. | | cwt. qrs. lbs. |
| 9.3.22+86 | £2.13.1¼ | 1.2.17+27 |
| 86 | 125 | 27 |
| 857.1.17 | £331.18.0¼ | 45.0. 9 |

## SUPPLEMENT TO THE MULTIPLICATION OF DENOMINATE NUMBERS.

6. How many seconds has a person lived who has completed his twentieth year, the year consisting of 365 days, 5 hours, 48 minutes, and 48 seconds? Ans. 662688000.

7. Bought 7 loads of hay, each weighing 1 ton, 3 cwt., 3 qrs., 12 lbs; what did the whole weigh?

8. If a man can reap 3 acres and 35 rods per day, how much will he reap in 30 days? Ans. 96 acres, 90 rods.

9. If a staemboat ply across a channel, the breadth of which is equal to 2°, 25′, 10″, what angular space has she traversed at the end of 20 trips? Ans. 48°, 23′.

10. If a man saves 3s. 9½d. a day, how much will he save in the year, omitting the Sabbaths? Ans. £59 6s. 9½d.

11. If 12 gallons, 3 quarts, 1 pint of molasses be used in a hotel in a week, how much would be used in a year at that rate?

Ans. 10 hhds., 39 gals., 2 qts.

12. If a man can saw one cord of wood in 8 hours, 45 minutes, 50 seconds, in what time will he saw 11 cords?

Ans. 4 days, 24 hours, 10 seconds.

13. If 13 waggons carry 3 tons, 15 cwt., 1 qr., 15 lbs. each, how much do they all carry? Ans. 49 tons, 0 cwt., 0 qr., 20 lbs.

14. If a man travel 20 miles, 5 furlongs, and 20 rods a day, how would he travel at that rate in a year?

Ans. 755 m., 7 fur., 20 rods.

15. There are 24 piles of wood, each containing 3 cords, 42 cubic feet; what is the whole quantity? Ans. 79 cords, 120 ft.

16. If 17 hhds. of sugar weigh 12 cwt., 1 qr., 20 lbs. each, how much will the whole weigh? Ans. 211 cwt., 2 qrs., 15 lbs.

17. Allowing 75 yards, 18 feet, for the surface of 9 rooms, how much paper would be required to cover the wall?

Ans. 693 sq. yards.

18. If 11 casks contain 54 gals., 3 qts., 1 pt., 2 gills each, how much would they all contain? Ans. 604 gals., 1 qt., 2 gills.

19. If the cars go 21 miles, 2 furlongs, 10 rods per hour, how far will they go in 15 hours? Ans. 319 miles, 1 fur., 30 rods.

20. If 1 silver cup weigh 3 oz., 15 dwts., 10 grs., how much will 10 such cups weigh? Ans. 3 lbs., 1 oz., 14 dwts., 4 grs.

## DIVISION.

In Division, all remainders are to be reduced to the next lower denomination, and in that form divided, to get the units of that denomination.

### EXERCISES.

1. A silversmith made half-a-dozen spoons weighing 2 lbs., 8 oz., 10 dwts.; what was the weight of each? Ans. 5 oz., 8 dwts., 8 grs.

2. If 45 waggons carry 685 bushels, 2 pecks, 4 quarts, how much does each carry on equal distribution? Ans. 15 bushels, $7\frac{5}{9}$ quarts.

3. If a labourer receives 149 lbs., 13 oz. of meat as payment for 26 days' work, how much is that per day, on an average?
Ans. 5 lbs., $12\frac{5}{26}$ oz.

4. If a steamer occupies 48 days, 17 hours, and 40 minutes, in making 121 trips, what is the average time? Ans. 9 h. 40 min.

5. If 98 bushels, 3 pecks, and 2 quarts of grain can be packed in 37 equal-sized barrels, how much will there be in each?
Ans. 2 bush., 2 pecks, $5\frac{17}{37}$ qts.

6. If a man has an income of £400 a year, how much has he each day? Ans. £1 1s. $11\frac{1}{73}$d.

7. An English nobleman has £200,000 a year; how much has he a day? Ans. £547 18s. $10\frac{1}{2}$d., nearly.

8. In a coal mine, 97 tons, 13 cwt., 2 qrs. were raised in 97 days; how much was that per day, on an average?
Ans. 13 cwt., 3 qrs., 22 lbs.+.

9. If 19 canisters of equal size contain 332 lbs., 8 oz., how much is in each? Ans. 17 lbs., 8 oz.

10. If $15.50 be the value of 1 lb. of silver, what will be the weight of $500000 worth?
Ans. 32258 lbs., 8 oz., 15 dwts., $11\frac{19}{31}$ grs.

11. If 1246 bushels of wheat are produced in a field of 16 acres what is the yield per acre? Ans. 77 bush., 3 pecks, 5 qts., $1\frac{3}{4}$ pts.

12. A gardener pulled 13500 bushels of apples off 60 trees; how many, on an average, were in each bushel? Ans. 230.

13. If 13 hogsheads of sugar weigh 6 tons, 8 cwts., 2 qrs., 7 lbs., what is the weight of each? Ans. 9 cwt., 3 qrs., 14 lbs.

14. What is the twenty-third part of 137 lbs., 9 oz., 18 dwts., 22 grs.? Ans. 5 lbs., 11 oz., 18 dwts., $5\frac{3}{23}$ grs.

15. A shipment of sugar consisted of 8003 tons, 17 cwt., 1 qr., 12 lbs., 10 oz., net weight; it was to be shared equally by 451 grocers; how much did each get?

Ans. 17 tons, 14 cwt., 3 qrs., 18 lbs. 14 oz.

16. If a horse runs 174 miles, 26 rods, in 14 hours, what is his speed per hour? Ans. 12 miles, 3 fur., 19 rods.

17. A farmer divided his farm, containing 322 acres, 2 roods, 10 rods, equally among his seven sons and 6 sons-in-law; what was the share of each? Ans. 24 acres, 3 roods, 10 rods.

18. If 132 bushels, 3 pecks, 7 quarts of corn be distributed equally among 23 poor persons; how much does each get?

Ans. 5 bushels, 3 pecks, 1 quart.

19. A man having purchased 119 cwt., 3 qrs., 23 lbs of hay, and drew home in 6 waggons; how much was on each waggon?

Ans. 19 cwt., 3 qrs., 23 lbs.

### MIXED EXERCISES ON DENOMINATE NUMBERS.

20. A gentleman, by his will, left an estate worth $2490, to be divided among his two sons and 3 daughters in the following proportions:—The widow was to receive *one-third* of the whole, less $346; the younger son $212 more than his mother; the older son as much as his mother and brother, lacking $335.50, and the three daughters were to have the remainder, share and share alike; what was the share of each?

Ans. The widow got $484; the older son got $$844\frac{1}{2}$; the younger son got $696; each daughter got $$155\frac{1}{6}$.

21. A gentleman left a property in land, consisting of 448 acres, 3 roods, 24 rods, to be divided among his four children in the following proportions:—The youngest was to get 4 acres, 3 roods, 6 rods more than the eighth part; the second youngest was to get one-fifth of the remainder; the oldest but one was to get one-third of the remainder, and the oldest the residue; what was the share of each?

Ans. The youngest got 60 acres, 3 roods, 24 rods; the next got 77 acres, 2 roods, 16 rods; the next got 103 acres, 1 rood, 34⅔ rods; the oldest got 206 acres, 3 roods, 29⅓ rods.

22. A ship made the following headway on six successive days: On Monday, 3°, 8′, 45″ south, and 1°, 51′ east; on Tuesday, 2°, 36′ south, and 2°, 1′, 15″ east; on Wednesday, 4°, 0′, 52″ south, and 1° east; on Thursday, 1°, 48′, 52″ south, and 3°, 16′, 22″ east; on Friday, 1°, 19′ south, and 48′, 29″ east; and on Saturday, 59′, 30″ south, and 3°, 52′, 11″ east; find her distances south and east from the port of departure.

Ans. South 13°, 52′, 59″; East 12°, 49′, 17″

23. A vintner sold in one week, 51 hogsheads, 53 gallons, 1 quart, 1 pint; in the next week, 27 hogsheads, 39 gallons, 3 quarts; in the next week, 19 hogsheads, 13 gallons, 3 quarts; how much did he sell in the three weeks?

Ans. 88 hogsheads, 43 gallons, 3 quarts, 1 pint.

24. In a pile of wood there are 37 cords, 119 cubic feet, 76 cubic inches; in another there are 9 cords, 104 cubic feet; in a third there are 48 cords, 7 cubic feet, 127 cubic inches, and in a fourth there are 61 cords, 139 cubic inches. Find the whole amount. Ans. 156 cords, 102 feet, 342 inches.

25. The following cargo was landed at Montreal from Liverpool: 78 tons, 3 cwt., 2 qrs., 26 lbs. of Irish pork; 125 tons, 15 cwt., 1 qr., 9 lbs. of iron; 90 tons, 12 cwt., 2 qrs., 20 lbs. of West of England cloth goods; 225 tons, 9 cwt., 12 lbs. of Scotch coal, and 106 tons, 1 qr. of Staffordshire pottery; what is the whole amount of the consignment? Ans. 636 tons, 1 cwt., 16 lbs.

26. If a man can count 100 one-dollar bills in a minute, and keep working 10 hours a day; how long will it take him to count a million? Ans. 16⅔ days.

27. The earth's equatorial diameter is 41847426 feet; how many miles? Ans. 7925 and 3426 feet.

28. The earth's polar diameter is 7899 miles, 900 feet; how many feet? Ans. 41707620 feet.

29. Sound is calculated to move 1130 feet per second; how far off is a cannon, the report of which is heard in 1′ 9″?

Ans. 77970 feet.

30. If the circumference of a waggon wheel be 14⅔ feet; how often will it turn round in a mile, (5280 feet)? Ans. 360 times.

## GREATEST COMMON MEASURE.

**13**.—When any quantity is contained an even number of times in a greater, the greater is called a multiple of the less, and the less a submultiple, measure or aliquot part of the greater. Thus: 48 is a multiple of 2, 3, 4, 6, 8, 12, 16 and 24, and each of these is a submultiple of 48.

When one quantity divides two or more others evenly it is called a common measure of those quantities, and the greatest number that will divide them all is called the greatest common measure. Thus: 7 is a common measure of 63 and 49, and it is also the greatest common measure, for no larger number will divide both evenly.

When any quantity is measured evenly by two or more others, it is called a common multiple of them. Thus: 24 is a common multiple of 2, 3, 4, 6, 8 and 12.

A number which can be divided into two equal integral parts is called an *even number*, and one which cannot be so divided is called an *odd number*. Hence all numbers of the series 2, 4, 6, 8, 10, 12, &c., are even, while those of the series 1, 3, 5, 7, 9, 11, &c., are odd. Hence the sum of any number of even quantities is even; also, the sum of any even number of odd quantities is even; but the sum of any odd number of odd quantities is odd. This principle is of great use in checking additions.

A prime number is one which has no integral factors except itself and unity; a composite number is one that has integral factors greater than unity, and numbers which have no common factor greater than unity are said to be *prime to each other*. Of the first kind are 1, 2, 3, 5, 7, 11, &c., of the second, 4, 6, 8, 9, 10, 12, &c.; also, 2 and 7 are prime to each other, and so are 6 and 7.

If one quantity measure another it will measure any multiple of it. Thus: since 3 measures 6, it will also measure 12, 18, 24, &c., because it is a factor of all these.

If one quantity measure two or more others, it will also measure their sum and difference, and also the sum and difference of any multiples of them, because it measures them when they are taken separately.

Hence, if one number divide the whole of another number, and also one part of it, it will divide the other part too. Thus: 6 divides 24 and 18, and so the other part, 6; 9 divides 45 and 27, and also the remainder, 18. Also, if a number be composed of several parts, each of which has a common factor, that factor will also measure their sum. Thus: 9 measures 18, 27, and 36, and their sum, 81.

From these principles we can deduce a rule for finding the greatest common measure of two or more quantities.

### RULE.

Divide the greater by the less, and then the less by the remainder, until nothing is left, and the last divisor will be the greatest common measure.

### EXAMPLE.

| | |
|---|---|
| 2145 | 3471 |
| 1326 | 2145 |
| 819 | 1326 |
| 507 | 819 |
| 312 | 507 |
| 195 | 312 |
| 117 | 195 |
| 78 | 117 |
| 39 | 78 |
| | 78 |

A concise form of the work is exhibited in the margin. The quotients are omitted as unnecessary. The last divisor, 39, is the G. C. M., as may be proved by trial. If it is required to find the G. C. M. of more than two numbers, first find the G. C. M. of two of them, and then the G. C. M. of that and another, and so on.

### EXERCISES.

Find the G. C. M. of the following quantities:

1. 247 and 323. Ans. 19.
2. 532 and 1274. Ans. 14.
3. 741 and 1273. Ans. 19.
4. 10416 and 25761 Ans. 93.
5. 468 and 1266. Ans. 6.
6. 285714 and 999999. Ans. 142857.
7. 15863 and 21489. Ans. 29.
8. 8280 and 11385. Ans. 1035.
9. 17222 and 32943. Ans. 79.
10. 19752 and 69132. Ans. 9876.

We may often find the G. C. M. by inspection. For example, in exercise 5, we see that 2 will measure both quantities (Art. 13), for both are even, and also that 3 will measure both, because it measures the sum of the digits (Art. 16.

The least common multiple of two or more numbers is the smallest number that is divisible by all of them. Thus: 48 is a common multiple of 2, 3, 4, 6, 8 and 12, but 24 is the *least* common multiple of them.

It is plain that the least common multiple of quantities that have no common factor is their product. Thus: the L. C. M. of 5, 7, 6 is 210. But if the quantities have a common factor, that factor is to be taken only once. Thus: 96, 48, 24, are all common multiples of 2, 3, 4, 6, 8, 12, but the least of these, 24, contains only the factors 3 and 8, which are prime to each other, for 2, 3, 4, 6 are all contained in 12, and 8 and 12 have a common factor, 4, which being left out of one of them, 8, gives 2×12=24, or, being left out of the other, 12, gives 8×3=24. From this we derive the

RULE:

Expunge all common factors and take the continued product of all the results and divisors. Thus, to find the L. C. M. of 2, 3, 4, 6, 9, 18, 27, 30, arrange them in a horizontal line, and as 2, 3, 6, 9 are all contained in 18, they may be omitted, as in the second line, then, as 2 is contained in 4, 18 and 30, it may be divided out, and as 9 in the third line is contained in 27, it may be omitted, as in the fourth line; and 27 and 15 being both divisible by 3, we obtain in the fifth line 2, 9, 5, all prime to each other, and the products of these and the divisors 3 and 2 is the L. C. M., 540.

2...3...4...6...9...18...27...30

2|4...18...27...30

2... 9...27...15

3|2...27...15

2... 9... 5

9

45

2

90

3

270

2

540

### EXERCISES

Find the L. C. M. of the following quantities:

1. 8, 12, 16, 24, 33. Ans. 528.
2. 35, 42, 45, 81, 100. Ans. 56700.
3. 2, 4, 8, 16, 32, 64, 128 Ans. 128.
4. 2, 3, 5, 7, 11. Ans. 2310.
5. 3, 9, 27, 81, 243, 729. Ans. 729.
6. 12, 16, 18, 30, 48. Ans. 720.
7. 3, 4, 5, 6, 7. Ans. 420.
8. 2, 3, 4, 5, 6, 7, 8, 9. Ans. 2520.
9. 2, 4, 7, 12, 16, 21, 56. Ans. 336.
10. 2, 9, 11, 33. Ans. 198.

### EXAMPLES FOR PRACTICE.

1. What will 320 caps cost at $7.50 each? Ans. $2400.

2. If you can purchase slates at 20 cents each; how many can you buy for $7.40? Ans. 37.

3. If you can walk 4 miles an hour; how far can you go in 24 hours? Ans. 96.

4. What will be the cost of 216 barrels of pork at $7.50 per barrel? Ans. $1620.

5. How many sheep can be bought for $560 at $3.50 per head? Ans. 160.

6. If 825 pounds of beef are consumed by a garrison in one day; what will be the cost for 6 days at 11 cents per pound for beef? Ans. $544.50.

7. A farmer sold 185 acres of land at $25 per acre, and received in payment 17 horses at $70 each, and 12 cows at $20 each; how much remains due? Ans. $3195.

8. A merchant bought 120 yards of Canadian tweed at $1.15 a yard; 60 yards of flannel at 95 cents per yard, and 13 dozen pairs of gloves at 35 cents per pair; what was the amount of his bill? Ans. $249.60.

9. At $2 per gallon; how much wine can be bought for $84? Ans. 42 gals.

10. A boy had $5.50, and he paid one dollar and five cents for a book; how much had he left? Ans. $4.45.

11. What will 18 cords of wood cost at $4.75 per cord? Ans. $85.50.

12. How many pounds of sugar can be bought for $9.35, at 11 cents per pound? Ans. 85 lbs.

13. What will a jury of 12 men receive for coming from Kingston to Toronto at 10 cents a mile each; the distance being 160 miles? Ans. $192.

14. A grocer bought a hogshead of molasses at 32 cents per gallon; but 18 gallons leaked out, and he sold the remainder at 55 cents per gallon; did he make or lose, and how much?

Ans. He gained $4.59.

15. If a clerk's salary is $600 a year, and his personal expenses $320; how many years before he will be worth $6600, if he has $1000 at the present time? Ans. 20 years.

16. A speculator bought 200 bushels of apples for $90, and sold the same for $120; how much did he make per bushel?

Ans. 15 cents.

17. A person sells 15 tons of hay at $22 per ton, and receives in payment a carriage worth $125, a cow worth $45, a colt worth $40, and the balance in cash; how much money ought he to receive?

Ans. $120.

18. How many pounds of butter, at 20 cents per pound, must be given for 18 pounds of tea worth 75 cents per pound?

Ans. $67\frac{1}{2}$ lbs.

19. A grocer bought 7 barrels of fish at $18 per barrel; but one barrel proved to be bad, which he sold for $5 less than cost, and the remainder at an advance of $3 per barrel; did he gain or lose, and how much? Ans. Lost $13.

20. A man bought a drove of cattle for $18130, and after selling 84 of them at $51 each, the rest stood him in $43 each; how many did he buy? Ans. 406.

21. What will 2 cwt. of cheese cost at $9\frac{1}{2}$ cents per pound?

Ans. $19 00.

22. A. is worth $960, B. is worth five times as much as A., less $600, and C. is worth three times as much as A. and B. and $300 more; what are B. and C. worth each, and how much are they all worth? Ans. B. $4200; C. $15780; all $20940.

23. A boy bought a dozen knives at 15 cents each, and after selling half of them at the rate of $2.22 per dozen, he lost three, and sold the balance at 25 cents each; did he make or lose, and how much? Ans. Gained 6 cents.

24. A labourer bought a coat worth $16, a vest worth $3, and a

pair of pants worth $5.50; how many days had he to work to pay for his suit; his services being worth 50 cents per day?

Ans. 49 days.

25. What will 14 bushels of clover seed cost at 12½ cents per pound? Ans. $105.

26. A farmer sold a load of oats weighing 1836 pounds, at 30 cents per bushel; how much did he receive for the same?

Ans. $16.20.

27. A produce dealer bought at one time, one load of wheat weighing 3240 pounds, at $1.05 per bushel; one load of barley weighing 2400 pounds, at 85 cents per bushel; one load of rye weighing 2800 pounds, at 65 cents per bushel; two loads of pease, each 2400 pounds, at 68 cents per bushel; three loads of buckwheat, each weighing 1400, at 55½ cents per bushel; and a quantity of oats weighing 578 pounds, at 33 cents per bushel; what had he to pay for the whole? Ans. $250.15½.

28. A farmer has 12 sheep worth $3.50 each; 9 pigs worth $4.65 each; one cow worth $35, and a fine horse valued at $150. He exchanges them with his neighbour for a yoke of oxen worth $75; two lambs worth $1.925 each; a carriage worth $100, and takes the balance in calves at $4.50; how many calves does he receive?

Ans. 20.

29. A and B sat down to count their money, and found that they had together $225, but A had $15 more than B; how much had each? Ans. A $120, B $105.

30. A miller bought 250 bushels of oats for $85 and sold 225 bushels for $70; what did the remainder cost him per bushel?

Ans. 60c.

31. A widow lady has a farm valued at $6720; also three houses, worth $12530, $11324, and $9875. She has a daughter and two sons. To the daughter she gives one-fourth the value of the farm, and one-third the value of the houses, and then divides the remainder equally among the boys, how much did each receive?

Ans. daughter $12923, each son, $13763.

32. A man went into business with a capital of $1500; the first year he gained $800, the second year $950, the third year $700, and the fourth year 625, when he invested the whole in a cargo of tea and doubled his money; what was he then worth. Ans. $9150.

33. A boy paid out 30 cents for apples, at the rate of 6 for 3 cents; how many apples did he purchase? Ans. 60.

34. A schoolboy bought 12 oranges at 3 cents each, and sold them for 12 cents more than he paid for them; how much did he sell them at each? Ans. 4c.

35. A clerk's income is $2698 a year, and his expenses $4.50 per day; how much will he save in two years? Ans. $2111.

36. A speculator bought 200 acres of land at $45 per acre, and afterwards sold 150 acres of it for $11550; the balance he sold at a gain of $5 per acre, and received in payment $250 cash, and the balance in sheep at $5 each; how many sheep did he receive? Ans. 450 sheep.

37. A butcher bought 9 calves for $54, and 9 lambs for $31.50; how much more did he pay for a calf than a lamb? Ans. $2.50.

38. A farmer sold to a grocer 380 pounds of pork, at 7 cents per pound; 150 pounds of butter, at 17 cents per pound, and one cheese weighing 53 pounds, at 9 cents per pound; and received in payment 22 pounds of sugar, at the rate of 11 pounds for a dollar; 150 pounds of nails, at 6 cents per pound; 15 pounds of tea, at 65 cents per pound; one half-barrel of fish, at $18 per barrel, and one suit of clothes worth $27; did the farmer owe the grocer, or the grocer the farmer, and how much? Ans. the grocer owed the farmer 12 cents.

39. A milkman sold 120 quarts of milk, at 5 cents per quart, and took in payment, one pig worth $1.50, and the balance in sheeting, at 10 cents per yard; how many yards did he receive? Ans. 45 yards.

40. How many pounds of cheese, at 9 cents per pound, must be given for 27 pounds of tea worth 80 cents per pound? Ans. 240.

## FRACTIONS.

**14.**—Vulgar or Common Fractions.—When we have divided any number by a less, and find no remainder, the quotient is called an integer, or whole number. When we have divided any number by a less as far as possible, and find a remainder still to be divided, but less than the divisor, and therefore not actually divisible by it, we must have recourse to some method of indicating this. We have seen already that the conventional sign of division is this mark (÷); thus, 3÷4 means that 3 is to be divided by 4, and this being impossible, we indicate the operation either as above or by writing the three in the place of the upper dot, and the 4 in the place of the lower, thus, $\frac{3}{4}$.

The nature of a fraction may be viewed in two ways. *First*, we may consider that a unit is divided into a certain number of equal parts and a certain number of these parts taken; or, *secondly*, that a number greater than unity is divided into certain equal parts, and *one* of these parts taken; thus, $\frac{3}{4}$ means either that a unit is divided into 4 equal parts and three of them taken, or that three is divided into 4 equal parts and one of them taken. For example, if a foot be divided into 4 equal parts, each of these parts will be 3 inches, and three of them will be nine inches; and since 3 feet make 36 inches, if we divide 3 feet into 4 equal parts, each of these parts will be 9 inches, and hence $\frac{3}{4}$ of $1=\frac{1}{4}$ of 3. The lower figure is called the denominator, because it shows the denomination or number of parts into which the unit is supposed to be divided, and the upper one is called the numerator, because it shows the number of those parts considered in any given question. When both are spoken of together they are called the *terms* of the fraction.

What may be considered the fundamental principle on which all the operations in fractions depend is this: that the form, but not the value of a fraction, is altered, if both the terms are either multiplied or divided by the same quantity. If we take the fraction $\frac{3}{4}$ and multiply its terms by 2, we get $\frac{6}{8}$. Now, the $\frac{1}{8}$ of a foot is an inch and-a-half, and therefore $\frac{6}{8}$ is 6 inches and 6 half-inches, or 9 inches; but we have seen that $\frac{3}{4}$ of a foot is 9 inches, therefore $\frac{3}{4}$ of a foot is the same as $\frac{6}{8}$ of a foot. So also $\frac{3}{4}$ of £1 and $\frac{6}{8}$ of £1 are both 15s. The same will hold good whatever the unit of measure may be, or whatever the fraction of that unit. Hence, universally the *form* of a fraction is altered if its terms be either multiplied or divided by the same number, but its *value* remains the same.

Again, if we multiply the numerator 3 by 2, but leave the denominator 4 unchanged, we obtain $\frac{6}{4}$, and, keeping to our first illustration, $\frac{6}{4}$ of a foot is 6 times three inches, or 18 inches, which is double of 9 inches, the value of $\frac{3}{4}$. We should have obtained the same result by taking $\frac{6}{8}$ and dividing its denominator by 2, without dividing its numerator. Hence, a fraction is multiplied by either multiplying its numerator or dividing its denominator. In like manner, if we take the fraction $\frac{6}{8}$ and divide its numerator by 2, we obtain $\frac{3}{8}$, and if we multiply the denominator of its equal $\frac{3}{4}$ by 2, we obtain the same rusult, $\frac{3}{8}$. Hence, $\frac{3}{8}$ is $\frac{1}{2}$ of $\frac{3}{4}$, and therefore a fraction is divided by either dividing its numerator or multiplying its denominator. These principles may also be referred to the obvious fact that in dividing any quantity the greater the divisor the less the quotient, and the less the divisor the greater the quotient. As it is always desirable to have the smallest numbers possible to handle, let the operator observe this as a universal rule—*divide when you can.*

Fractions are classified in four different ways, according to four different circumstances.

### I. They are divided into Proper and Improper Fractions.

A proper fraction is one whose numerator is less than its denominator. In strictness such alone is a fraction. An improper fraction is one whose numerator is greater than its denominator. Strictly this is not really a fraction, but only a certain quantity expressed in the fractional form.

### II. Simple and Compound Fractions.

The term simple fraction, as opposed to compound fraction, expresses that the fraction is multiplied by unity alone, as $\frac{5}{8}$, which means either $\frac{5}{8}$ of 1 or $\frac{1}{8}$ of 5, or $\frac{5}{8}\times1=\frac{1}{8}\times5$.

A compound fraction is one that is multiplied by some other quantity. A fraction is called compound if either multiplier or multiplicand, or both, be fractional. Thus: $\frac{3}{4}$ of $\frac{5}{6}$ and $\frac{7}{8}$ of 11 are both compound, and are written $\frac{3}{4}\times\frac{5}{6}$ and $\frac{7}{8}\times11$.

### III. Simple and Complex Fractions.

The term simple fraction, as opposed to complex fraction, means that there is only one division. Thus: $\frac{15}{16}$ means that a single number, 15, is divided by a single number, 16.

A complex fraction is one of which either the numerator or denominator, or both, are fractional, that is, it indicates a division, when either the given product or given factor, or both, are fractional. Thus: $\frac{3}{4}\div\frac{7}{11}$, or $\frac{\frac{3}{4}}{\frac{7}{11}}$ and $\frac{8}{\frac{5}{9}}$ and $\frac{\frac{11}{12}}{7}$ are complex fractions and exhibit the only three posssible forms.

### IV. Vulgar, or Common, and Decimal Fractions.

Decimal fractions are those expressed with a denominator, 10, or a power of 10, *e. g.*, $\frac{7}{10}$, $\frac{19}{100}$, $\frac{21}{1000}$.

Any fraction not so expressed is called vulgar or common. Thus: $\frac{3}{4}$ would be called a common fraction, but its equivalent, $\frac{75}{100}$, would be called a decimal fraction, and is written ·75, the denominator being omitted, but its existence being indicated by the mark (·), called the decimal point.

A mixed quantity is one expressed partly by a whole number and partly by a fraction, as $4\frac{7}{8}$, $12\frac{1}{2}$. This is not another kind of fraction, but simply another mode of writing an improper fraction when the division indicated has been performed as far as possible Thus: $\frac{39}{8}=4\frac{7}{9}$, and $\frac{25}{2}=12\frac{1}{2}$.

It is often said that there are six kinds of fractions—proper improper, simple, compound, complex, and mixed. This is logically incorrect, for a proper fraction is simple, and a mixed quantity is an improper fraction in another form.

**15.**—Operations in Common Fractions.—From the principles laid down (Art. 21,) we can deduce rules for all the operations in fractions.

I. An improper fraction is reduced to a mixed quantity by performing the division indicated, as $\frac{217}{9}=24\frac{1}{9}$.

II. A mixed quantity is reduced to an improper fraction by multiplying the integral part by the denominator and adding in the numerator, as $12\frac{7}{8}=\frac{103}{8}$.

So also an integer may be expressed in the fractional form by writing 1 as a denominator, and multiplying the terms by whatever number will bring it to any required denomination. Thus: to reduce 7 to the same denamination as $\frac{5}{6}$, write $\frac{7}{1}$ and multiply the terms by 6, and the result, $\frac{42}{6}$, will be equivalent to the integer 7, and of the same form as $\frac{5}{6}$.

### EXERCISES.

1. Express $\frac{441}{9}$ as a whole or mixed number. Ans. 49.
2. Express $\frac{87}{16}$ as a whole or mixed number. Ans. $5\frac{7}{16}$.
3. Express $\frac{781}{11}$ as a whole or mixed number. Ans. 71.
4. Express $\frac{129}{22}$ as a whole or mixed number. Ans. $5\frac{19}{22}$.
5. Express $\frac{19876}{3579}$ as a whole or mixed number Ans. $5\frac{1981}{3579}$.
6. Express $\frac{859}{78}$ as a whole or mixed number. Ans. $11\frac{1}{78}$.
7. Express $\frac{365}{52}$ as a whole or mixed number. Ans. $7\frac{1}{52}$.
8. Express $\frac{89}{12}$ as a whole or mixed number. Ans. $7\frac{5}{12}$.
9. Express $\frac{1157}{13}$ as a whole or mixed number. Ans. 89.
10. Express $\frac{117}{11}$ as a whole or mixed number. Ans. $10\frac{7}{11}$.
11. Express $\frac{149}{63}$ as a whole or mixed number. Ans. $2\frac{23}{63}$.
12. Express $\frac{176}{9}$ as a whole or mixed number. Ans. $19\frac{5}{9}$.
13. Express $\frac{211}{17}$ as a whole or mixed number. Ans. $12\frac{7}{17}$.
14. Express $\frac{29}{7}$ as a whole or mixed number. Ans. $4\frac{1}{7}$.

15. Express $\frac{97}{4}$ as a whole or mixed number. Ans. $24\frac{1}{4}$.
16. Express $\frac{118}{21}$ as a whole or mixed number. Ans. $5\frac{13}{21}$.
17. Express $\frac{99}{31}$ as a whole or mixed number. Ans. $3\frac{6}{31}$.
18. Express $\frac{11}{2}$ as a whole or mixed number. Ans. $5\frac{1}{2}$.
19. Express $\frac{121}{4}$ as a whole or mixed number. Ans. $30\frac{1}{4}$.
20. Express $\frac{1331}{16}$ as a whole or mixed number. Ans. $83\frac{3}{16}$.
21. Express $\frac{181}{20}$ as a whole or mixed number. Ans. $9\frac{1}{20}$.
22. Express $27\frac{1}{2}$ as an improper fraction. Ans. $\frac{55}{2}$.
23. Express $66\frac{1}{9}$ as an improper fraction. Ans. $\frac{595}{9}$.
24. Express $15\frac{17}{19}$ as an improper fraction. Ans. $\frac{302}{19}$.
25. Express $7\frac{3}{4}$ as an improper fraction. Ans. $\frac{31}{4}$.
26. Express 49 as a fraction with the same denominator as $\frac{12}{13}$. Ans. $\frac{637}{13}$.
27. Express 19s. as a fraction of £1. Ans. $\frac{19}{20}$.
28. Express 11 inches as a fraction of a foot. Ans. $\frac{11}{12}$.
29. Bring $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{12}$ to the same denomination. Ans. $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, $\frac{2}{12}$, $\frac{1}{12}$.
30. Express 11 as a fraction having the same denominator as $\frac{17}{701}$. Ans. $\frac{7711}{701}$.

III. To reduce a fraction to its lowest terms or simplest form, divide the terms by their greatest common measure. This is often readily done by inspection, as $\frac{48}{72}=\frac{4}{6}=\frac{2}{3}$, but in such questions as $\frac{1092}{6552}$, the most secure and speedy method is to find the G. C. M. of the terms and divide them by it. Thus: the G. C. M. of the fraction $\frac{1092}{6552}$ is 1092, and the terms of the fraction divided by this give $\frac{1}{6}$, the simplest form.

### EXERCISES.

1. Reduce $\frac{4536}{13608}$ to its lowest terms or simplest form. Ans. $\frac{1}{3}$.
2. Reduce $\frac{5971}{9383}$ to its lowest terms or simplest form. Ans. $\frac{7}{11}$.
3. Reduce $\frac{2920}{3285}$ to its lowest terms or simplest form. Ans. $\frac{8}{9}$.
4. Reduce $\frac{72000}{960000}$ to its lowest terms or simplest form. Ans. $\frac{3}{40}$.
5. Reduce $\frac{3580}{4296}$ to its lowest terms or simplest form. Ans. $\frac{5}{6}$.
6. Reduce $\frac{87615}{175230}$ to its lowest terms or simplest form. Ans. $\frac{1}{2}$.
7. Reduce $\frac{987}{12831}$ to its lowest terms or simplest form. Ans. $\frac{1}{13}$.
8. Reduce $\frac{3333}{4444}$ to its lowest terms or simplest form. Ans. $\frac{3}{4}$.
9. Reduce $\frac{5034}{6712}$ to its lowest terms or simplest form. Ans. $\frac{3}{4}$.

10. Reduce $\frac{13594}{20391}$ to its lowest terms or simplest form. Ans. $\frac{2}{3}$.

11. Reduce $\frac{33595}{114223}$ to its lowest terms or simplest form. Ans. $\frac{5}{17}$.

12. Reduce $\frac{6935}{7300}$ to its lowest terms or simplest form. Ans. $\frac{19}{20}$.

13. Reduce $\frac{571428}{1999998}$ to its lowest terms or simplest form. Ans. $\frac{2}{7}$.

14. Reduce $\frac{31185}{50457}$ to its lowest terms or simplest form. Ans. $\frac{945}{1529}$.

15. Reduce $\frac{16338}{21006}$ to its lowest terms or simplest form. Ans. $\frac{7}{9}$.

16. Reduce $\frac{1827}{3045}$ to its lowest terms or simplest form. Ans. $\frac{3}{5}$.

17. Reduce $\frac{272}{425}$ to its lowest terms or simplest form. Ans. $\frac{16}{25}$.

18. Reduce $\frac{873}{3395}$ to its lowest terms or simplest form. Ans. $\frac{9}{35}$.

19. Reduce $\frac{41472}{96768}$ to its lowest terms or simplest form. Ans. $\frac{3}{7}$.

20. Reduce $\frac{63624}{69408}$ to its lowest terms or simplest form. Ans. $\frac{11}{12}$.

21. Reduce $\frac{789057010000}{1578114020000}$ to its lowest terms or simplest form. Ans. $\frac{1}{2}$.

IV. To multiply one fraction by another, muitiply numerator by numerator and denominator by denominator. Thus: $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$. To illustrate that $\frac{1}{3}$ of $\frac{1}{3}$ is $\frac{1}{9}$, take a line and let it be divided into 3 parts, and each of those again into 3 parts, as in the margin, we find that the result is 9 parts, each, of course, being $\frac{1}{9}$ of the unit.

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We have seen that a fraction is multiplied by multiplying the numerator or dividing the denominator. Now, if it were required to multiply $\frac{3}{4}$ by $\frac{5}{7}$, we could not divide the denominator, as 5 is not contained in 4, and therefore we multiply the numerator and obtain $\frac{15}{4}$, but we have multiplied by a quantity equal to 7 times the given one, and therefore we must divide the product by 7, *i. e.* (Art. 21,) we must multiply the denominator 4 by 7, which gives $\frac{15}{28}$ for the correct product.

### EXERCISES.

1. Multiply $\frac{5}{12}$ by $\frac{13}{17}$? Ans. $\frac{65}{204}$.
2. What is the product of $\frac{3}{4}$ by $\frac{11}{13}$? Ans. $\frac{33}{52}$.
3. What is the product of $\frac{7}{12}$ by $\frac{5}{6}$? Ans. $\frac{35}{72}$.
4. What is the product of $\frac{5}{6}$ by $\frac{13}{14}$? Ans. $\frac{65}{84}$.
5. What is the product of $\frac{1}{2}$ by $\frac{19}{20}$? Ans. $\frac{19}{40}$.

6. What is the product of $\frac{7}{8}$ by $\frac{9}{10}$? Ans. $\frac{63}{80}$.
7. What is the product of $\frac{99}{100}$ by $\frac{7}{10}$? Ans. $\frac{693}{1000}$.
8. What is the product of $\frac{36}{55}$ by $\frac{7}{11}$? Ans. $\frac{252}{605}$.
9. What is the product of $\frac{8}{9}$ by $\frac{4}{5}$? Ans. $\frac{32}{45}$.
10. What is the product of $\frac{12}{13}$ by $\frac{7}{11}$? Ans. $\frac{84}{143}$.

When the product has been obtained it should be reduced to its lowest terms. Thus: the product of $\frac{7}{11}$ by $\frac{11}{13}$ is $\frac{77}{143}$, the terms of which are both divisible by 11, and so we get the equivalent fraction $\frac{7}{13}$. But we might as well have divided by 11 before multiplying, for by this method we should at once have found the fraction in its simplest form, viz., $\frac{7}{13}$. In the same manner any number or numbers which are factors of both numerator and denominator, may be omitted in the operation. This we call cancelling in preference to the excessively awkward term "cancellation." This method will be clearly seen in exercise 11.

If either the multiplier or multiplicand be a mixed quantity, it must be reduced to an improper fraction before the multiplication is performed. Thus: $8\frac{3}{4}\times5\frac{5}{6}=\frac{35}{4}\times\frac{35}{6}=\frac{1225}{24}=51\frac{1}{24}$.

11. What fraction is equal to $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$? Ans. $\frac{1}{9}$.
12. What quantity is equal to $12\frac{1}{2}$ multiplied by $7\frac{5}{6}$? Ans. $97\frac{11}{12}$.
13. What quantity is equal to $19\frac{1}{8}$ multiplied by $1\frac{15}{17}$? Ans. 36.
14. What is the value of $\frac{5}{6}$ of $\frac{4}{5}$ of $\frac{18}{23}$ of $\frac{1}{8}$? Ans. $\frac{21}{46}$.
15. What is the value of $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{8}{9}$ of $\frac{11}{13}$? Ans. $\frac{440}{2457}$.
16. What is the product of $27\frac{5}{8}$ by $3\frac{8}{9}$? Ans. $107\frac{31}{72}$.
17. What is the product of $\frac{18}{19}$ by $\frac{19}{20}$? Ans. $\frac{9}{10}$.
18. What is the product of $5\frac{1}{2}$ by $5\frac{1}{2}$? Ans. $30\frac{1}{4}$.
19. Find the square and cube of $\frac{17}{22}$? Ans. $\frac{289}{484}$ and $\frac{4913}{10648}$.
20. What is the cube of $\frac{39}{40}$? Ans. $\frac{59319}{64000}$.
21. Multiply 27 by $\frac{1}{27}$? Ans. 1.

## V.—DIVISION OF FRACTIONS.

To divide one fraction by another, multiply by the reciprocal of the divisor; or, in other words, invert the divisor and multiply. In the language of science, the reciprocal of a fraction is the fraction with its terms inverted. Thus: $\frac{8}{7}$ is the reciprocal of $\frac{7}{8}$; $\frac{4}{3}$ of $\frac{3}{4}$. To find the reciprocal of a whole number, we must first represent it as having a denominator 1,—thus $4=\frac{4}{1}$; $6=\frac{6}{1}$, and therefore the reciprocals are $\frac{1}{4}$ and $\frac{1}{6}$. The rule for division may be proved in two ways:

First proof.—Let it be required to divide $\frac{7}{11}$ by $\frac{5}{6}$. If we had been required to divide by the whole number 5, we should either have divided (Art. 14,) the numerator, or multiplied the denominator,—as the numerator is not divisible by 5, we multiply the denominator, and obtain $\frac{7}{55}$; but we have divided by a quantity equal to six times the given one, and therefore, to compensate, we must multiply the result by 6, which gives $\frac{42}{55}$.

Second proof.—Write the question in the complex form—$\frac{\frac{7}{11}}{\frac{5}{6}}$, then (Art. 14,) multiply both terms by 11, and $\frac{7}{\frac{55}{6}}$ is obtained; and again multiply the terms by 6, and $\frac{42}{55}$ is the result as before.—The two operations are virtually the same, though exhibited in different forms, and both are equivalent to the technical rule, "Invert the divisor and multiply."

Mixed quantities must be reduced to improper fractions as in multiplication. The expressions *multiplication* and *division*, as applied to fractions, are extensions of the ordinary meanings of those terms, for in their original meaning, the former implies increase, and the latter decrease; but when two proper fractions are multiplied together, the product is less than either of the factors, and when one proper fraction is divided by another, the quotient is greater than either the divisor or dividend. This will be seen by the annexed examples:

$\frac{3}{4}\times\frac{7}{8}=\frac{21}{32}$. But $\frac{3}{4}=\frac{24}{32}$ and $\frac{7}{8}=\frac{28}{32}$, both greater than $\frac{21}{32}$.

Also, $\frac{7}{8}\div\frac{3}{4}=\frac{7}{8}\times\frac{4}{3}=\frac{28}{24}$. But $\frac{7}{8}=\frac{21}{24}$ and $\frac{3}{4}=\frac{18}{24}$, both less than $\frac{28}{24}$.

If two fractions have a common denominator, their quotient is the quotient of their numerators. We have placed multiplication and division of fractions before addition and subtraction, because, as in whole numbers, multiplication and division are deduced from addition and subtraction, so conversely in fractions, addition and subtraction are to be deduced from multiplication and division, for a fraction is produced by division, and the multiplication of a fraction is merely the repeating of the divided unit a certain number of times. Thus: $\frac{1}{8}$ is a unit divided into 8 equal parts, and $\frac{7}{8}$ is that fraction repeated 7 times.

### EXERCISES.

1. Divide $\frac{3}{11}$ by $\frac{2}{3}$; $\frac{3}{11}\div\frac{2}{3}=\frac{3}{11}\times\frac{3}{2}$. Ans. $\frac{9}{22}$
2. What is the quotient of $\frac{13}{14}$ divided by $\frac{13}{15}$? Ans. $\frac{15}{14}=1\frac{1}{14}$
3. What is the quotient of $\frac{7}{22}$ divided by $\frac{113}{355}$? Ans. $\frac{2485}{2486}$
4. What is the quotient of $\frac{29}{45}$ divided by $\frac{29}{33}$? Ans. $\frac{11}{15}$.
5. What is the quotient of $\frac{115}{246}$ divided by $\frac{23}{27}$? Ans. $\frac{45}{82}$.
6. What is the quotient of 36 divided by $19\frac{1}{8}$? Ans. $1\frac{15}{17}$.
7. What is the quotient of $3\frac{5}{9}$ divided by $2\frac{5}{8}$? Ans. $1\frac{67}{189}$.
8. What is the quotient of $4\frac{1}{2}$ divided by 15? Ans. $\frac{3}{10}$.
9. What is the quotient of $\frac{50}{17}$ divided by $2\frac{17}{18}$? Ans. $\frac{900}{901}$.
10. What is the quotient of $75\frac{7}{10}$ divided by 9? Ans. $8\frac{37}{90}$.
11. What is the quotient of $6\frac{19}{21}$ divided by $9\frac{2}{3}$? Ans. $\frac{5}{7}$.
12. What is the quotient of $5\frac{4}{7}$ divided by $8\frac{7}{12}$? Ans. $\frac{468}{721}$.
13. Divide the product of $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{9}$ by the product of $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{8}{11}$? Ans. $\frac{11}{9}=1\frac{2}{9}$.
14. What is the quotient of $\frac{7}{11}$ of $\frac{11}{13}\div\frac{3}{8}$ of $\frac{11}{13}$ of $\frac{7}{11}\div\frac{9}{10}$ of $\frac{5}{8}$? Ans. $4\frac{20}{27}$.
15. How many $\frac{1}{54}$ are there in $\frac{3}{19}$? Ans. $8\frac{10}{19}$.
16. What is the value of $\frac{5}{6}$ of $\frac{7}{8}\div\frac{3}{4}$ of $\frac{11}{12}$? Ans. $1\frac{2}{33}$.
17. Divide 27 by $\frac{1}{27}$? Ans. 729.

Hence, any quantity divided by its reciprocal gives the square of that number, and exercise 21, of multiplication, shows that any quantity multiplied by its own reciprocal gives unity.

18. Divide $\frac{80}{561}$ by $\frac{4}{9}$, and the quotient by $\frac{3}{17}$? Ans. $1\frac{9}{11}$.
19. Divide $\frac{4}{7}$ by $\frac{7}{11}$, and the quotient by $\frac{22}{23}$? Ans. $\frac{46}{49}$.
20. Divide $\frac{76}{79}$ by $\frac{13}{47}$? Ans. $3\frac{491}{1027}$.
21. Divide $\frac{11}{18}$ by $\frac{44}{36}$? Ans. $\frac{1}{2}$.

## VI.—ADDITION OF FRACTIONS.

We have seen that no quantities can be added together except they are in the same denomination. We can add $\frac{4}{7}$, $\frac{3}{7}$, $\frac{5}{7}$ and $\frac{11}{7}$, as they are all of the same denomination, *sevenths*, and we find $\frac{23}{7}$. We can easily see that to add $\frac{3}{4}$ and $\frac{7}{8}$, we have only to alter the form of $\frac{3}{4}$ to $\frac{6}{8}$, and we have both fractions of the same denomination, and therefore can add them,—$\frac{6}{8}+\frac{7}{8}=\frac{13}{8}$. So, also, $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{6}+\frac{7}{12}=\frac{6}{12}+\frac{8}{12}+\frac{9}{12}+\frac{10}{12}+\frac{7}{12}=\frac{40}{12}=\frac{10}{3}$. But we cannot always tell thus by inspection, and therefore must be guided by some rule. To find the value of $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{2}{9}+\frac{7}{12}$.

By Art. 13 we find the L. C. M. of 4, 6, 8, 9, 12 to be 72, and the rest of the common operation is equivalent to multiplying the terms of each fraction by 72. Thus: if the terms of $\frac{3}{4}$ be both multiplied by 72, we get $\frac{216}{288}=\frac{54\times4}{72\times4}=\frac{54}{72}$, but we might as well have divided 72 by 4 before multiplying, and, to balance that, have multiplied the numerator 3, not by 72, but by the fourth part of 72, viz., 18, giving $\frac{54}{72}$, as the following scheme will show:— $\frac{3\times72}{4\times72}=\frac{3\times18\times4}{4\times18\times4}=\frac{3\times18}{4\times18}=\frac{54}{72}$. The other fractions being altered in the same manner, we get $\frac{54}{72}+\frac{60}{72}+\frac{63}{72}+\frac{16}{72}+\frac{42}{72}$, and as these are now all of the same denomination, though not altered in value, we can add them, and we find $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{2}{9}+\frac{7}{12}=\frac{54}{72}+\frac{60}{72}+\frac{63}{72}+\frac{16}{72}+\frac{42}{72}=\frac{235}{72}$. Hence the

## RULE.

Find the L. C. M. of all the denominators, which will be the common denominator; divide this common multiple by each denominator, and multiply the quotient by each numerator in succession for new numerators; add all these new numerators together, and place the common denominator below the sum, and the fraction thus obtained will be the sum of the given fractions. If the numerator, thus obtained, be greater than the denominator, the resulting fraction may be reduced to a whole or a mixed number by division.

## EXERCISES.

1. Express $\frac{1}{15}+\frac{2}{15}+\frac{4}{15}+\frac{7}{15}$ as a single fraction? Ans. $\frac{14}{15}$.
2. Find the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$? Ans. $2\frac{3}{4}$.
3. Add together $4\frac{7}{8}$, $1\frac{17}{18}$, $2\frac{23}{24}$, $3\frac{20}{27}$ and $5\frac{1}{12}$? Ans. $18\frac{65}{108}$.
4. What fraction is equal to $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}$? Ans. $\frac{63}{64}$.
5. What fraction is equal to $1\frac{1}{2}+2\frac{2}{3}+3\frac{3}{4}+4\frac{4}{5}+5\frac{5}{6}+6\frac{6}{7}$? Ans. $25\frac{171}{420}$.
6. Express $\frac{1}{2}$ of $\frac{3}{4}+\frac{2}{3}$ of $\frac{5}{6}+\frac{3}{4}$ of $\frac{1}{2}$ as a single fraction? Ans. $\frac{47}{36}=1\frac{11}{36}$.
7. Find the sum of $1\frac{14}{15}$, $8\frac{2}{5}$, $3\frac{9}{22}$ and $4\frac{3}{8}$? Ans. $18\frac{31}{264}$.
8. Find the sum of $\frac{1}{8}$ of $\frac{4}{5}+\frac{1}{12}$ of $\frac{4}{7}+\frac{3}{8}$ of $\frac{8}{9}$? Ans. $\frac{101}{210}$.
9. What single fraction is equivalent to $\frac{1}{2}$ of $\frac{1}{3}+\frac{1}{3}$ of $\frac{1}{4}+\frac{1}{4}$ of $\frac{1}{5}$? Ans. $\frac{3}{10}$.
10. What single fraction is equivalent to $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{1}{5}+\frac{1}{8}$ of $\frac{2}{3}$ of $\frac{1}{4}+\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{1}{7}$? Ans. $\frac{3}{16}$.
11. What single fraction is equivalent to $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}+\frac{2}{3}$ of $\frac{6}{8}$ of $\frac{5}{7}$? Ans. $\frac{89}{112}$.

12. Simplify $\frac{4\frac{1}{20}+8\frac{2}{15}}{4\frac{1}{15}+8\frac{1}{10}}$? Ans. $1\frac{1}{730}$.

13. Find a single fraction equivalent to $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{2}{3}+\frac{3}{5}$ of $\frac{7}{8}$? Ans. $\frac{289}{360}$.

14. Divide the sum of $\frac{5}{11}$ and $\frac{3}{7}$ by the sum of $\frac{4}{5}$ and $\frac{3}{7}$? Ans. $\frac{340}{473}$.

15. Simplify $\frac{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}}{\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\frac{31}{32}+\frac{63}{64}+\frac{127}{128}}$? Ans. $\frac{127}{769}$.

16. Simplify $\frac{\frac{1}{3}+\frac{4}{7}+\frac{21}{32}}{\frac{2}{5}+\frac{23}{36}+\frac{3}{5}}$? Ans. $\frac{3147}{3304}$.

## VII.—SUBTRACTION OF FRACTIONS.

What we have said of addition enables us to give at once the

### RULE FOR SUBTRACTION.

Reduce the given fractions, if necessary, to new ones having a common denominator, as in addition, and subtract the numerator of the less from that of the greater, and place the common denominator below the remainder, and the resulting fraction will be the difference between the given fractions.

EXAMPLES.—(1.) To subtract $\frac{5}{11}$ from $\frac{7}{11}$. Here the denominations being the same, we can subtract at once, and find the difference to be $\frac{2}{11}$. (2.) To find the value of $\frac{8}{9}-\frac{6}{7}$. These fractions brought to a common denominator, as in addition, become $\frac{56}{63}$ and $\frac{54}{63}$, and therefore the difference is $\frac{2}{63}$. (3.) To find the excess of $12\frac{1}{3}$ above $7\frac{5}{8}$, we find new fractions with a common denominator, viz., $\frac{8}{24}$ and $\frac{15}{24}$, and we write $12\frac{8}{24}-7\frac{15}{24}$. Now we are required first to subtract $\frac{15}{24}$ from $\frac{8}{24}$, but as we cannot do this directly, we take one of the 12 preceding units, and call it $\frac{24}{24}$, (for $\frac{24}{24}=1$,) then $\frac{24}{24}+\frac{8}{24}=\frac{32}{24}$, and $\frac{32}{24}-\frac{15}{24}=\frac{17}{24}$, then we subtract the 7 from the remaining 11; or, as in simple subtraction, 8 from 12, and we find the total excess to be $4\frac{17}{24}$. In practice it is most convenient to subtract 15 from 24, and add 8; thus $24-15=9$, and $9+8=17$, and the answer is $4\frac{17}{24}$.

### EXERCISES.

1. $\frac{5}{6}-\frac{3}{6}=\frac{2}{6}=\frac{1}{3}$. 2. $\frac{9}{11}-\frac{5}{11}=\frac{4}{11}$. 3. $\frac{10}{13}-\frac{3}{13}=\frac{7}{13}$. 4. $\frac{7}{8}-\frac{19}{24}=\frac{1}{12}$.

5. What is the difference between $\frac{3}{8}$ and $\frac{29}{32}$? Ans. $\frac{17}{32}$.

6. What is the difference between $\frac{449}{756}$ and $\frac{65}{189}$? Ans. $\frac{1}{4}$.

7. What is the difference between $\frac{59}{60}$ and $\frac{8}{15}$? Ans. $\frac{9}{20}$.

8. What is the excess of $20\frac{4}{9}$ above $9\frac{19}{22}$? Ans. $10\frac{115}{198}$.

9. From $5\frac{23}{32}$ take $3\frac{2}{7}$? Ans. $2\frac{97}{224}$.
10. What is the difference between $5\frac{7}{12}$ and $6\frac{431}{2520}$? Ans. $\frac{1481}{2520}$.
11. What is the value of $\frac{3}{4}+\frac{5}{6}-\frac{7}{8}+\frac{7}{12}-\frac{1}{2}$? Ans. $\frac{19}{24}$.
12. What is the difference between $100\frac{3}{19}$ and $50\frac{40}{41}$? Ans. $49\frac{142}{779}$.
13. What is the difference between $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{1}{2}$? Ans. 0
14. What is the difference between $\frac{3}{8}$ of $\frac{7}{10}$ and $\frac{5}{6}$ of $\frac{4}{9}$? Ans. $\frac{233}{2160}$.
15. What is the value of $\frac{1}{2}+\frac{2}{3}-\frac{3}{4}-\frac{5}{6}+\frac{11}{12}$? Ans. $\frac{1}{2}$.

## VIII.—DENOMINATE FRACTIONS.

Hitherto we have treated of fractions abstractly, and we must now apply the principles laid down to denominate numbers, and show how a fraction may be transformed from one denomination to another of the same kind *e. g.*, how a fraction of a shilling may be expressed as a fraction of a pound, and *vice versa.*

### RULE.

(1.) *Reduce the given quantity to the lowest denomination which it expresses.* (2.) *Reduce the unit in the terms of which it is to be expressed to the same denomination, and* (3,) *make the former the numerator and the latter the denominator, and the fraction will be expressed in the required terms.*

### EXAMPLES.

1. To express 16s. 8d. as a fraction of £1: Reducing 16s. 8d. to pence, we get 200, and reducing £1 to pence, we get 240, and, therefore, 16s. 8d. is £$\frac{200}{240}$, which, in its lowest terms, is £$\frac{5}{6}$.

2. In like manner, to express 17s. 6d. as a fraction of £1, we reduce 17s. 6d. to pence, and find 210, which, divided by 240, the number of pence in £1, gives £$\frac{210}{240}$=£$\frac{7}{8}$.

3. So also, 12s. 6d., expressed as a fraction of £1, is £$\frac{5}{8}$.

4. 18s. 4d., expressed as a fraction of £1, is £$\frac{11}{12}$.

5. 12s. 6d., expressed as a fraction of £1, is £$\frac{5}{8}$.

6. 13s. 4d., expressed as a fraction of £1, is £$\frac{2}{3}$.

### EXERCISES.

1. Express 3s. 9d. as a fraction of £1. Ans. £$\frac{3}{16}$.
2. Express 4s. 4d. as a fraction of £1. Ans. £$\frac{13}{60}$.
3. Express $4\frac{1}{2}$d. as a fraction of 1s. Ans. $\frac{3}{8}$s.
4. Express 1 oz. troy as a fraction of 1 lb. Ans. $\frac{1}{12}$.

5. Express 40 lbs. as a fraction of 1 cwt. Ans. $\frac{2}{5}$ cwt.

6. Express 50 lbs. as a fraction of 1 ton. Ans. $\frac{1}{40}$ ton.

7. Express 72 lbs. as a fraction of 1 cwt. Ans. $\frac{18}{25}$ cwt.

8. A day is 23 hours, 56 minutes, 48 seconds, nearly; what fraction of this will 7 hours be? Ans. $\frac{525}{1796}$.

9. Express 95 square yards as a fraction of an acre. Ans. $\frac{19}{968}$.

10. Express 14 yards as a fraction of a mile. Ans. $\frac{7}{880}$.

11. What fraction of a year ($365\frac{1}{4}$ days) is one month (30 days?) Ans. $\frac{40}{487}$.

12. Express 100 yards as a fraction of a mile. Ans. $\frac{5}{88}$.

13. Express 45 cents as a fraction of a dollar. Ans. $\frac{9}{20}$.

14. Express 60 lbs. as a fraction of a cwt. Ans. $\frac{3}{5}$.

15. A man has an income of \$3610 a year and saves $\frac{3}{7}$ of it; how much does he spend? Ans. \$$2062\frac{6}{7}$.

To find the value of a fraction in the denominations which the integer contains, reduce the numerator to the next lower denomination, and divide the result by the denominator; if there be a remainder, reduce to the next denomination, and divide again, and continue the same operation till there is either no remainder, or down to the lowest denomination by which the integer is counted. Thus, $\frac{6}{7}$ of £1 is 120 shillings divided by 7, which gives 17 shillings and 1 shilling, or 12 pence remainder, and $12\div7=1\frac{5}{7}$, so that £$\frac{6}{7}$=17 shillings and $1\frac{5}{7}$d.

### EXERCISES.

1. What is the value of £$\frac{7}{12}$ Stg.? Ans. 11s. 8d.

2. What is the value of $\frac{9}{10}$ of a yard? Ans. 2 ft., $8\frac{2}{5}$ in.

3. What is the value of $\frac{13}{24}$ of a mile? Ans. 4 fur., 13 rods, 1 yd., 2 ft., 6 in.

4. What is the value of $\frac{19}{20}$ of a shilling Stg.? Ans. $11\frac{2}{5}$d.

5. What is the value of $\frac{4}{7}$ of a ton? Ans. 11 cwt., 1 qr., $17\frac{6}{7}$ lbs.

6. What is the value of $\frac{2}{3}$ lb. troy? Ans. 8 oz.

7. What is the value of $\frac{6}{13}$ of a shilling? Ans. $5\frac{7}{13}$d.

8. What is the value of \$$\frac{8}{9}$? Ans. $88\frac{8}{9}$ cts.

9. What is the value of $\frac{4}{5}$ of \$6? Ans. \$4.80.

10. What is the value of $\frac{17}{20}$ of \$8? Ans. \$6.80.

To change a fraction to one of a lower denomination, reduce the numerator to that denomination and divide by the denominator. Thus, to express £$\frac{7}{145}$ as a fraction of a shilling, reduce £7 to shillings, which gives 140 shillings, and $\frac{140}{145}=\frac{28}{29}$ of a shilling is obtained.

EXERCISES.

1. Express $\frac{7}{100}$ of a foot as a fraction of an inch. Ans. $\frac{21}{25}$.
2. Express $\frac{1}{120}$ of a cwt. as a fraction of a lb. Ans. $\frac{5}{6}$.
3. Express $\frac{1}{20}$ of a lb. as a fraction of an oz. Ans. $\frac{4}{5}$.
4. Express $\frac{3}{4}$ of $\frac{5}{12}$ of a yard as a fraction of a foot. Ans. $\frac{15}{16}$.
5. Express $\frac{1}{27}$ of a rod as a fraction of a yard. Ans. $\frac{22}{27}$.
6. Express $\frac{1}{2}$ of $\frac{1}{8}$ of an acre as a fraction of a rood. Ans. $\frac{1}{4}$.
7. Reduce $\frac{3}{26}$ cwt. to the fraction of a pound. Ans. $11\frac{7}{13}$ lb.
8. Reduce $\frac{1}{21}$ of a day to the fraction of a minute.
   Ans. $68\frac{4}{7}$ min.
9. What part of a second is the *one-millionth* part of a day?
   Ans. $\frac{54}{625}$ sec.
10. Reduce £$\frac{1}{36}$ to the fraction of a penny. Ans. $6\frac{2}{3}$d.
11. Reduce $\frac{1}{24}$ of a pound avoirdupois to the fraction of an oz.
    Ans. $\frac{2}{3}$ oz.

The reducing of a denominate fraction from one of a lower to one of a higher denomination being the converse of the last rule, we must perform the same operation on the denominator as was there performed on the numerator.

Thus, $\frac{5}{8}$d. is £$\frac{1}{384}$, for £$\frac{5}{8\times12\times20}$=£$\frac{5}{1920}$=£$\frac{1}{384}$.

EXERCISES.

1. What part of 1 lb. troy is $\frac{3}{5}$ of a grain? Ans. $\frac{1}{9600}$.
2. What part of 4 days is $\frac{3}{4}$ of a minute? Ans. $\frac{1}{7680}$.
3. What part of 5 bushels is $\frac{2}{3}$ of $\frac{3}{4}$ of a pint? Ans. $\frac{1}{640}$.
4. What part of a rod is $2\frac{3}{4}$ of $\frac{7}{12}$ of an inch? Ans. $\frac{7}{864}$.
5. What part of 2 weeks is $\frac{5}{14}$ of a day? Ans. $\frac{5}{196}$.

## DECIMAL FRACTIONS.

**16.**—WE have seen already (Art. 3,) that every figure to the right is *one-tenth* the value it would have if removed one place to the left. Thus, resuming our former example, 8 standing alone means 8 *units*, but if we place another 8 after it, thus 88, it now means 8 *tens*, so that the last 8 is *one-tenth* of the first. Now, since the 8 to the right expresses units, another 8 placed to the right will express *eight-tenths* of the same unit, and another subjoined will express $\frac{8}{100}$ of the unit. Thus we see that the decimal notation is directly an extension of the Arabic. Hence arose the convenient mode of writing $8\frac{7}{10}$ in the form 8.7, by which is indicated that all the figures before the decimal point (.) represent integers, and all after it fractions, each being *one-tenth* of what it would be if *one* place further to the left. Therefore 888.888 is *eight hundreds, eight tens, eight units,—eight-tenths, eight one-hundredths*, and *eight one-thousandths;* or, $\frac{8}{10}+\frac{8}{100}+\frac{8}{1000}$. These added will give $\frac{800}{1000}+\frac{80}{1000}+\frac{8}{1000}$, or $\frac{888}{1000}$, which, for brevity, is written .888, and may be read eight hundred and eighty-eight one-thousandths; or, as is usual, *point* 888, or *decimal* 888, but never properly eight hundred and eighty-eight. In the same manner as 80 means 8 tens and no units, so .08 means no tenths, but 8 hundredths, and .008 means no tenths, no hundredths, but eight one-thousandths, &c.—Hence we see that for every cipher in the denominator, which is always 10 of a power of 10, there must be a figure in the numerator when expressed decimally. Thus: $\frac{8}{1000}$ must be written decimally .008. From this we see that removing the decimal point one place to the right is the same as multiplying by 10, and removing it one place to the left is the same as dividing by 10; so, also, removing the point two places to the right is the same as multiplying by 100, and removing it two places to the left is the same as dividing by 100. This is the principle already laid down for the reduction of dollars to cents, and cents to dollars.

I.—REDUCTION OF COMMON FRACTIONS TO DECIMALS.—Let it now be required to express the common fraction $\frac{7}{8}$ as a decimal. We have seen (Art. 14,) that we may multiply the terms of any fraction by the same number without changing the value of the fraction. Let us then multiply the terms of $\frac{5}{8}$ by 1000, and we get $\frac{5000}{8000}$. On the same principle we can divide the terms by the same number without altering the value. Let us then divide by 8, and we get $\frac{625}{1000}$, where the denominator is a power of 10, and therefore the fraction is in the decimal form, and may be written .625, the denominator being omitted. But as it is not always apparent by what power of 10 we must multiply, so that when the terms are divided by the given denominator, that denominator may be transformed into 10 or a power of 10, *i. e.*, into 1 followed by a certain number of ciphers, we may as well add ciphers, one by one, as we proceed. This is exhibited in the annexed example. From these principles we can deduce a rule for reducing a common fraction to a decimal.

```
8)50(0.625
  48
  ——
   20
   16
   ——
    40
    40
```

### RULE.

```
16)110(.6875
    96
    ——
    140
    128
    ———
     120
     112
     ———
      80
      80
```

```
7)10(.1̇42857̇
   7
   —
   30
   28
   ——
    20
    14
    ——
     60
     56
     ——
      40
      35
      ——
       50
       49
       ——
        1
```

*Divide the numerator, with a cipher or ciphers annexed, by the denominator. Thus $\frac{11}{16}$ will give, as in the margin,* .6875. In the examples given we find that the addition of three ciphers to the first, and four to the second, makes the numerator divisible by the denominator without remainder. Such fractions are called terminating decimals. From this we see that there are common fractions whose terms can be multiplied by such powers of 10 as will make the numerator divisible by the denominator without remainder, but it often happens that no power of ten will effect this, and that remainders occur which cannot be made divisible evenly by the denominator, by the addition of any number of ciphers. Such fractions will never terminate, and therefore are called interminate, and the common fraction can never be expressed exactly in the decimal form, and all we can do is to make an approximation more or less close, according to the number of decimal places to which we carry it. Let us take the fraction $\frac{1}{9}$.—First, 9 is not contained in 1, and therefore we place the decimal point in the quotient, and add a cipher to the numerator, and we find that 9 is contained *once* in 10, with a remainder 1,—annexing another cipher, we again obtain 1 in the quotient, and this will obviously continue *ad infinitum.*—This recurrence is marked by a dot or dash over the figure, thus: .$\dot{1}$ or .1′. If we express $\frac{1}{7}$ as a decimal, we find that after we have got six figures in the quotient, we have a remainder 1, the same as the original numerator, and therefore we should again obtain the same quotient .142857, and hence this is called a circulating or periodic decimal, and the first and last of the recurring figures are marked with a point or trait. Thus: .$\dot{1}4285\dot{7}$ or .1′42857′. Again, it often happens that some figures do not recur whilst others following them do, as in the annexed example, after we have got

$$\begin{array}{rr}
\frac{4111}{33300}) & 41110(.12\dot{3}4\dot{5}. \\
 & 33300 \\ \hline
 & 78100 \\
 & 66600 \\ \hline
 & 115000 \\
 & 99900 \\ \hline
 & 151000 \\
 & 133200 \\ \hline
 & 178000 \\
 & 165500 \\ \hline
 & 11500
\end{array}$$

five figures the 11500 which gave us the third figure 3, in the quotient recurs, and by pursuing the division we should find 345 recurring without end. When all the figures recur, the fraction is called a pure periodic decimal; when only some of them recur, it is called mixed, and the term rapeater is applied when only one figure recurs, as $\frac{1}{9}$=.1111, &c.=$.\dot{1}$ or $\frac{7}{12}$=.58333, &c.=$.58\dot{3}$. Since the denominator is always 10, or a power of 10, and since 10 has no factors but 2 and 5, and therefore powers of 10 no factors but 2 and 5, or powers of these, it follows that no decimal will terminate except the denominator be expressed by either or both of these, or some power or product of them. Hence all terminating decimals are derived from common fractions having for denominator some figure of the series 2, 4, 8, 16, 32, &c., or 5, 25, 125, &c., or, 10, 20, 40, 50, 60, 80, 100, &c.

### EXERCISES.

1. Reduce the common fraction $\frac{1}{4}$ to a decimal. Ans. .25.
2. Reduce the common fraction $\frac{1}{2}$ to a decimal. Ans. .5.
3. Reduce the common fraction $\frac{3}{4}$ to a decimal. Ans. .75.
4. Reduce the common fraction $\frac{1}{3}$ to a decimal. Ans. $.\dot{3}$.
5. Reduce the common fraction $\frac{1}{9}$ to a decimal. Ans. $.\dot{1}$.
6. Reduce the common fraction $\frac{1}{8}$ to a decimal. Ans. .125.
7. Reduce the common fraction $\frac{1}{6}$ to a decimal. Ans. $.1\dot{6}$.
8. Reduce the common fraction $\frac{1}{7}$ to a decimal. Ans. $.\dot{1}4285\dot{7}$.
9. Reduce the common fraction $\frac{1}{5}$ to a decimal. Ans. .2.
10. Reduce the common fraction $\frac{1}{10}$ to a decimal. Ans. .1.
11. Reduce the common fraction $\frac{1}{11}$ to a decimal. Ans. $.\dot{0}\dot{9}$.
12. Reduce the common fraction $\frac{1}{12}$ to a decimal. Ans. $.08\dot{3}$.
13. Reduce the common fraction $\frac{2}{3}$ to a decimal. Ans. $.\dot{6}$.
14. Reduce the common fraction $\frac{4}{5}$ to a decimal. Ans. .8.

15. Reduce the common fraction $\frac{5}{6}$ to a decimal. Ans. $.8\dot{3}$.
16. Reduce the common fraction $\frac{3}{8}$ to a decimal. Ans. .375.
17. Reduce the common fraction $\frac{5}{8}$ to a decimal. Ans. .625.
18. Reduce the common fraction $\frac{7}{8}$ to a decimal. Ans. .875.
19. Reduce the common fraction $\frac{4}{9}$ to a decimal. Ans. $.\dot{4}$.
20. Reduce the common fraction $\frac{5}{7}$ to a decimal. Ans. $.\dot{7}1428\dot{5}$.
21. Reduce the common fraction $\frac{10}{11}$ to a decimal. Ans. $.\dot{9}\dot{0}$.
22. Reduce the common fraction $\frac{11}{12}$ to a decimal. Ans. $.91\dot{6}$.
23. Reduce the common fraction $\frac{12}{13}$ to a decimal. Ans. $.\dot{9}2307\dot{6}$.
24. Reduce the common fraction $\frac{11}{14}$ to a decimal.
Ans. $.\dot{7}85714\dot{2}$.
25. Reduce the common fraction $\frac{11}{16}$ to a decimal. Ans. .6875.
26. Reduce the common fraction $\frac{5}{66}$ to a decimal. Ans. $07\dot{5}$.
27. Reduce the common fraction $\frac{11}{32}$ to a decimal. Ans. .34375.
28. Reduce the common fraction $\frac{5}{1025}$ to a decimal.
Ans. $.004\dot{8}780\dot{4}$.
29. Reduce the common fraction $\frac{37}{79}$ to a decimal.
Ans. $.\dot{4}68354430379\dot{7}$.
30. Reduce the common fraction $\frac{4}{909}$ to a decimal. Ans. $.\dot{0}04\dot{4}$.
31. Reduce the common fraction $\frac{1}{49}$ to a decimal.
Ans. $.\dot{0}2040816326530612244897959183673469387755\dot{1}$.
32. Express $\frac{1}{99}$ decimally. Ans. $.\dot{0}\dot{1}$.
33. Express $\frac{1}{999}$ decimally. Ans. $.\dot{0}0\dot{1}$.
34. Express $\frac{1}{999}$ decimally. Ans. $.\dot{0}00\dot{1}$.
35. Express $\frac{6}{10001}$ decimally. Ans. $.\dot{0}00599\dot{4}$.

To reduce a denominate number to the form of a decimal fraction, *reduce it to the lowest denomination which it contains; reduce the integral unit to the same denomination, and divide the former by the latter.*

Thus, to express 18s. 4d. as a decimal of £1, we must reduce it to pence, the lowest denomination given, and divide it by 240, the number of pence in £1, which gives the fraction $\frac{220}{240}=\frac{22}{24}=\frac{11}{12}$, and this reduced to a decimal, gives .716 or £.916. In like manner 15s. 10½d. is reduced to half-pence, viz., 381, and the half-pence in £1 are 480, and $\frac{381}{480}=\frac{127}{160}$, which expressed decimally is .79375.

### EXERCISES.

1. What decimal of £1 is 11s. 4½d.? Ans. .56875.

2. Express 15s. 9¾d. as a decimal of £1. Ans. .790625.

3. What decimal of a square mile is an acre? Ans. .0015625.

4. Express 1 pound troy as a decimal of 1 pound, avoirdupois.* Ans. $.8\dot{2}8571\dot{4}$.

5. Reduce 17 cwt. to the decimal of a ton. Ans. .85.

6. Express $\frac{15}{16}$ of a cwt. as a decimal of a ton. Ans. .046875.

oz.
11÷16=.6875
lbs.
22.6875÷25=.9075
qrs.
2.9075÷4=.726875
cwt.
11.726875÷20=.58634375

---

16)11

25)22.6875

4)2.9075

20)11.726875

.58634375

The operation annexed is often convenient in practice. To reduce 11 cwt., 2 qrs., 22 lbs., 11 oz., to the decimal of a ton. First, we divide the 11 oz. by 16, the number of oz. in 1 lb., and then annex the 22 lbs., and divide by 25, the lbs in a qr., and so on. The first form of the work is best suited for illustration, the second is neater in practice. The principle is the same as that implied in the general rule given above.

### ADDITIONAL EXERCISES.

7. Reduce 10 drams to the decimal of 1 lb. Ans. .0390625.

8. Reduce 11 dwt. to the decimal of 1 lb. Ans. $.0458\dot{3}$.

9. Express 1 oz., avoirdupois, as a fraction of 1 oz., troy, (see note.) Ans. $.911458\dot{3}$.

10. Reduce 5 hours, 48 minutes, 49.7 seconds to the decimal of a day. Ans. .2422419.

---

* A caution seems necessary here, for since the pound (troy,) contains 12 ounces, and the pound (avoirdupois,) 16, the natural conclusion would be that the pound (troy) is $\frac{12}{16}$ or $\frac{3}{4}$ of the pound avoirdupois. This is not correct, for the ounce troy exceeds the ounce avoirdupois by 42½ grains, though the pound avoirdupois (7000 grs.) exceeds the pound Troy (5740 grs.) by 1240 grains. This will be manifest from the operation on the margin, where the standard weights according to Act of Parliament, dating from A. D. 1826, are given.

5760÷12=480
7000÷16=437½

difference.. 42½

II.—REDUCTION OF DECIMALS TO COMMON FRACTIONS.—To find the common fraction corresponding to any given decimal.—This involves three cases according as the fraction is a terminating decimal, a pure circulating decimal, or a mixed circulating decimal. The first case scarcely requires proof. We give it, however, in order to assist those unaccustomed to the algebraic notation, to understand more clearly the form of illustration used in the other cases.

Let us take the fraction .9375, and use d for decimal. We now write d=.9375, and multiplying both terms by 10000, we obtain 10000 d=9375, and therefore $d=\frac{9375}{10000}$, which reduced to its lowest terms is $\frac{15}{16}$, the common fraction required. This is simply putting for denominator 1, followed by a cipher for each figure in the decimal.

$$\begin{array}{l} d=.666+ \\ \hline 9\ d=6 \end{array}$$

To find the value of a pure circulator, suppose $.\dot{6}$. Put $d=.\dot{6}$, or d=.666, and multiply by 10, which gives 10 d=6.66, and writing the former expression beneath, and subtracting, we get 9 d=6, and consequently $d=\frac{6}{9}$ or $\frac{2}{3}$, the common fraction sought.

$$\begin{array}{r} d=.\dot{7}\dot{2} \\ 100\ d=72.\dot{7}\dot{2} \\ \hline 99\ d=72 \end{array}$$

Let us now seek the vulgar fraction corresponding to $.\dot{7}\dot{2}$. Put $d=.\dot{7}\dot{2}$, multiply by 100, and subtract as before, and there results a remainmainder of 99 d=72, or $d=\frac{72}{99}=\frac{8}{11}$.

$$\begin{array}{r} d=.56\dot{8}\dot{1} \\ 10000\ d=5681.\dot{8}\dot{1} \\ 100\ d=\quad 56.\dot{8}\dot{1} \\ \hline 9900\ d=5625 \end{array}$$

Again, to find the vulgar fraction coresponding to $.568\dot{1}$. Multiply first by 10000, and then by 100, and subtract the latter from the former, and you obtain 9900 d=5625, and hence $d=\frac{5625}{9900}$, which reduced to its lowest terms is $\frac{25}{44}$.

From these investigations the three following rules for the three cases mentioned are derived :

I. *If the fraction be a terminating decimal make it the numerator, and for denominator write* 1, *followed by as many ciphers as there are figures in the decimal.*

II. *If the decimal be a pure circulator, make the digits of the decimal the numerator, and for denominator write as many nines as there are figures in the period*

III. *If the decimal be a mixed circulator, subtract the non-circulating part from the whole decimal to the end of the first period, both being treated as whole numbers; make the remainder the numerator, and for denominator write as many nines as there are circulating figures, and after them as many ciphers as there are non-circulating figures. In all cases reduce to the lowest terms.*

EXERCISES.

1. Find the vulgar fraction corresponding to $.\dot{0}\dot{4}$. Ans. $\frac{4}{99}$.
2. Find the vulgar fraction corresponding to $.\dot{5}\dot{4}$. Ans. $\frac{6}{11}$.
3. Find the vulgar fraction corresponding to $.24\dot{5}\dot{7}$. Ans. $\frac{811}{3300}$.
4. Find the vulgar fraction corresponding to $.\dot{1}$. Ans. $\frac{1}{9}$.
5. Find the vulgar fraction corresponding to $.\dot{3}$. Ans. $\frac{1}{3}$.
6. Find the vulgar fraction corresponding to $.\dot{7}$. Ans. $\frac{7}{9}$.
7. Find the vulgar fraction corresponding to $.75$. Ans. $\frac{3}{4}$.
8. Find the vulgar fraction corresponding to $.47\dot{5}4\dot{3}$. Ans. $\frac{3958}{8325}$.
9. Find the vulgar fraction corresponding to $.\dot{4}68354430379\dot{7}$. Ans. $\frac{37}{79}$.
10. Find the vulgar fraction corresponding to $.4\dot{9}$. Ans. $\frac{1}{2}$.
11. Find the vulgar fraction corresponding to $.\dot{1}6\dot{2}$. Ans. $\frac{18}{111}$.
12. Find the vulgar fraction corresponding to $.1\dot{4}$. Ans. $\frac{13}{90}$.
13. Find the vulgar fraction corresponding to $.013\dot{8}$. Ans. $\frac{1}{72}$.
14. Find the vulgar fraction corresponding to $.56\dot{8}\dot{1}$. Ans. $\frac{25}{44}$.
15. Find the vulgar fraction corresponding to $.\dot{5}9\dot{2}$. Ans. $\frac{16}{27}$.

The last rule may be deduced from the other two in the following manner:—Let us take the mixed circulator $.4\dot{1}\dot{8}$, and this being multiplied by 10, the four becomes a whole number, and to preserve the same value, 10 is put as a divisor, which gives $\frac{4.\dot{1}\dot{8}}{10}$ or $\frac{4+.\dot{1}\dot{8}}{10}$, but by rule II. we have $.\dot{1}\dot{8}=\frac{18}{99}$, and hence the whole may be written $\frac{4+\frac{18}{99}}{10}=\frac{396+18}{990}=\frac{414}{990}=\frac{23}{55}$, and this result corresponds to rule III.

## IV.—ADDITION & SUBTRACTION OF DECIMALS.

From what has been said, it is plain that decimals can be added and subtracted just as whole numbers, care being taken to keep the decimal points in the same vertical line. In all operations into which repetends enter, it should be observed that in order to have a result true to any given number of places, it is generally desirable to carry out the repetend to one or two places more than the required number. It is often sufficient, however, to allow for what would be carried, which can usually be done by inspection. In all cases, respect should be had to the degree of exactness which the nature of the calculation requires. The figures beyond those required can be estimated and added in. Thus, if only five places are required, and the calculation be carried to six places, and the seventh figure is a large *one*, it should be added to the sixth figure.

This may be stated in the form of a

### RULE.

*Add and subtract as in whole numbers, keeping the decimal points in the same vertical line.*

### EXERCISES.

| (1.) | (2.) | (3.) |
|---|---|---|
| 1.78645 | 8.58333333+ | 51.250000000 |
| 3.97863 | 17.74747474+ | 3.444444444+ |
| 7.84396 | 112.08080808+ | 7.637373737+ |
| 4.32782 | 6.12500000 | .885555555+ |
| 9.54179 | 15.66666667 | 11.875000000 |
| 11.69857 | .76969697 | 7.875875875+ |
| 5.48491 | 11.00000000 | 7.111111111+ |
| 44.66213 | 171.97297979 | $90.079\dot{3}6072\dot{4}$ |

In exercise 2, the eighth figure of each of the fifth and sixth lines is made 7 instead of 6, which renders it unnecessary to make any allowance for the repetends that would follow, but this change is not made on any of the last figures of exercise 3, and therefore we add 2 for what would be carried from the tenth decimal place to the ninth.

4. Find in the decimal form the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$. Ans. $2.31\dot{6}$.

5. Find in the decimal form the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, $\frac{31}{32}$, $\frac{63}{64}$, $\frac{127}{128}$.
Ans. 6.0078125.

6. Find in the decimal form the sum of $\frac{23}{25}$, $\frac{2}{5}$, $\frac{7}{24}$, $\frac{11}{15}$.
Ans. 2.345.

7. Find in the decimal form the sum of $2\frac{3}{5}$, $4\frac{7}{8}$, $5\frac{3}{10}$.
Ans. 12.775.

8. What is the sum of .786425, .975324, .176009, .32, .62519375, .4? Ans. 3.28295175.

9. Add to 6 places $18.1\dot{2}7\dot{6}$, $11.\dot{3}4\dot{9}$, $12.1\dot{4}\dot{5}$, $8.\dot{6}4\dot{8}$, $15.2\dot{3}$.
Ans. 65.504414.

10. Find to 6 places the sum of $15.\dot{7}$, $12.\dot{4}$, $18.\dot{3}8\dot{7}$, $.41\dot{6}$, $.74\dot{6}8\dot{7}$, $.\dot{9}$, $.\dot{4}\dot{5}$, $10.\dot{4}\dot{5}$, $.12\dot{3}4\dot{5}$. Ans. 59.351152.

11. What is the sum of $.\dot{7}\dot{6}$, $.41\dot{6}$, $.\dot{4}\dot{5}$, $.\dot{6}4\dot{8}$, $.2\dot{3}$ to five places of decimals? Ans. 2.52087.

12. Reduce to decimals, and find the sum of $\frac{3}{8}$, $\frac{1}{22}$, $\frac{7}{24}$, $\frac{22}{88}$.
Ans. $1.41\dot{6}$.

13. Find the sum of $.4\dot{2}\dot{7}$, $.41\dot{6}$, $1.\dot{3}2\dot{8}$, $3.0\dot{2}\dot{9}$, 5.476 to six places of decimals. Ans. 10.678037.

14. Required the sum of 1.25, $1.\dot{4}$, $1.6\dot{3}\dot{7}$, $1.88\dot{5}$, 1.684, 1.937, 1.148 and 1.764085. Ans. 12.750458.

15. Find the sum of $.4\dot{6}32\dot{1}$, $.81\dot{5}3\dot{2}$, $.1549\dot{2}\dot{6}$, $.75\dot{3}\dot{2}$ to true to four places. Ans. 2.1867.

16. From 3.468 substract 1.2591, and you find the excess 2.2089.

17. What is the excess of 10.008576 above 5.789?
Ans. 4.219576.

18. From $11.\dot{4}$ take $1.\dot{4}\dot{8}$, and there remains to six places 9.959596.

19. What is the excess of 7.8 above $1.37546\dot{5}\dot{8}$?
Ans. $6.42453\dot{4}\dot{1}$.

20. What is the difference between $9.46\dot{5}7\dot{4}$, and $4.18\dot{3}4\dot{5}$?
Ans. $5.28\dot{2}2\dot{9}$.

21. Express, decimally, the difference between $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$, and $\frac{1}{3}+\frac{4}{7}+\frac{21}{32}$. Ans. 2.34613+.

22. What is the difference, according to the decimal notation, between $\frac{7}{9}$ and $\frac{13}{90}$ true to six places of decimals? Ans. .636363.

23. What is the difference between $\frac{1}{2}$ and $\frac{4}{7}$ expressed decimally true to six decimal places? Ans. .071428.

24. What is the difference between the vulgar fractions corresponding to $.4\dot{9}$ and .5? Ans. 0.

25. Find the value of .786425+.975324+.176009+.32+.62519375—3.28295175+.4. Ans. 0.

26. What is the difference between 138.6012, and 128.8512? Ans. 9.75

27. What is the excess of 31.6322 above 5.674+1.83+.3125+18.62+4.3+.395—.5. Ans. 1.0007.

28. What is the excess, expressed decimally, of 5.83 above $4\frac{17}{99}$. Ans. $1.65\dot{8}\dot{2}$.

29. What is the difference between 8.375 and $7\frac{3}{7}$ true to six decimal places? Ans. .946428.

30. What is the value of 601.050725—441.001—.00625—3.818475—156.1+.125. Ans. .25.

## V.—MULTIPLICATION OF DECIMALS.

$$\begin{array}{r} 5.678 \\ 6 \\ \hline 34.068 \end{array}$$

$$\begin{array}{r} 5678 \\ .6 \\ \hline 3406.8 \end{array}$$

If we multiply a decimal by a whole number, the process is precisely the same as if the multiplicand were a whole number, but care must be taken to keep the decimal point in the same relative position. Thus, in the annexed example, as there are three decimal places in the multiplicand, we make three also in the product. If we have to multiply a whole number by a decimal, we must mark off a decimal in the product for each decimal in the multiplier.—The reason of this will be manifest from the consideration that if we multiply 8 units by .6, or $\frac{6}{10}$, we get $\frac{48}{10}$, or 4.8, *i. e.*, 4 units and 8-tenths; and again, when we multiply 7 tens by .6 or $\frac{6}{10}$, we get $\frac{420}{10}$=42 units, which with the 4 units already obtained, make 46 units, and we now have arrived at whole numbers. The same illustration will apply to multiplying by .66, which requires two decimal places to be laid off from the right. Therefore, for every decimal place in the multiplier one must be cut off in the product, and we saw already that for every decimal place in the multiplicand, a decimal place must be cut off in the product, and therefore we conclude that for every decimal place in both factors, a decimal place must be marked in the product. It may be well to vary the illustration by observing that as the tenth of a tenth is a one-hundredth, tenths multiplied by tenths give hundredths; so also the product of tenths and hundredths is thousandths, and so on. Thus: .2 or $\frac{2}{10}$, multiplied by .3 or $\frac{3}{10}$, is $\frac{6}{100}$. Now, .6 would not represent this, for that would mean $\frac{6}{10}$; hence, it is necessary to prefix a cipher, and write .06, and this agrees with what has been already noted (Art. 3) regarding whole numbers, viz., that we are compelled by the nature of the notation to introduce a zero character, and in the present instance the cipher means that there are no *tenths*, just as it indicated in the case referred to that there were no tens. So, also, $\frac{6}{1000}$ would be written decimally .006, which would mean that there are no tenths, no hundredths, but 6 thousandths. From these explanations we deduce the

RULE:

*Multiply, as in whole numbers, and cut off from the right a decimal place for every one in both multiplier and multiplicand.*

EXAMPLES.

Multiply .78 by .42. Here we multiply as if the quantities were whole numbers, and in the product point off a decimal figure for each one in both multiplier and multiplicand. In Ex. 1, the number of figures in the product is the same as the number in both factors, and therefore we have no whole number in the result, but four decimal places. In Ex. 2 there are four decimal places in the factors, and there are six figures in the product, and consequently two figures represent whole numbers. In Ex. 3, when we multiply 6 by 3, we obtain 18, but if we had carried the repitend out one place farther we should have had 5 to be multiplied by 3, and consequently 1 to carry, so we add 1 to the 18, and in like manner we must allow 2 when multiplying by 4, and 1 when multiplying by 2.

$$(1.)\quad \begin{array}{r} .78 \\ .42 \\ \hline 156 \\ 312\phantom{0} \\ \hline .3276 \end{array}$$

$$(2.)\quad \begin{array}{r} .674 \\ 34.6\phantom{0} \\ \hline 4044 \\ 2696\phantom{0} \\ 2022\phantom{00} \\ \hline 23.3204 \end{array} \qquad (3.)\quad \begin{array}{r} 4.\dot{5}\dot{6} \\ 2.43 \\ \hline 1369 \\ 1826\phantom{0} \\ 913\phantom{00} \\ \hline 11.0929 \end{array}$$

### EXERCISES.

1. Multiply 7.49 by 63.1. Ans. 472.619.
2. Multiply .156 by .143. Ans. .022308.
3. Multiply 1.05 by 1.05, and the product by 1.05.
Ans. 1.157625.
4. Find the continual product of .2, .2, .2, .2, .2, .2.
Ans. .000064.
5. Multiply .0021 by 21. Ans. .0441.
6. Multiply 3.18 by 41.7. Ans. 132.606.
7. Multiply .08 by .036. Ans. .00288.
8. Multiply .13 by .7. Ans. .091.
9. Multiply .31 by .32 Ans. .0992.
10. Find the continual product of 1.2, 3.25, 2.125.
Ans. 8.2875.
11. Multiply 11.4 by 1.14. Ans. 12.996.
12. Find the continual product of .1, .1, .1, .1, .1, .1.
Ans. .000001.
13. Multiply 1240 by .008. Ans. 9.92.
14. Find the continual product of .101, .011, .11, 1.1 and 11.
Ans. .001478741.
15. Multiply $7.4\dot{3}$ by .862 to six places of decimals.
Ans. $.6408\dot{3}\dot{9}$.
16. Multiply 3.18 by 11.7, and the product by 1000.
Ans. 132606.
17. Multiply .144 by .144. Ans. .020736.
18. What is the continual product of 13.825, 5.128 and .001 ?
Ans. .0708946.
19. What is the continual product of 4.2, 7.8 and .01 ?
Ans. .3276.
20. What is the continual product of .0001, 6.27 and 15.9 ?
Ans. .0099693.

CONTRACTED METHOD.—In many instances where long lines of figures are to be multiplied together, the operation may be very much shortened, and yet sufficient accuracy attained. We may instance what the student will meet with hereafter, calculations in compound interest and annuities, involving sometimes most tedious operations. By the following method the results in such cases may be obtained with great ease, and correct to a very minute fraction. If we are computing dollars and cents, and extend our calculation to four places of decimals, we are treating of the one-hundredth part of a cent, or the ten-thousandth part of a dollar, a quantity so minute as to become relatively valueless. Hence we conclude that three or four decimal places are sufficient for all ordinary purposes. There are cases, indeed, in which it is necessary to carry out the decimals farther, as, for instance, in the case of Logarithms to be considered hereafter. The principle of the contracted method will be best explained by comparing the two subjoined operations on the same quantities.

Let it be required to find the product of 6.35642 and 47.6453, true to four places of decimals:

| EXTENDED OPERATION. | | CONTRACTED OPERATION. |
|---:|:---|---:|
| 6.35642 | | 6.35642 |
| 47.6453 | | 3546.74 |
| 19 | 06926 | 2542568 |
| 317 | 8210 | 444949 |
| 2542 | 568 | 38138 |
| 38138 | 52 | 2542 |
| 444949 | 4 | 317 |
| 2542568 | | 19 |
| | | 2 carried. |
| 302.8535 | 37826 | 302.8535 |

### RULE FOR THE CONTRACTED METHOD.

*Place the units' figure of the whole number under the last required decimal place of the multiplicand, and the other integral figures to the right of that in an inverted order, and the decimal figures, also in an inverted order, to the left of the integral unit; multiply by each figure of the inverted multiplier, beginning with the figure of the multiplicand immediately above it, omitting all figures to the right, but allowing for what would have been carried if the decimal had been carried out one place farther—place the first figure of each partial product in the same vertical column, and the others in vertical columns to the left; the sum of these columns will be the required product.*

Thus, in the above example, we are required to find the product correct to four decimal places, therefore we set the units' figure, 7, under the fourth decimal figure, and the tens' figure, 4, to the right, and the decimal figures, 6453, to the left in reversed order; then we multiply the whole line by 4, and then we multiply by 7, omitting the 2 which stands to the right, but allowing 1 for what would have been carried, that is, we say 7 times 4 is 28, and 1 is 29, and we write the nine under the 8, the first figure of the first partial product. By comparing the contracted method with the figures of the extended form, which are to the left of the vertical line drawn after the fourth decimal figures, it will be seen that the figures of each column are the same but placed in reversed order, which makes no difference in the sum, as $5+3=3+5=8$. This is the same principle as the contracted method of multiplying by 17, 71, &c., suggested in the article on simple multiplication.*

The object of writing the multiplier in a reversed order is simply to make the work come in the usual form, as otherwise we should be crossing and recrossing, so to speak, as will be seen by the operation in the margin.—Beginning with the left hand figure of the multiplier, and the right hand figure of the multiplicand, we find the first partial product; then taking the second figure of the multiplier from the left, (7) and the second figure of the multiplicand from the right, we get the second partial product, and so on, moving one place each time towards the right in the multiplier, and one place towards the left in the multiplicand. This is so different from the ordinary mode of operation, as to be excessively awkward and puzzling, and this gave rise to the idea of reversing the order of the digits. We append this remark as most persons cannot at first sight comprehend the reason of the inversion.

```
  6.35642
 47.6453
 ________
2542568
 444949
  38138
   2542
    317
     19
      2 allowed.
 _______________
302.8535
```

---

* Let the learner observe that all the figures of the first column are of the same rank, viz., ten-thousandths, and therefore may be added together, and as the value of each figure is increased or decreased 10 times according to its position to left or right, it follows that all figures at equal distances from the decimal point, whether to right or left, are of the same rank, *i. e.*, units will be under units, tens under tens, tenths under tenths, hundredths under hundredths, &c., &c. The contracted method is not of much use in terminating decimals which extend to only a few places, but it saves a vast deal of labour in questions which involve either repetends or terminating decimals expressed by a long line of decimal figures

### ADDITIONAL EXERCISES.

21. Multiply .26736 by .28758 to four decimal places
Ans. .0769.

22. Multiply $7.\dot{2}8571\dot{4}$ by 36.74405 to five decimal places.
Ans. 267.70665.

23. Multiply 2.656419 by $1.\dot{7}2\dot{3}$ to six decimal places.
Ans. 4.578932.

24. What decimal fraction, true to six places, will express the product of $\frac{9}{14}$ multiplied by $\frac{3}{17}$ ? Ans. .113445.

25. What decimal fraction is equivalent to $\frac{23}{27}\times\frac{45}{82}$ ?
Ans. .46748.

26. What is the second power of .841 ? Ans. .707281.

27. What is the product of $1.\dot{6}\dot{5}$ by $1.\dot{4}\dot{8}$, true to five places?
Ans. 2.45975.

28. Express decimally $2\frac{3}{13}\times\frac{8}{9}$. Ans. 2.393162.

29. What is the product of $73.6\dot{3}7\dot{1}$ by 8.143?
Ans. $599.6272\dot{0}7\dot{7}$.

30. .681472×.01286, true to five places, will give .00876.

$$\begin{array}{r}
.681472 \\
68210.0 \\
\hline
681 \\
136 \\
55 \\
4 \\
\hline
.00876
\end{array}$$

In the last exercise it must be observed that since there is no whole number, and five decimal places are required, we must place a cipher under the fifth decimal figure, and write .01286 in reversed order. That the result is a sufficiently close approximation will be evident from the consideration that the last figure 6 is only six one-hundred-thousandths of the unit, and consequently the next figure would be only one-millionth part of the unit.

## VI.—DIVISION OF DECIMALS.

We have already seen (1) that we cannot perform any operation except the numbers concerned are of the same denomination, or one of them be abstract; (2) that when a denominate number is used either as a multiplier or a divisor, it ceases to be denominate, and becomes abstract, and (3) that the rules for addition, subtraction, multiplication and division of integers apply equally to decimals, the only additional requirement being the placing and moving of the decimal point.

Suppose then we are required to divide 1.2321 by 11.1, we must by (1) bring both quantities to the same denomination. Now the dividend is carried down to ten-thousands for $1.2321=1+\frac{2321}{10000}$, and therefore we express 11.1 in the corresponding form, ten-thousandths or $11+\frac{1000}{10000}$, or 11.1000, so that we change the form, but not the value of 11.1, the divisor. Again, by (2) the .1, which originally expressed *a tenth* of some unit, and therefore was in reality denominate, now becomes abstract as one of the figures of the given factor of 1.2321, by means of which we are to find the other factor. Hence by (3) we can now divide 1.2321 by 11.1000, as if both were whole numbers, and this is the reason for omitting the decimal point when we have made the number of decimal places equal. Beginners generally feel a difficulty in conceiving how a fraction divided by a fraction can give a whole number. The difficulty may be easily removed by noticing that $\frac{1}{4}$ is contained *twice* in $\frac{1}{2}$ for $\frac{2}{4}=\frac{1}{2}$, *e. g.*, a half dollar contains, or is equivalent to, two quarter dollars. Thus the fraction $\frac{1}{2}$ divided by the fraction $\frac{1}{4}$, gives the whole number 2. So, also, $\frac{1}{8}$ is contained 4 times in $\frac{1}{2}$, and therefore $\frac{1}{2}\div\frac{1}{8}=4$, a whole number. Hence, when we have reduced the divisor and dividend to the same denomination, we may omit the decimal point, as we have only to find *how often the one is contained in the other.* Hence the

### RULE.

*If the number of decimal places in the divisor and dividend be not equal, make them equal by supplying ciphers or repetends, and then divide as in whole numbers, and the quotient so far will be a whole number, but if there is a remainder, annex ciphers or repetends, and the part of the quotient thus obtained will be a decimal.*

The decimal places may be supplied as the work proceeds, as it is easy to see how many ciphers or repetends must be supplied; for we have seen in multiplication that the number of decimal places in any product must be equal to all the decimal places in the factors, and, since a dividend must always be viewed as a product, it follows that the difference between the number of decimal places in dividend and divisor will indicate how many ciphers or repetends must be supplied.

### EXERCISES.

1. Divide 47.58 by 26.175 to six decimal places.
Ans. 1.817765.

2. Divide 70.8946 by 13.825 to three places.
Ans. 5.128.

3. Divide $468.\dot{7}$ by 3.365 to six places of decimals.
Ans. 139.309889.

4. Express decimally $1 \div \frac{3}{700}$. Ans. $233.\dot{3}$.

5. Express in the decimal form $\frac{7}{8}$ of $\frac{6}{7} \div \frac{4}{5}$ of $\frac{8}{9}$ true to six places of decimals. Ans. 1.054687.

6. Divide the whole number 9 by the fraction .008. Ans. 1125.

7. What is the quotient of $5.\dot{0}\dot{9}$ by $6.\dot{2}$? Ans. $.\dot{8}\dot{1}$ nearly.

8. Divide .54439 by 7777. Ans. .00007.

9. What decimal is obtained by dividing 1 by 10.473654 ?
Ans. .09547766.

10. What is the difference between $\frac{2}{3} \div \frac{5}{7}$ and $\frac{5}{6} \div \frac{10}{11}$ in the decimal form? Ans. $.2458\dot{3}$.

## CONTRACTED METHOD.

The work may often be much abbreviated in the manner exhibited by the following example:

```
.14736).23748|(1.611              14736)23748(1.611
        14736|                      ···· 14736
        -----|-                          -----
         9012|0                           9012
         8841|6                           8842
         ----|--                          ----
          170|40                           170
          147|36                           147
          ---|---                          ---
           23|040                           23
           14|736                           15
           --|---                           --
            8|304                            8
```

Here it is required to divide .23748 by .14736. Since both divisor and dividend contain the same number of decimal places, no alteration is needed, and so we can at once reject the decimal point, and divide as in whole numbers. The principle of the contraction is simply what has been already explained, viz., that all we look for in such calculations is a sufficiently close approximation, by which we mean an approximation sufficient for all practical purposes. For this reason, when we have obtained the integral part of the quotient, we may omit one figure of the divisor in succession after each operation, as the value of each figure decreases in a tenfold degree as we descend towards the right, and after three decimal figures the error, or deficit rather, becomes only thousandths, which are very rarely worth taking into account. For example, if the calculation regards dollars and cents, the error at the fourth decimal place would be only the *one-thousandth* part of a cent.

### RULE.

*Arrange the fractions as in the ordinary mode; find the first figure of the quotient and the first remainder; then, instead of annexing a periodic figure or a cipher, cut off the right hand figure of the divisor, and use the remaining figures to find the next figure of the quotient, and so on.*

It is usual to mark the figures as they are successively cut off by placing a point below each. In multiplying by each figure of the quotient, allowance must be made for what would have been carried from the figure of the divisor last cut off, had it been used in the division.

The vertical line drawn through the ordinary form shows how closely the two modes correspond. As has already been remarked, it is desirable, in order to secure accuracy, to carry the figures of repetends to one or two places more than are required.

### EXERCISES.

(1.)

```
43232323)73640000(170.3355.
 ······  43232323
         --------
         30407677
         30262626
         --------
          145051
          129697
          ------
           15354
           12970
          ------
            2384
            2162
            ----
             222
             216
             ---
               6
```

(2.)

```
54637)43682(.7995
 ····  38246
       -----
        5436
        4917
        ----
         519
         491
         ---
          28
          27
          --
           1
```

Divide 73.64 by .432, and .43682 by .54637 to 4 decimal places each. To show that there will be three integral places in the quotient of Ex. 1, we must consider that there are two places of whole numbers in the dividend and none in the divisor, and, therefore, if we divide 73 and 6, the first decimal place of the dividend by .4, the first figure of the divisor, we get three integral places. Hence, since we are to have four decimal places, we shall have seven figures in all. This contraction is extremely useful when there are many decimal places.

3. Find the quotient of 8.6134÷7.3524 to four decimal places. Ans. 1.1715.
4. Divide $.\dot{6}\dot{1}$ by 13.543516 to five decimal places. Ans. .04549.
5. Divide .58 by 77.482 to five decimal places. Ans. .00756.
6. Divide .812.54567 by $7.\dot{3}\dot{4}$ to three decimal places. Ans. 110.649.
7. Divide 1 by 10.473654 to six decimal places. Ans. .09547.
8. Divide 7.126491 by .531 to six decimal places. Ans. 13.420887.
9. Divide 1.77975 by the whole number 25425. Ans. .00007.
10. Divide to eight places .879454 by .897. Ans. .98043924.

## VII.—DENOMINATE DECIMALS.

*To express one denominate number as a fraction of another of the same kind, reduce both to the lowest denomination contained in either, make the former the numerator and the latter the denominator of a common fraction, and reduce the fraction so found to a decimal in the manner already pointed out.*

### EXAMPLES.

To express 16 cents as a fraction of a dollar: Here the lowest denomination mentioned is cents, and we reduce a dollar to cents and write $\frac{16}{100}=\frac{4}{25}$, and, dividing 4 by 25, we get .16. To express 11s. $4\frac{1}{2}$d. as a decimal of £1, we reduce both to half-pence, and obtain $\frac{273}{480}=\frac{91}{160}$, which, reduced to a decimal, is .56875.

### EXERCISES.

1. Reduce 5s. $10\frac{1}{2}$d. to the decimal of £1. Ans. £ .29375.
2. Reduce $10\frac{1}{2}$d. to the decimal of £1. Ans. £ .04375.
3. Reduce 15s. $9\frac{3}{4}$d. to the decimal of £1. Ans. £ .790625.
4. Express 3 roods and 11 rods as a decimal of an acre. Ans. .81875.

5. Express 3 cwt., 1 qr., 7 lbs., as a decimal of a ton. Ans. .166.

6. Reduce 37 rods to the decimal of a mile. Ans. .115625.

7. Reduce 7 oz., 4 dwts., to the decimal of a pound. Ans. .6.

8. Reduce a pound troy to the decimal of a pound avoirdupois; correct to six decimal places.* Ans. .822857+.

9. Reduce 5 hours, 48 minutes, 49.7 seconds, to the decimal of a day, taken as 24 hours. Ans. .2422419.

10. Express an ounce avoirdupois as a decimal of a pound troy. Ans. $.911458\dot{3}$.

## VIII.—REDUCTION TO DENOMINATIONS.

*To find the value of a fraction in the lower denominations, expressed as a decimal of any given denomination, multiply in succession by the numbers which express the given and lower denominations, and after each multiplication cut off from the right as many decimal figures as are contained in the given decimal, and the figures to the left of the decimal point will give the required value.*

EXAMPLES.

1. To find the value of £.79375; multiply by 20, by 12 and by 2, which gives £0.15.10½.—Repetends must either be reduced to common fractions or found approximately.

$$\begin{array}{r} .79375 \\ 20 \\ \hline 15.87500 \\ 12 \\ \hline 10.50000 \\ 2 \\ \hline 1.00000 \end{array}$$

2. To find the value of £$.\dot{7}$:

$$\begin{array}{rl} .77777 & \multirow{2}{*}{\}\text{carry 1.}} \\ 20 & \\ \hline 15.55550 & \\ 12 & \\ \hline 6.66600 & \\ 4 & \\ \hline 2.66400 & \quad £0.15.6\tfrac{3}{4}\text{ nearly.} \end{array}$$

---

* The standard pounds are meant here, viz.: troy, 5760 grains, and avoirdupois 7000 grains. Taking the ounces would give $\frac{12}{16}=\frac{3}{4}=.75$.

### EXERCISES.

1. What is the value of £.475 ? Ans. 9s. 6d.

2. What is the value of $.\dot{7}$ of a cwt. ?
Ans. 3 qrs., 3 lbs., 1 oz., $12\frac{4}{7}$ drs.

3. What is the value of $.541\dot{6}$ of a shilling sterling ? Ans. $6\frac{1}{2}$d.

4. What is the value of .6845 of s cwt. ?
Ans. 2 qrs., 20 lbs., 10 oz., $9\frac{123}{125}$.

5. What is the value of $.\dot{4}$ of 9s. $4\frac{1}{2}$d ? We have $.\dot{4}=\frac{4}{9}$ and 9s. $4\frac{1}{2}$d., multiplied by 4, and the product, divided by 9, gives 4s. 2d., the exact value.

6. What is the value of $.02\dot{6}$ of $1^\circ\ 15'$ ? Reducing $.02\dot{6}$ to a vulgar fraction, we get $\frac{24}{900}=\frac{2}{75}$, and multiplying $1^\circ\ 15'$ by 2, and dividing by 75, we find $2'$.

## RATIO AND PROPORTION.

**17.**—Ratio is the relation which one quantity bears to another of the same kind with respect to magnitude, or the number of times that the less is contained in the greater. Thus, the ratio 7 to 21 is 3, because 7 is contained 3 times in 21, or 21 is 3 times 7. The same result is obtained if we divide 7 by 21, for we then find $\frac{7}{21}=\frac{1}{3}$, which means that 7 is $\frac{1}{3}$ of 21, and this expresses the very same relation as before; for, to say that 7 is $\frac{1}{3}$ of 21 is precisely the same as to say that 21 is 3 times 7. (See note under Inverse Proportion.) And, therefore, 3 is called the measure of the ratio. The numbers thus compared are called the terms of the ratio—the first the antecedent and the second the consequent, and the relation is written 7 : 21. The sign ( : ) originally indicated division.

That the magnitudes must be of the same kind will be obvious from the consideration that 7 bags of flour could have no ratio to 21 dollars, for multiplying 7 bags of flour by 3 would not make them 21 dollars, but 21 bags of flour, and multiplying 7 dollars by 3 would not make them 21 bags of flour, but 21 dollars. Hence, the less could not be increased to make the greater, except they are homogeneous, or of the same kind.

Proportion is the equality of ratios.

The ratio of 9 to 27 is 3, but we have seen that the ratio of 7 to 21 is also 3, therefore the ratios of 7 to 21 and of 9 to 27 are the same, or $7\div21=9\div27$, and these quantities are, therefore, called proportionals. The sign ( : : ) was formerly used for equality, and is still retained for equality of ratios, and the sign (=) is used for the actual equality of quantities, though occasionally used for equality of ratios. Hence, the usual mode of writing the equality of two ratios is 7 : 21 : : 9 : 27. Such a statement is called a proportion, or an analogy, and is read—7 is to 21 as 9 to 27, *i. e.*, 27 exceeds 9 as many times as 21 exceeds 7, and this is expressed by saying 27 is the same multiple of 9 that 21 is of 7, or that 9 is the same sub-multiple, measure, or aliquot part of 27 that 7 is of 21. The four quantities are called the *terms* of the proportion; the first and last are called the *extremes*, and second and third the *means;* also, the first and third are called *homologous*, or of the same name, *i. e.*, both are antecedents, and so the second and fourth are homologous, for they are both consequents. The last term is called a fourth proportional to the other three, and we shall denote it by F. P. There are two simple ways of testing the correctness of an analogy. The first is to divide the second term by the first, and the fourth by the third, and if the quotients are equal, the analogy is correct. This is manifest from what has been already said. The second principle is, that, if the analogy be correct, the product of the extremes is equal to the product of the means. To prove this, let us resume the analogy, 7 : 21 : : 9 : 27. We have seen that $21\div7=27\div9$, or $3=3$. Now, if each be multiplied by 63, we have (by Ax. II., Cor.,) $189=189$. But 189 is the product of 27 by 7, the extremes, and also of 21 by 9, the means—these products then are always equal. From this simple principle we readily deduce a rule for finding a fourth proportional to three given quantities. Let the quantities be 48, 96, and 132, written thus: 48 : 96 : : 132 : , the required quantity. Now, $132\times96=12672$, the product of the means are therefore equal to the product of the extremes. We have, therefore, a product, 12672, and one of its factors, 48, hence, dividing this product by the given factor, we find the other factor to be 264, which is therefore the fourth proportional, or fourth term of the proportion, and we can now write the whole analogy, thus:— 48 : 96 : : 132 : 264. To prove the correctness of the operation, multiply 264 by 48, and 12672 is obtained, the same as before. Hence,

### THE RULE.

*Divide the product of the second and third terms by the first, and the quotient will be the required fourth term.*

To show the order in which the three given quantities are to be arranged, let it be required to find how much 730 yards of linen will cost at the rate of $30 for 50 yards. It is plain that the answer, or fourth term, must be dollars, for it is a price that is required, and in order that the third term may have a ratio to the fourth, the $30 must be the third term. Again, since 730 yds. will cost more than 50 yds., the fourth term will be greater than the third, and therefore the second must be greater than the first, and therefore the statement is $\overset{\text{yds.}}{50} : \overset{\text{yds.}}{730} :: \overset{\$}{30}$ : 4th proportional, and by the rule $\frac{730\times30}{50}=\frac{21900}{50}=438$, the fourth term, and we can now write the whole analogy, 50 yds : 730 yds : : $30 : $438.

This may be called the ascending scale, for the second is greater than the first, and the fourth greater than the third. If the question had been to find what 50 yards of linen will cost at the rate of $438 for 730 yards, we still find that the answer will be dollars, and that therefore, as before, dollars must be in the third place, but we see that the answer will now be less than 438, as 50 yards, of which the price is required, will cost much less than 730 yards, of which the price is given, and that therefore the second term must be less than the first. Hence the statement is 730 yds : 50 yds : : $438 : F. P., and by the rule $\frac{438\times50}{730}=30$, the fourth proportional. We now have the full analogy 730 yds : 50 yds : : $438 : $30. As the second is less than the first, and the fourth less than the third, this may be called the descending scale. If the first should turn out to be equal to the second, and therefore the third equal to the fourth, we should say that the quantities were to each other in the ratio of *equality*.

### RULE FOR THE ORDER OF THE TERMS.

*If the question implies that the consequent of the second ratio must be greater than the antecedent, make the greater term of the first ratio the consequent, and the less the antecedent, and vice versa.*

The questions hitherto considered belong to what is called *Direct Proportion*, to distinguish it from another kind called *Inverse Proportion;* because, in the former, the greater the number given, the less will be the corresponding number required, and *vice versa;*

whereas, in the latter, the greater the number given, the less will be the number required, and *vice versa.* To illustrate this, let it be required to find how long a stack of hay will feed 12 horses, if it will feed 9 horses for 20 weeks. Here the answer required is time, and therefore 20 weeks will be the antecedent of the second ratio; but the greater the number of horses, the shorter time will the hay last, and therefore the fourth term will be less than the third, and therefore the statement will not be 9 : 12, but the reverse, 12 : 9; and hence the name INVERSE, because the term 9, for which the time (20 weeks,) is given, and which therefore we should expect to be in the first place, has to be put in the second; and the term 12, for which the time is required, and which therefore we should expect to be in the second place, has to be put in the first, and thus the whole analogy is 12 : 9 : : 20 : 15.*

The principal changes that may be made in the order of the terms, will be more readily and clearly understood by the subjoined scheme, than by any explanation in words:

Original Analogy: 8 : 6 : : 12 : 9 for $8\times9=72=6\times12$.

Alternately: 8 : 12 : : 6 : 9 for $8\times9=72=6\times12$.

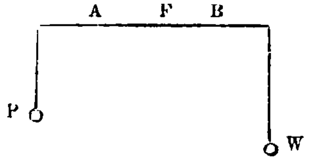
By Inversion: 6 : 8 : : 9 : 12 for $6\times12=72=8\times9$.

By Composition: $8+6$ : 6 : : $12+9$ : 9 or 14 : 6 : : 21 : 9 for $14\times9=126=6\times21$.

By Division: $8-6$ : 6 : : $12-9$ : 9 or 2 : 6 : : 3 : 9 for $2\times9=18=6\times3$.

By Conversion: 8 : $8-6$ : : 12 : $12-9$ or 8 : 2 : : 12 : 3 for $8\times3=24=2\times12$.

Simple transposition is often of the greatest use. Let us take an easy practical example. In calculating what power will balance a given weight, when the arms of the lever are known, let P be the power, W the weight, A the arm of power, and B the arm of weight. The rule is, that the power and weight are inversely as the arms. This solves all the four possible cases by transposition.



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* Inverse ratio is sometimes spoken of, but in reality there is no such thing. It is true that Inverse Proportion requires the terms of one of the ratios to be inverted, but that is a matter of analogy, not of ratio, for we have seen already that 7÷21 expresses the very same relation as 21÷7.—(See in-

A : B : : W : P, gives the power when the others are known,
B : A : : P : W gives the weight when the others are known,
W : P : : A : B gives the arm of weight when the others are known,
P : W : : B : A gives the arm of power when the others are known.

The work may often be contracted in the following manner :—Resuming our example 48 : 96 : : 132 : fourth proportional, we see that 96 is double of 48, and therefore the ratio of 48 to 96 is the same as that of any two numbers, the second of which is double the first, and 48 : 96 is the same as 1 : 2, and we reduce the analogy to the simple form of 1 : 2 : : 132 : 4th prop., and we have $\frac{132\times2}{1}=264$, the term required, as before. In the example 50 : 730 : : 30 : 4th term, we have $\frac{730\times30}{50}=\frac{73\times30}{5}=\frac{73\times6\times5}{5}=73\times6=438$. This is equivalent to dividing the first and second by 10, and the first and third by 5. Hence we may divide the first and second, or first and third by any number that will measure both. The same principle will also be illustrated by the consideration that the second and third are multipliers, and the first a divisor; and if we first multiply, and then divide by the same quantity, the one operation will manifestly neutralize the other. Thus : 48 : 96 : : 132 : F. P. may be written 1×48 : 2×48 : : 132 : F. P. ; where it is plain that since by first multiplying 132 by 48, and then dividing by the same, the one operation would neutralize the other, both may be omitted.

In proportion, when the means are equal, such as 4 : 12 : : 12 : 36, it is usual to write the analogy thus—4 : 12 : 36, and 12 is called a mean proportional between 4 and 36. To prove the correctness of this statement, we multiply 36 by 4 and 12 by itself, and as both give 144, the analogy is correct. Now, as 144 is the square or second power of 12, so 12 is called the second root, or square root of 144, or that which produced it, or the *root* from which it *grew* ; hence, to find a mean proportional between two given quantities, we have the following

### RULE.

*Multiply them together, and take the square root of the product.*

Thus, in the above example, 4×36=144, the square root of which is 12. Again, to find a mean proportional between 9 and 49, we mul-

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troductory remarks.) The term *Reciprocal Ratio* is liable to the same objection, for though 3 and ⅓ are reciprocals, yet they express the same relation. When the expression *Inverse Ratio* is legitimately used, it does not refer to a *single* ratio, but means that *two* ratios are so related that *one* of them must be inverted.

tiply 49 by 9, which is 441, the square root of which is 21, which is a mean proportional between 9 and 49, *i. e.*, 9 : 21 : 49, or, written at full length, 9 : 21 : : 21 : 49. Proof: 49×9=441 and 21×21=441. As the learner is not supposed, at this stage, to know the method of finding the roots of quantities beyond the limits of the multiplication table, we append a table of squares and roots at the end of the book.

When each quantity in a series is a mean proportional between two adjacent quantities, the quantities are said to be continued, or continual proportionals. Thus: 2 : 4 : 8 : 16 : 32 : 64 : 128, and 3 : 9 : 27 : 81 : 243, are series in which each is a mean proportional between two adjacent ones. Let us take 16 and the two adjacent ones, 8 and 32—the analogy is 8 : 16 : : 16 : 32. Proof: 8×32=256, and 16×16=256. So also, 27 and the adjacent terms, 9 and 81. The analogy is 9 : 27 : : 27 : 81, and the proof, 9×81=729, and 27×27=729.

This subject will be treated of at length in a subsequent part of the work, but this explanation has been introduced here to fill up the outline and let the learner understand the *nature* of continued proportionals.

### EXERCISES.

1. If 6 barrels of flour cost $32, what will 75 barrels cost? Ans. $400.

2. If 18 yards of cloth cost $21, what must be paid for 12 yards? Ans. $14.

3. How much must be paid for 15 tons of coal, if 2 tons can be purchased for $15? Ans. $112.50.

4. If you can walk 84 miles in 28 hours, how many minutes will you require to walk 1 mile? Ans. 20.

5. What will 14 horses cost, if 3 of the average value can be bought for $270? Ans. $1260.

6. What must be paid for a certain piece of cloth, if $\frac{2}{3}$ of it cost $9. Ans. $13.50.

7. If 5 men are required to build a wall in 5 days, how many men will do the same in 2$\frac{1}{2}$ days? Ans. 10.

8. If 16 sheep are $\frac{2}{3}$ of a flock, how many are there in the same? Ans. 24.

9. What must be paid for 4$\frac{1}{2}$ cords of wood, if the cost of 3 cords is $10? Ans. $15.

10. What is the height of a tree which casts a shadow of 125 feet, if a stake 6 feet high produces a shadow of 8 feet? Ans. $93\frac{3}{4}$.

11. How long will it take a train to run from Toronto to Hamilton (a distance of 39 miles), at the rate of 5 miles in $15\frac{5}{13}$ minutes? Ans. 2 hours.

12. If 15 men can build a bridge in 10 days, how many men will be required to erect three of the same dimensions in $\frac{1}{2}$ the time? Ans. 90.

13. If a man receive $4.50 for 3 days' work, how many days ought he to remain in his place for £6 4s. 6d.? Ans. $16\frac{3}{5}$.

14. How much may a person spend in 94 days if he wishes to save 18 guineas out of a salary of $500 per annum? Ans. $109.297+.

15. If 3 cwt. 3 qrs. 14 lbs. of sugar cost $36.50, what will 2 qrs. 2 lbs. cost? Ans. $4.879+.

16. 5 men are employed to do a piece of work in 5 days, but after working 4 days they find it impossible to complete the job in less than 3 days more, how many additional men must be employed to do the work in the time agreed upon at first? Ans. 10.

17. A watch is 10 minutes too fast at 12 o'clock (noon) on Monday, and it gains 3 minutes 10 seconds a day, what will be the time by the watch at a quarter past 10 o'clock, A. M., on the following Saturday? Ans. 10 h. 40m. $36\frac{7}{48}$s.

18. A bankrupt owes $972, and his property, amounting to $607.50, is distributed among his creditors; what does one receive whose demand is $$11.33\frac{1}{3}$? Ans. $7.083+.

19. What is the value of .15 of a hhd. of lime, at $2.39 per hhd.? Ans. $.3585.

20. A garrison of 1200 men has provisions for $\frac{3}{4}$ of a year, at the rate of $\frac{7}{8}$ of a pound per day; how long will the provisions last at the same allowance if the garrison be reinforced by 400 men? Ans. $6\frac{3}{4}$ months.

21. If a piece of land 40 rods in length and 4 in breadth make an acre, how long must it be when it is 5 rods $5\frac{1}{2}$ feet wide? Ans. 30 rods.

22. A borrowed of B £175 5s. for 102 days, and afterwards would return the favor by lending B £210 6s.; for how long should he lend it? Ans. 85 days.

23. If a man can walk 300 miles in 6 successive days, how many miles has he to walk at the end of 5 days? Ans. 50.

24. If 495 gallons of wine cost $394; how much will $72 pay for? Ans. 90 gal.

25. If 112 head of cattle consume a certain quantity of hay in 9 days; how long will the same quantity last 84 head? Ans. 12 days.

26. If 171 men can build a house in 168 days; in what time will 108 men build a similar house? Ans. 266 days.

27. It has been proved that the diameter of every circle is to the circumference as 113 : 355; what then is the circumference of the moon's orbit, the diameter being, in round numbers, 480,000 miles?
Ans. 1,507,964$\frac{68}{113}$ m.

28. A round table is 12 ft. in circumference; what is its diameter?
Ans. 3 ft. 9$\frac{297}{355}$ in.

29. A was sent with a warrant; after he had ridden 65 miles, B was sent after him to stop the execution, and for every 16 miles that A rode, B rode 21; How far had each ridden when B overtook A?
Ans. 273 miles.

30. Find a fourth proportional to 9, 19 and 99. Ans. 209.

31. A detective chased a culprit for 200 miles, travelling at the rate of 8 miles an hour, but the culprit had a start of 75 miles; at what rate did the latter travel? Ans. 5 miles an hour.

32. How much rum may be bought for $119.50, if 111 gallons cost $89.625? Ans. 148 gallons.

33. If 110 yards of cloth cost $18; what will $63 pay for?
Ans. 385 yards.

34. If a man walk from Toronto to Hamilton, a distance of (say) 38 miles in 13 hours, 18 minutes; in what time will he walk at the same rate from St. Catherines to London, supposing the distance to be 102 miles? Ans. 35 hours, 42 min.

35. A butcher used a false weight 14$\frac{3}{4}$ oz., instead of 16 oz. for a pound, of how many ℔s. did he defraud a customer who bought 112 just lbs. from him? Ans. 9$\frac{29}{59}$ lbs.

36. If 123 yards of muslin cost $205; how much will 51 yards cost? Ans. $85.

37. In a copy of Milton's Paradise Lost, containing 304 pages, the combat of Michael and Satan commences at the 139th page; at what page may it be expected to commence in a copy containing 328 pages?

Ans. The fourth proportional is 149$\frac{37}{38}$; and hence the passage will commence at the foot of page 150.

38. Suppose a man, by travelling 10 hours a day, performs a

journey in four weeks without desecrating the Sabbath; now many weeks would it take him to perform the same journey, provided he travels only 8 hours per day, and pays no regard to the Sabbath?

Ans. 4 weeks, 2 days.

39. A cubic foot of pure fresh water weighs 1000 oz., avoirdupois; find the weight of a vessel of water containing $217\frac{1}{2}$ cubic in.

Ans. 7 lbs., $13\frac{125}{144}$ oz.

40. Suppose a certain pasture, in which are 20 cows, is sufficient to keep them 6 weeks; how many must be turned out, that the same pasture may keep the rest 6 months? Ans. 15.

41. A wedge of gold weighing 14 lbs., 3 oz., 8 dwt., is valued at £514 4s.; what is the value of an ounce? Ans. £3.

42. A mason was engaged in building a wall, when another came up and asked him how many feet he had laid; he replied, that the part he had finished bore the same proportion to one league which $\frac{3}{17}$ does to 87; how many feet had he laid? Ans. $32\frac{192}{1479}$.

43. A farmer, by his will, divides his farm, consisting of 97 acres, 3 roods, 5 rods, between his two sons so that the share of the younger shall be $\frac{3}{4}$ the share of the elder; required the shares.

Here the ratio of the shares is 4 : 3, and we have shown that if four magnitudes are proportionals, the first term increased by the second is to the second as the third increased by the fourth is to the fourth. Now, 97 acres, 3 roods, 5 rods, being the sum of the shares, we must take the sum of 4 and 3 for first term, and either 4 or 3 for the second, and therefore 7 : 4 : : 97 acres, 3 roods, 5 rods : F.P., *i. e.*, the sum of the numbers denoting the ratio of the shares is to one of them as the sum of the shares is to one of them. This gives for the elder brother's share, 55 acres, 3 roods, 20 rods, and the younger's share is found either by repeating the operation, or by subtracting the share thus found from the whole, giving 41 acres, 3 roods, 25 rods.

44. A legacy of $398 is to be divided among three orphans, in parts which shall be as the numbers 5, 7, 11, the eldest receiving the largest share; required the parts?

23 : 5 : : 398 : $86\frac{12}{23}$, the share of the youngest.

23 : 7 : : 398 : $121\frac{3}{23}$, the share of the second.

23 : 11 : : 398 : $190\frac{8}{23}$, the share of the eldest.

45. Three sureties on $5000 are to be given by A, B and C, so that B's share may be one-half greater than A's, and C's one-half greater than B's; required the amount of the security of each?

Ans. A's share, \$1052.63$\frac{3}{19}$; B's, \$1578.94$\frac{14}{19}$; C's, \$2368.42$\frac{2}{19}$.

46. Suppose that one man starts from Montreal, and walks 6 miles an hour, and another at the same time from Newtonville, (5 miles west of Port Hope), at the rate of 5 miles an hour, when will they meet, the whole distance being 285 miles?

Ans. $\frac{5}{11}$ of a mile west of Gananoque, which is 155 miles west of Montreal by the Grand Trunk line.

47. A certain number of dollars is to be divided between two persons, the less share being $\frac{2}{3}$ of the greater, and the difference of the shares \$800; what are the shares, and what is the whole sum to be divided? Ans. Less share, \$1600; greater, \$2400; total, \$4000.

48. A certain number of acres of land are to be divided into two parts, such that the one shall be $\frac{3}{7}$ of the other; required the parts and the whole, the difference of the parts being 716 acres?

Ans. the less part 537 acres; the greater, 1253 acres; the whole, 1790.

49. A mixture is made of copper and tin, the tin being $\frac{1}{3}$ of the copper, the difference of the parts being 75; required the parts and the whole mixture? Ans. tin, 37$\frac{1}{2}$; copper, 112$\frac{1}{2}$; the whole, 150.

50. Pure water consists of two gasses, oxygen and hydrogen; the hydrogen is about $\frac{2}{15}$ of the oxygen; how many ounces of water will there be when there are 764$\frac{12}{17}$ oz. of oxygen more than of hydrogen?

Ans. 1000 oz.

## COMPOUND PROPORTION.

Proportion is called simple when the question involves only one condition, and compound when the question involves more conditions than one. As each condition implies a ratio, simple proportion is expressed, when the required term is found, by two ratios, and compound, by more than two. Thus, if the question be, How many men would be required to reap 65 acres in a given time, if 96 men, working equally, can reap 40 acres in the same time? Here there is but one condition, viz., that 96 men can reap 40 acres in the given time, which implies but one ratio, and when the question has been stated 40 : 65 : : 96 : F.P., and the required term is found to be 156, and the proportion 40 : 65 : : 96 : 156, we have the proportion, expressed by two ratios. But, suppose the question were, If a man, walking 12 hours a day, can accomplish a journey of 250 miles in 9 days, how many days would he require, walking at the same rate, 10 hours each day, to travel 400 miles? Here there are two conditions, viz.: *first*, that, in the one case, he travels 12 hours a day, and in the other 10 hours; and, *secondly*, that the distances are 250 and 400 miles. The statement, as we shall presently show, would be

$$\left.\begin{array}{r}10 : 12\\ 250 : 400\end{array}\right\} :: 9 : 17\tfrac{7}{25}.$$

Here each condition implies one ratio, 10 : 12 and 250 : 400, and when the required term, which is $17\frac{7}{25}$, is found, there are four ratios, viz., the two already noted, and 9 : $17\frac{7}{25}$, gives two more, one in relation to 10 : 12, and one in relation to 250 : 400. This will be evident, when we have shown the method of statement and operation.

EXPLANATORY STATEMENT AND OPERATION.

$$\begin{array}{l}11 : 33 :: 12 : \text{F.P.}\\ 1 : 3 \;\; : 12 : 36\\ \hline 18 : 5 \;\; :: 36 : \text{F.P.}\\ 1 : 5 \;\; :: \;\; 2 : 10.\end{array}$$

PRACTICAL STATEMENT AND OPERATION.

$$\begin{array}{l}\left.\begin{array}{r}11 : 33\\ 18 : 5\end{array}\right\} :: 12 : \text{F. P.}\\ \hline \left.\begin{array}{r}1 : 3\\ 3 : 5\end{array}\right\} :: \;\; 2 : \text{F. P.}\\ \hline \left.\begin{array}{r}1 : 1\\ 1 : 5\end{array}\right\} :: \;\; 2 : 10.\end{array}$$

Let the question be, How many men would be required to reap 33 acres in 18 days, if 12 men, working equally, can reap 11 acres in 5 days?

We first proceed, as on the left margin, as if there were only one condition in the question; or, in other words, as if the number of days were the same in both cases, and the question were—If 12 men can reap 11 acres in a given time, how many men will be required to reap 33 acres in the same time. This, then, is a question in simple proportion, and by that rule we have the statement—11 : 33 : : 12 : F. P., which, by contraction, becomes 1 : 3 : : 12 : F. P.; and thus, we find F. P. to be 36, the number of men required, if the time were the same in both cases. The question is now resolved into this: How many men will be required to reap, in 18 days, the *same quantity* of crop that 36 men can reap in 5 days? This is obviously a case of inverse proportion, for the longer the time allowed the less will be the number of men required, and hence the statement, 18 : 5 : 36 : F. P., which, by contraction, becomes 1 : 5 : : 2 : F. P., which gives 10 for the number of men. The work may be shortened by making the two statements at once, as on the right margin. We first notice that the last term is to represent a certain number of men, and, therefore, we place 12 in the third place; next, we see that, *other things being equal*, it will take more men to reap 33 than to reap 11 acres, and that, therefore, as far as that is concerned, the fourth term will be greater than the third, and so we put 11 in the first place, and 33 in the second. Again we see that, *other things being equal*, a less number of men will be required when 18 days are allowed for doing the work, than when it is required to be done in 5 days, and that therefore the fourth term, as far as that is concerned, will be less than the third, and therefore we write 18 : 5 below the other ratio as on the margin. Then by contraction we get $\left.\begin{array}{l}1:3\\3:5\end{array}\right\} :: 2 : \text{F. P.}$ Now, as 3 in the first term is to be a multiplier, and 3 in the second a divisor, we may omit these also, and we obtain $\left.\begin{array}{l}1:1\\1:5\end{array}\right\} :: 2 : 10$, the answer as before.

$$\frac{11\times18}{198} : \frac{33\times5}{165} :: 12 : \text{F. P.}$$

$$\frac{165\times12}{198}=10$$

The full uncontracted operation would be to multiply 18 by 11, which gives 198, then to multiply 33 by 5, which gives 165. then multiply 165. the product of the two second terms, by 12, and divide the result, 1980, by 198, the product of the two first terms, which gives 10 as before.

Because in the analogy 198 : 165 : : 12 : 10, the first two terms are products, this kind of proportion has been called *compound*, and the ratio of 19 to 165 is called a *compound ratio*. We can show the strict and original meaning of the term *compound ratio* more easily by an example, than by any explanation in words. Let us take any series of numbers, whole, fractional or mixed, say 5, $\frac{7}{8}$, $\frac{3}{5}$, 19, 12. 1, 17, 11, $\frac{15}{16}$, 25, then the ratio of the first to the last is said to be compounded of the ratio of the first to the second, the second to the third, the third to the fourth, &c., &c., &c., to the end. Now the ratio of 5 to 25 is $\frac{25}{5}=5$, and the several ratios are in this order, $\frac{\frac{7}{8}}{5}\times\frac{\frac{3}{5}}{\frac{7}{8}}\times\frac{19}{\frac{3}{5}}\times\frac{12}{19}\times\frac{1}{12}\times\frac{17}{1}\times\frac{11}{17}\times\frac{\frac{15}{16}}{11}\times\frac{25}{\frac{15}{16}}$ which leaving finally $\frac{25}{5}=5$ as before. If we took them in reverse order, viz., $\frac{5}{25}=\frac{1}{5}$, it is obvious that all therein could be cancelled, as each would in succession be a multiplier and a divisor.

We would also remark that compound proportion is nothing else than a number of questions in simple proportion solved by one operation. This will be evident from our second example by comparing the two operations on the opposite margins. Again, we remarked that every condition implies a ratio, and that therefore the third and fourth terms of our first example really involve two ratios, one in relation to each of the preceding. Hence universally the number of ratios, expressed and implied, must always be double the number of conditions, and therefore always even. As the third ratio is only written once, the number of ratios appears to be odd, but is in reality even.

### RULE:

*Place, as in simple proportion, in the third place the term that is the same as the required term. Then consider each condition separately to see which must be placed first, and which second, other things being equal.*

### EXAMPLE.

1. If $35.10 pay 27 men for 24 days; how much will pay 16 men 18 days? Here we first observe that the answer will be money, and therefore $35.10 must be in the third place. Again, it will take less money to pay 16 men than 27 men, and therefore, other things being equal, the answer, as far as this is concerned, will be less than $35.10, and therefore we put the less quantity, 16, in the second place. So also because it will take less to pay any given number of men for 18 days than for 24 days, therefore we put the less quantity in the second place, which the statement shows in the margin.

$$
\begin{array}{rrl}
27 : & 16 : : & \$35.10 \\
24 : & 18 & \\
\hline
3 : & 2 & \\
3 : & 2 & \\
\hline
9 : & 5 : : & \$35.10 \\
 & & \phantom{\$35.1}4 \\
\hline
 & & 9)140.40 \\
\hline
 & \text{Ans.} & \$15.60
\end{array}
$$

### EXERCISES.

1. If 15 men, working 12 hours a day, can reap 60 acres in 16 days; in what time would 20 boys, working 10 hours a day, reap 98 acres, if 7 men can do as much as 8 boys in the same time?

Ans. $26\frac{22}{25}$ days.

2. If 15 men, by working $6\frac{2}{3}$ hours a day, can dig a trench 48 feet long, 8 feet broad, and 5 feet deep, in 12 days; how many hours a day must 25 men work in order to dig a trench 36 feet long, 12 feet broad, and 3 feet deep, in 9 days? Ans. $3\frac{3}{5}$.

3. If 48 men can build a wall 864 feet long, 6 feet high, and 3 feet wide, in 36 days; how many men will be required to build a wall 36 feet long, 8 feet high, and 4 feet wide, in 4 days. Ans. 32.

4. In what time would 23 men weed a quantity of potato ground which 40 women would weed in 6 days, if 7 men can do as much as 9 women? Ans. $8\frac{8}{69}$ days.

5. Suppose that 50 men can dig in 27 days, working 5 hours a day, 18 cellars which are each 48 feet long, 28 feet wide, and 15 feet deep; how many days will 50 men require, working 3 hours each day, to dig 24 cellars which are each 36 feet long, 21 feet wide, and 20 feet deep. Ans. 45 days.

6. If 15 bars of iron, each 6 ft. 6 in. long, 4 in. broad, and 3 in. thick, weigh 20 cwt., 3 qrs., (28 lbs.) 16 lbs.; how much will 6 bars 4 ft. long, 3 in. broad, and 2 in. thick, weigh?

Ans. 2 cwt., 2 qrs., 8 lbs.

7. If 112 men can seed 460 acres, 3 roods, 8 rods, in 6 days; how many men will be required to seed 72 acres in 5 days?

Ans. 21.

8. If the freight, by railway, of 13 cwt. for 65 miles be 45 shillings; how far should $35\frac{5}{24}$ be carried for 75 shillings?

Ans. 40 miles.

9. If a family of 9 persons can live comfortably in London, England, for 1560 guineas a year; what will it cost a family of 8 to live in London, C. W., in the same style, for seven months, prices supposed to be $\frac{3}{5}$ of what they would be in England?

Ans. £509.12

10. If 63 lbs. of tea cost £20 10s. 6d.; what will 70 lbs. of a different quality cost, 9 lbs. of the former being equal in value to 10 lbs. of the latter? Ans. £20 10s. 6d.

11. If 120 yards of carpeting, 5 quarters wide, cost $60; what will be the price of 36 yards of the same quality, but 7 quarters wide? Ans. $25.20

12. If 48 men, in 5 days of $12\frac{1}{2}$ hours each, can dig a canal $139\frac{3}{4}$ yards long, $4\frac{1}{2}$ yards wide, and $2\frac{1}{2}$ yards deep; how many hours per day must 90 men work for 42 days to dig $491\frac{1}{16}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{5}$ yards deep? Ans. 4.

13. A, standing on the bank of a river, discharges a cannon, and B, on the opposite bank, counts six pulsations at his wrist between the flash and the report; now, if sound travels 1142 feet per second.

and the pulse of a person in health beats 75 strokes in a minute, what is the breadth of the river? Ans. 1 mile, $201\frac{3}{5}$ feet.

14. If 264 men, working 12 hours a day, can make 240 yards of a canal, 3 yards wide, and 12 yards deep, in 5 days; how long will it take 24 men, working 9 hours a day, to make another portion 420 yards long, 5 yards wide, and 3 yards deep?

Ans. $320\frac{5}{6}$ days.

15. If the charge per freight train for 10800 lbs. of flour be $16 for 20 miles; how much will it be for 12500 lbs. for 100 miles?

Ans. $$92\frac{16}{27}$.

16. If $42 keep a family of 8 persons for 16 days; how long, at that rate, will $100 keep a family of 6 persons? Ans. $50\frac{50}{63}$ days.

17. If a mixture of wine and water, measuring 63 gallons, consist of four parts wine, and one of water, and be worth $138.60; what would 85 gallons of the same wine in its purity be worth?

Ans. $233.75.

18. If I pay 16 men $62.40 for 18 days work; how much must I pay 27 men at the same rate? Ans. $140.40.

19. If 60 men can build a wall 300 feet long, 8 feet high, and 6 feet thick, in 120 days, when the days are 8 hours long; in what time would 12 men build a wall 30 feet long, 6 feet high, and 3 feet thick, when the days are 12 hours long? Ans. 15 days.

20. If 24 men, in 132 days, of 9 hours each, dig a trench of four degrees of hardness, $337\frac{1}{2}$ feet long, $5\frac{2}{3}$ feet wide, and $3\frac{1}{2}$ feet deep; in how many days, of 11 hours each, will 496 men dig a trench of 7 degrees of hardness, 465 feet long, $3\frac{2}{3}$ feet wide, and $2\frac{1}{3}$ feet deep?

Ans. $5\frac{1}{2}$.

21. If 50 men, by working 3 hours each day, can dig, in 45 days, 24 cellars, which are each 36 feet long, 21 feet wide, and 20 feet deep; how many men would be required to dig, in 27 days, working 5 hours each day, 18 cellars, which are each 48 feet long, 28 feet wide, and 15 feet deep? Ans. 50.

22. If 15 men, 12 women, and 9 boys, can complete a certain piece of work in 50 days; what time would 9 men, 15 women, and 18 boys, require to do twice as much, the parts performed by each, in the same time, being as the numbers 3, 2 and 1? Ans. 104 days.

23. If 12 oxen and 35 sheep eat 12 tons, 12 cwt. of hay, in 8 days; how much will it cost per month (of 28 days,) to feed 9 oxen and 12 sheep, the price of hay being $40 per ton, and 3 oxen being supposed to eat as much as 7 sheep? Ans. $924.

24. A vessel, whose speed was $9\frac{1}{4}$ miles per hour, left Belleville at 8 o'clock, a. m., for Gananoque, a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 is to 5, starting from the same place, arrived 5 minutes before the first; what time did the second vessel leave Belleville? Ans. 55 min. past 10 o'clock, a. m.

25. If 9 compositors, in 12 days, working 10 hours each day, can compose 36 sheets of 16 pages to a sheet, 50 lines to a page, and 45 letters in a line; in how many days, each 11 hours long, can 5 compositors compose a volume, consisting of 25 sheets, of 24 pages in a sheet, 44 lines in a page, and 40 letters in a line? Ans. 16 days.

### MISCELLANEOUS EXERCISES ON THE PRECEDING RULES.

1. What is the value of .7525 of a mile?
Ans. 6 fur., 0 rd, 4 yds, 1 ft., $2\frac{2}{5}$ in.

2. What is the value of .25 of a score? Ans. 5.

3. Reduce 1 ft. 6 in. to the decimal of a yard. Ans. .5.

4. What is the value of 14 yards of cloth, at $3.375 per yard?
Ans. $47.15.

5. What part of 2 weeks is $\frac{5}{14}$ of a day? Ans. $\frac{5}{196}$.

6. What part of £1 is 13s. 4d? Ans. $\frac{2}{3}$.

7. Reduce $\frac{9}{75}$ of a day to hours, minutes and seconds.
Ans. 2 hours, 52 min., 48 sec.

8. Add $\frac{2}{3}$ of a furlong to $\frac{8}{9}$ of a mile.
Ans. 7 fur., 31 rds, 0 yd., 1 ft., 10 in.

9. What is the value of $.857\frac{1}{7}$ of a bushel of rye?
Ans. 48 pounds.

10. Reduce 47 pounds of wheat to the decimal of a bushel.
Ans. $.783\frac{1}{3}$.

11. Reduce 9 dozen to the decimal of a gross. Ans. .75.

12. Add $\frac{7}{10}$ of a cwt. to $\frac{3}{5}$ of a quarter. Ans. 3 qrs., 10 lbs.

13. Subtract $\frac{7}{8}$ of a day from $\frac{5}{7}$ of a week. Ans. 4 days, 3 hrs.

14. From $\frac{11}{18}$ of 5 tons take $\frac{3}{7}$ of 9 cwt.
Ans. 2 tons, 17 cwt., 1 qr., $\frac{25}{63}$ lbs.

15. How many yards of cloth, at $$3\frac{1}{2}$ a yard, can be bought for $$48\frac{1}{7}$? Ans. $13\frac{37}{49}$ yards.

16. A man bought $\frac{7}{8}$ of a yard of cloth for $2.80; what was the rate per yard? Ans. $3.20.

17. How many tons of hay, at $$16\frac{1}{2}$ per ton, can be bought for $$196\frac{1}{8}$? Ans. $11\frac{39}{44}$ tons.

18. At $\$17\frac{3}{5}$ per week, how many weeks can a family board for $\$765\frac{3}{5}$? Ans. $43\frac{1}{2}$ weeks.

19. What number must be added to $26\frac{3}{5}$, and the sum multiplied by $7\frac{3}{4}$, that the product may be 496? Ans. $37\frac{2}{5}$.

20. A man owns $\frac{3}{4}$ of an oil well. He sells $\frac{2}{3}$ of his share for \$3500; what part of his share in the well has he still, and what is it worth at the same rate? Ans. $\frac{1}{4}$ left, and worth \$875.

21. How long will $119\frac{3}{7}$ hhds. of water last a company of 30 men, allowing each man $\frac{2}{5}$ of a gallon a day? Ans. 627 days.

22. Reduce $\frac{2}{7}$ of $2\frac{4}{5}$, $\frac{8}{12}$ of $1\frac{5}{6}$, and $3\frac{1}{2}$ of $2\frac{2}{7}$, to equivalent fractions having the *least* common denominator. Ans. $\frac{36}{45}$, $\frac{55}{45}$, $\frac{360}{45}$.

23. From $\frac{2}{7}$ of $2\frac{4}{5}$ of 4, take $\frac{9}{17}$ of $6\frac{4}{5}$ of $\frac{1}{9}$. Ans. $2\frac{4}{5}$.

24. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{9}$? Ans. $1\frac{2089}{2520}$.

25. What is the sum of $\frac{21}{35}$ of $3\frac{5}{8}+\frac{11}{45}$ of 85? Ans. $22\frac{2401}{2520}$.

26. How long will it take a person to travel 442 miles, if he travels $3\frac{1}{4}$ miles per hour, and $8\frac{1}{2}$ hours a day? Ans. 16 days.

27. Find the sum of $2\frac{1}{2}$ of $\frac{7}{10}$, $3\frac{1}{3}$ of $\frac{7}{9}$ of $\frac{6}{17}$ of $4\frac{1}{4}$ and $\frac{1}{2}$.
Ans. $6\frac{5}{36}$.

28. A has $2\frac{1}{7}$ times $8\frac{5}{6}$ dollars, and B $6\frac{1}{2}$ times $9\frac{3}{4}$ dollars; how much more has B than A? Ans. $\$44\frac{25}{56}$.

29. If I sell hay at \$1.75 per cwt.; what should I give for $9\frac{3}{5}$ tons, that I may make \$7 on my bargain. Ans. \$329.

30. If 7 horses eat $93\frac{1}{3}$ bushels of oats in 60 days; how many bushels will one horse eat in $87\frac{3}{7}$ days? Ans. $19\frac{3}{7}$.

31. Bought $14\frac{7}{10}$ yards of broadcloth for \$102.90; what was the value of $87\frac{3}{7}$ yards of the same cloth? Ans. \$612.

32. How many bushels of wheat, at $\$2\frac{2}{3}$ per bushel, will it require to purchase $168\frac{8}{99}$ bushels of corn worth 75 cents per bushel?
Ans. $47\frac{3}{11}$.

33. If in $82\frac{1}{2}$ feet there are 5 rods; how many rods in one mile?
Ans. 320.

34. Suppose I pay \$55 for $\frac{5}{8}$ of an acre of land; what is that per acre? Ans. \$88.

35. If $\frac{5}{6}$ of a pound of tea cost $\$1.66\frac{1}{3}$; what will $\frac{7}{9}$ of a pound cost? Ans. $\$1.55\frac{11}{45}$.

36. Subtract the sum of $2\frac{1}{2}$ and $1\frac{1}{12}$, from the sum of $\frac{3}{4}$, $7\frac{4}{9}$ and 3, and multiply the remainder by $3\frac{3}{11}$. Ans. $24\frac{10}{11}$.

37. If $\frac{7}{8}$ lb. cost $23\frac{3}{14}$ cents; what will $2\frac{11}{12}$ cost?
Ans. $77\frac{8}{21}$ cents.

38. What is the difference between $2\frac{1}{10}\times3\frac{1}{9}$ and $2\frac{1}{9}\times3\frac{1}{10}$ ?
Ans. $\frac{1}{90}$.

39. If $\frac{7}{8}$ lb. cost $\$\frac{3}{8}$ ; what will $\frac{11}{12}$ lb. cost? Ans. $39\frac{2}{7}$ cents.

40. What is the difference between $\frac{3}{4}$ of $\frac{1}{3}+\frac{1}{5}+\frac{1}{7}\times\frac{1}{9}$, and $\frac{1}{4}+\frac{1}{6}+\frac{1}{8}$ ? Ans. $\frac{191}{2520}$.

41. If $4\frac{7}{11}$ yards cost $\$1\frac{1}{33}$ , what will $2\frac{1}{8}$ yards cost?
Ans. $47\frac{2}{9}$ cents.

42. Bought $\frac{3}{7}$ of 2000 yards of ribbon, and sold $\frac{2}{3}$ of it; how much remains? Ans. $285\frac{5}{7}$ yards.

43. Divide the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, $\frac{31}{32}$, $\frac{63}{64}$, $\frac{127}{128}$ by the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$, and divide the quotient by $6\frac{7}{127}$, and multiply the result by $\frac{3}{4}$ of $\frac{5}{6}$. Ans. $\frac{5}{8}$.

44. I bonght $\frac{7}{8}$ of a lot of wood land, consisting of 47 acres, 3 roods, 20 rods, and have cleared $\frac{1}{2}$ of it; how much remains to be cleared? Ans. 20 acres, 3 roods, $31\frac{1}{4}$ rods.

45. What is the difference between $1\frac{25}{168}$ and $1\frac{33}{36}$ ? Ans. $\frac{247}{504}$.

46. If $\$\frac{11}{12}$ pay for a $1\frac{1}{2}$ st. of flour; for how much will $\$\frac{5}{8}$ pay?
Ans. $1\frac{1}{44}$ st.

47. Mount Blanc, the highest mountain in Europe, is 15,872 feet above the level of the sea; how far above the sea level is a climber who is $\frac{3}{16}$ of the whole height from the top, *i. e.*, $\frac{3}{16}$ of perpendicular hight? Ans. 12896 feet.

48. What will 45.94375 tons cost if 12.796875 tons cost $54.64?
Ans- $196.17.

49. If I gain $37.515625 by selling goods worth $324.53125; what shall I gain by selling a similar lot for $\$520.663541\dot{6}$. ?
Ans. $60.1884.

50. If 52.815 cwt. cost $22.345; what will 192.664 cwt. cost at the same rate? Ans. $81.512+

51. Required, the sum of the surfaces of 5 boxes, each of which is $5\frac{1}{2}$ feet long, $2\frac{1}{4}$ feet high, and $3\frac{1}{6}$ feet wide, and also the number of cubic feet contained in each box. The box supposed to be made from inch lumber? Ans. Surface, $390\frac{5}{6}$ ft.; each box $42\frac{863}{864}$ C. F.

52. If I pay $\$\frac{9}{10}$ for sawing into three pieces wood that is 4 ft. long; how much more should I pay, per cord, for sawing into pieces of the same length, wood that is 8 feet long? Ans. $22\frac{1}{2}$ cents.

53. A sets out from Oshawa, on a journey, and travels at the rate of 20 miles a day; 4 days after, B sets out from the same place, and travels the same road, at the rate of 25 miles per day; how many days before B will overtake A? Ans. 16.

54. A farmer having $56\frac{1}{2}$ tons of hay, sold $\frac{3}{5}$ of it at \$$10\frac{5}{8}$ per ton, and the remainder at \$9.75 per ton; how much did he receive for his hay? Ans. \$$580\frac{43}{80}$.

55. If the sum of $87\frac{11}{12}$ and $117\frac{16}{27}$ is divided by their difference; what will be the quotient? Ans. $6\frac{593}{841}$.

56. If $8\frac{3}{4}$ yards of silk make a dress, and 9 dresses be made from a piece containing 80 yards; what will be the remnant left? Ans. $1\frac{1}{4}$ yards.

57. A merchant expended \$840 for dry goods, and then had remaining only $\frac{37}{49}$ as much money as he had at first; how much money had he at first? Ans. \$3430.

58. If a person travel a certain distance in 8 days and 9 hours, by travelling 12 hours a day; how long will it take him to perform the same journey, by traveling $8\frac{3}{4}$ hours a day? Ans. 12 days.

59. If 15 horses, in 4 days, consume 87 bushels, 6 qrts: of oats; how many horses will 610 bushels, 1 peck, 2 qrts, keep for the same time? Ans. 105.

60. Reduce 1 pound troy, to the fraction of one pound avoirdupois. Ans. $\frac{144}{175}$.

61. Reduce $\frac{\frac{2}{5}\times\frac{10}{12}}{4\frac{1}{2}\text{ of }\frac{2}{9}}$ to a simple fraction. Ans. $\frac{1}{3}$.

62. What will be the cost of 8 cwt., 3 qrs., $12\frac{1}{2}$ lbs. of beef, if 4 cwt. cost \$34? Ans. \$$75\frac{7}{16}$.

63. If 4 men, working 8 hours a day, can do a certain piece of work in 15 days; how long would it take one man, working 10 hours a day, to do the same piece of work? Ans. 48 days.

64. Divide \$1728 among 17 boys and 15 girls, and give each boy $\frac{7}{11}$ as much as a girl; what sum will each receive?

Ans. Each girl, \$$66\frac{66}{71}$; each boy, \$$42\frac{42}{71}$.

65. If A can cut 2 cords of wood in $12\frac{1}{3}$ hours, and B can cut 3 cords in $17\frac{1}{2}$ hours; how many cords can they both cut in $24\frac{1}{2}$ hours? Ans. $8\frac{32}{185}$.

66. If it requires 30 yards of carpeting, which is $\frac{3}{4}$ of a yard wide, to cover a floor; how many yards, which is $1\frac{1}{4}$ yards wide, will be necessary to cover the same floor? Ans. 18.

67. A person bought 1000 gallons of spirits for \$1500; but 140 gallons leaked out; at what rate per gallon must he sell the remainder so as to make \$200 by his bargain? Ans. \$2 per gallon.

68. What must be the breadth of a piece of land whose length is $40\frac{1}{2}$ yards, in order that it may be twice as great as another piece of

land whose length is $14\frac{5}{8}$ yards, and whose breadth is $13\frac{2}{13}$ yards?
Ans. $9\frac{1}{2}$ yards.

69. If 7 men can reap a rectangular field whose length is 1,800 feet, and breadth 960 feet, in 9 days of 12 hours each; how long will it take 5 men, working 14 hours a day, to reap a field whose length is 800 feet, and breadth 700 feet? Ans. $3\frac{1}{2}$ days.

70. 124 men dug a trench 110 yards long, 3 feet wide, and 4 feet deep, in 5 days of 11 hours each; another trench was dug by one-half the number of men in 7 days of 9 hours each; how many feet of water was it capable of holding? Ans. 2268 cubic feet.

71. If 100 men, by working 6 hours each day, can, in 27 days, dig 18 cellars, each 40 feet long, 36 feet wide, and 12 feet deep; how many cellars, that are each 24 feet long, 27 feet wide, and 18 feet deep, can 240 men dig in 81 days, by working 8 hours a day?
Ans. 256.

72. A gentleman left his son a fortune, $\frac{1}{5}$ of which he spent in 2 months, $\frac{1}{4}$ of the remainder lasted him 3 months longer, and $\frac{2}{3}$ of what then remained lasted him 5 months longer, when he had only $895.50 left; how much did his father leave him? Ans. $4477.50.

73. A farmer having sheep in two different fields, sold $\frac{1}{4}$ of the number from each field, and had only 102 sheep remaining. Now 12 sheep jumped from the first field into the second; then the number remaining in the first field, was to the number in the second field as 8 to 9; how many sheep were there in each field at first?
Ans. 80 in first field; 56 in second.

74. A and B paid $120 for 12 acres of pasture for 8 weeks, with an understanding that A should have the grass that was then on the field, and B what grew during the time they were grazing; how many oxen, in equity, can each turn into the pasture, and how much should each pay, providing 4 acres of pasture, together with what grew during the time they were grazing, will keep 12 oxen 6 weeks, and in similar manner, 5 acres will keep 35 oxen 2 weeks?

Ans. { A should turn into the field 18 oxen, and pay $72.
B should turn into the field 12 oxen, and pay $48.

## ANALYSIS AND SYNTHESIS.

Analysis is the act of separating and comparing all the different parts of any compound, and showing their connection with each other, and thereby exhibiting all its elementary principles.

The converse of Analysis is Synthesis. The meaning and use of these terms will probably be most readily comprehended by reference to their derivation.

They are both pure Greek words. Analysis means *loosing up.* The general reader would here probably expect *loosing down,* as employed in most popular definitions; but we may illustrate the Greek term, *loosing up,* by our own everyday phrase, *tearing up,* which means *rending into shreds,* the English *up* conveying the same idea here as the Greek *ana* in analysis. The Greek synthesis means literally placing together; that is, the component parts being known, the word synthesis indicates the act of combining them into one. We might give many illustrations, but one will suffice, and we choose the one which will be most generally understood. When we analyse a sentence, we loose it *up,* or tear it *up,* into its component parts, and by synthesis we write or compose, *i. e.,* put together the parts, which, by analysis, we have found it to consist of.

When we commence to analyse a *problem* we reason from a given quantity to its unit, and then from this unit to the required quantity; hence, all our deductions are self-evident, and we therefore require *no rule* to solve a problem by analysis.

Although this part of arithmetic is usually called analysis, yet, as it is really both analysis and synthesis, we have given it a title in accordance with the principles now laid down.

### EXAMPLE.

1. If 12 pounds of sugar cost \$1.80, what will 7 pounds cost?

### SOLUTION.

$$\begin{array}{r} 12)1.80 \\ \hline .15 \\ 7 \\ \hline \$1.05 \end{array}$$

If 12 lbs. cost \$1.80, one pound will cost the $\frac{1}{12}$ of \$1.80=15 cents. Now, if 1 lb. cost 15 cents, 7 lbs. will cost 7 times 15 cents=to \$1.05. Therefore, 7 lbs. of sugar will cost \$1.05, if 12 lbs. cost \$1.80.

NOTE.—The work may be somewhat shortened, especially in long questions, by arranging it in the following manner, so as to admit of cancelling, if possible:—

$$\frac{1}{\cancel{12}}\times\frac{\overset{15}{\cancel{180}}}{1}\times\frac{7}{1}=\frac{105}{1}=\$1.05.\quad \text{Ans.}$$

2. If 5 bushels of pease cost \$5.50, for what can you purchase 19 bushels? Ans. \$20.90.

3. If 9 men can perform a certain piece of labor in 17 days, how long will it take 3 men to do it? Ans. 51 days.

4. How many pigs, at \$2 each, must be given for 7 sheep, worth \$4 a head? Ans. 14.

5. If \$100 gain \$6 in 12 months, how much would it gain in 40 months? Ans. \$20.

6. If $4\frac{2}{3}$ bushels of apples cost $\$3\frac{1}{9}$, what will be the cost of $7\frac{1}{2}$ bushels?

SOLUTION.

In the first place, $4\frac{2}{3}$ bushels$=\frac{14}{3}$ bushels, and $\$3\frac{1}{9}=\$\frac{28}{9}$. Now, since $\frac{14}{3}$ bushels cost $\$\frac{28}{9}$, one bushel will cost $\$\frac{28}{9}\div\frac{14}{3}=\frac{28}{9}\times\frac{3}{14}=\$\frac{2}{3}$, and $7\frac{1}{2}$ or $\frac{15}{2}$ bushels will cost $\frac{15}{2}$ times $\$\frac{2}{3}=\frac{2}{3}\times\frac{15}{2}=$ \$5, the value of $7\frac{1}{2}$ bushels of apples, if $4\frac{2}{3}$ bushels are worth $\$3\frac{1}{9}$.

OPERATION.

$$\frac{\overset{\cancel{2}}{\cancel{28}}}{\underset{\cancel{3}}{\cancel{9}}}\times\frac{\cancel{3}}{\cancel{14}}\times\frac{\overset{5}{\cancel{15}}}{\cancel{2}}=\frac{5}{1}=\$5.$$

7. If $\frac{2}{3}$ of $3\frac{3}{4}$ lbs. of tea cost $\$1\frac{7}{8}$; what will be the cost of $5\frac{1}{2}$ pounds? Ans. $\$4.12\frac{1}{2}$.

8. 100 is $\frac{2}{3}$ of what number? Ans. 150.

9. If $\frac{4}{9}$ of a mine cost \$2800; what is the value of $\frac{2}{3}$ of it? Ans. \$4200.

10. $\frac{2}{3}$ of 24 is $1\frac{3}{5}$ times what number? Ans. 10.

11. $\frac{3}{4}$ of 40 is $\frac{5}{12}$ of how many times $\frac{1}{2}$ of $\frac{4}{5}$ of 20? Ans. 9.

12. A is 16 years old, and his age is $\frac{2}{3}$ times $\frac{2}{3}$ of his father's age; how old is his father? Ans. 36.

13. A and B were playing cards; A lost \$10, which was $\frac{1}{3}$ times $\frac{3}{5}$ as much as B then had; and when they commenced $\frac{2}{3}$ of A's money was equal to $\frac{3}{4}$ of B's; how much had each when they began to play? Ans. A \$50; B \$40.

14. A man willed to his daughter \$560, which was $\frac{1}{3}$ of $\frac{3}{4}$ of what he bequeathed to his son; and 4 times the son's portion was $\frac{2}{3}$ the value of the father's estate; what was the value of the estate? Ans. \$13,440.

15. A gentleman spent $\frac{1}{4}$ of his life in Montreal, $\frac{1}{3}$ of it in Ottawa, and the remainder of it, which was 25 years, in Belleville; what age was he when he died? Ans. 60 years.

16. A owns $\frac{1}{8}$, and B $\frac{1}{12}$ of a ship; A's part is worth $650 more than B's; what is the value of the ship? Ans. $15,600.

17. A post stands $\frac{1}{4}$ in the mud, $\frac{1}{3}$ in the water, and 15 feet above the water; what is the length of the post? Ans. 36 feet.

18. A grocer bought a firkin of butter containing 56 pounds, for $11.20, and sold $\frac{3}{4}$ of it for \$$8\frac{2}{5}$; how much did he get a pound?
Ans. 20 cents.

19. The head of a fish is 4 feet long, the tail as long as the head and $\frac{1}{2}$ the length of the body, and the body is as long as the head and tail; what is the length of the fish? Ans. 32 feet.

20. A and B have the same income; A saves $\frac{1}{7}$ of his; B, by spending $65 a year more than A, finds himself $25 in debt at the end of 5 years; what did B spend each year? Ans. $425.

21. A can do a certain piece of work in 8 days, and B can do the same in 6 days; A commenced and worked alone for 3 days, when B assisted him to complete the job; how long did it take them to finish the work?

### SOLUTION.

If A can do the work in 8 days, in one day he can do the $\frac{1}{8}$ of it, and if B can do the work in 6 days, in one day he can do the $\frac{1}{6}$ of it, and if they work together, they would do $\frac{1}{8}+\frac{1}{6}=\frac{7}{24}$ of the work in one day. But A works alone for 3 days, and in one day he can do $\frac{1}{8}$ of the work, in 3 days he would do 3 times $\frac{1}{8}=\frac{3}{8}$ of the work, and as the whole work is equal to $\frac{8}{8}$ of itself, there would be $\frac{8}{8}-\frac{3}{8}=\frac{5}{8}$ of the work yet to be completed by A and B, who, according to the conditions of the question, labour together to finish the work. Now A and B working together for one day can do $\frac{7}{24}$ of the entire job, and it will take them as many days to do the balance $\frac{5}{8}$ as $\frac{7}{24}$ is contained in $\frac{5}{8}$, which is equal $\frac{5}{8}\times\frac{24}{7}=7\frac{1}{7}$ days.

22. A and B can build a boat in 18 days, but if C assists them, they can do it in 8 days; how long would it take C to do it alone?
Ans. $14\frac{2}{5}$ days.

23. A certain pole was $25\frac{1}{2}$ feet high, and during a storm it was broken, when $\frac{3}{4}$ of what was broken off, equalled $\frac{2}{3}$ of what remained; how much was broken off, and how much remained?
Ans. 12 feet broken off, and $13\frac{1}{2}$ remained.

24. There are 3 pipes leading into a certain cistern; the first will fill it in 15 minutes, the second in 30 minutes, and the third in one hour; in what time will they all fill it together?
Ans. 8 min., $34\frac{2}{7}$ sec,

25. A and B start together by G. T. R. from Cobourg to Brockville, a distance of 138 miles. A goes by freight train, at the rate of 12 miles per hour, and B by mixed train, at the rate of 18 miles per hour. C leaves Brockville for Cobourg at the same time by express train, which runs at the rate of 22 miles per hour; how far from Cobourg will A and B each be when C meets them?

Ans. A $48\frac{12}{17}$ miles; B $62\frac{1}{10}$ miles.

26. A cistern has two pipes, one will fill it in 48 minutes, and the other will empty it in 72 minutes; what time will it require to fill the cistern when both are running? Ans. 2 hours, 24 min.

27. If a man spends $\frac{5}{12}$ of his time in working, $\frac{1}{3}$ in sleeeping, $\frac{1}{16}$ in eating, and $1\frac{1}{2}$ hours each day in reading; how much time will be left? Ans. 3 hours.

28. A wall, which was to be built 32 feet high, was raised 8 feet by 6 men in 12 days; how many men must be employed to finish the wall in 6 days? Ans. 30 men.

29. A and B can perform a piece of work in $5\frac{5}{11}$ days; B and C in $6\frac{2}{3}$ days; and A and C in 6 days; in what time would each of them perform the work alone, and how long would it take them to do the work together?

Ans. A, 10 days; B, 12 days; C, 15 days; and A, B, and C, together, in 4 days.

30. My tailor informs me that it will take $10\frac{1}{4}$ square yards of cloth to make me a full suit of clothes. The cloth I am about to purchase is $1\frac{7}{8}$ yards wide, and on sponging it will shrink $\frac{1}{20}$ in width and length; how many yards of this cloth must I purchase for my "new suit?" Ans. $6\frac{62}{1083}$ yards.

31. If A can do $\frac{2}{3}$ of a certain piece of work in 4 hours, and B can do $\frac{3}{4}$ of the remainder in 1 hour, and C can finish it in 20 min.; in what time will they do it all working together?

Ans. 1 hour, 30 min.

32. A certain tailor in the City of Hamilton bought 40 yards of broadcloth, $2\frac{1}{4}$ yds wide; but on sponging, it shrunk in length upon every 2 yards, $\frac{1}{16}$ of a yard, and in width, $1\frac{1}{2}$ sixteenths upon every $1\frac{1}{2}$ yards. To line this cloth he bought flannel $1\frac{1}{4}$ yards wide, which, when wet, shrunk $\frac{1}{2}$ the width on every 10 yards in length, and in width it shrunk $\frac{1}{2}$ of a sixteenth of a yard; how many yards of flannel had the tailor to buy to line his broadcloth?

Ans. $71\frac{7}{13}$ yards.

33. If 6 bushels of wheat are equal in value to 9 bushels of barley, and 5 bushels of barley to 7 bushels of oats, and 12 bushels of

oats to 10 bushels of pease, and 13 bushels of pease to $\frac{1}{2}$ ton of hay, and 1 ton of hay to 2 tons of coal, how many tons of coal are equal in value to 80 bushsls of wheat?

### SOLUTION.

If 6 bushels of wheat are equal in value to 9 bushels of barley, or 9 bushels of barley to 6 bushels of wheat, one bushel of barley would be equal to $\frac{1}{9}$ of 6 bushels of wheat, equal to $\frac{6}{9}$, or $\frac{2}{3}$ of a bushel of wheat, and 5 bushels of barley would be equal to 5 times $\frac{2}{3}$ of a bushel of wheat, equal to $\frac{2}{3}\times5=\frac{10}{3}=3\frac{1}{3}$ bushels of wheat. But 5 bushels of barley are equal to seven bushels of oats; hence, 7 bushels of oats are equal to $3\frac{1}{3}$ bushels of wheat, and one bushel of oats would be equal to $3\frac{1}{3}\div7=\frac{10}{21}$ bushels of wheat, and 12 bushels of oats would be equal to 12 times $\frac{10}{21}=\frac{120}{21}=5\frac{5}{7}$ bushels of wheat. But 12 bushels of oats are equal in value to 10 bushels of pease, hence, 10 bushels of pease are equal to $5\frac{5}{7}$ bushels of wheat, and one bushel of pease would equal $5\frac{5}{7}\div10=\frac{4}{7}$ of a bushel of wheat, and 13 bushels of pease would equal $\frac{4}{7}\times13=\frac{52}{7}=7\frac{3}{7}$ bushels of wheat. But 13 bushels of pease equal in value $\frac{1}{2}$ ton of hay, hence, $\frac{1}{2}$ ton of hay equals $7\frac{3}{7}$ bushels of wheat, and one ton would equal $7\frac{3}{7}\times2=14\frac{6}{7}$ bushels of wheat. But one ton of hay equals 2 tons of coal, hence, 2 tons of coal are equal in value to $14\frac{6}{7}$ bushels of wheat, and one ton would equal $14\frac{6}{7}\div2=7\frac{3}{7}$ bushels of wheat. Lastly, if $7\frac{3}{7}$ bushels of wheat be equal in value to one ton of coal, it would take as many tons of coal to equal 80 bushels of wheat, as $7\frac{3}{7}$ is contained in 80, which gives $10\frac{10}{13}$ tons of coal.

NOTE.—This question belongs to that part of arithmetic usually called Conjoined Proportion, or, by some, the "Chain Rule," which has each antecedent of a compound ratio equal in value to its consequent. We have thought it best not to introduce such questions under a head by themselves, on account of their *theory* being more easily understood when exhibited by Analysis than by Proportion. Questions that do occur like this will most probably relate to Arbitration of Exchange. Although they may all be worked by Compound Proportion as well as by Analysis, yet the most expeditious plan, and the one generaly adopted, is by the following

### RULE.

*Place the antecedents in one column and the consequents in another, on the right, with the sign of equality between them. Divide the continued product of the terms in the column containing the odd term by the continued product of the other column, and the quotient will be the answer.*

Let us now take our last example (No. 33), and solve it by this rule:

6 bushels of wheat=9 bushels of barley
5 bushels of barley=7 bushels of oats.
12 bushels of oats=10 bushels of pease.
13 bushels of pease=$\frac{1}{2}$ ton of hay.
1 ton of hay=2 tons of coal.
— tons of coal=80 bushels of wheat.

$$\frac{\overset{\cancel{3}}{\cancel{9}},\ 7,\ \overset{\cancel{2}}{\cancel{10}},\ \cancel{1}\text{-}\cancel{2},\ \cancel{2},\ \overset{\overset{20}{\cancel{40}}}{\cancel{80}}}{\underset{\cancel{2}}{\cancel{6}},\ \cancel{5},\ \underset{\underset{\cancel{3}}{\cancel{6}}}{\cancel{12}},\ 13,\ \underset{\cancel{2}}{\cancel{1}},}=\frac{140}{13}=10\tfrac{10}{13}.\quad\text{Ans.}$$

34. If 12 bushels of wheat, in Toronto, are equal in value to $12\frac{1}{2}$ bushels in Hamilton, and 14 bushels in Hamilton are worth $14\frac{1}{2}$ bushels in Woodstock, and 12 bushels in Woodstock are worth $12\frac{1}{2}$ bushels in Guelph, and 25 bushels in Guelph are worth 28 bushels in Barrie, how many bushels in Barrie are worth 60 bushels in Toronto? Ans. $75\frac{25}{48}$.

35. If 12 shillings in Massachusetts are worth 16 shillings in New York, and 24 shillings in New York are worth $22\frac{1}{2}$ shillings in Pennsylvania, and $7\frac{1}{2}$ shillings in Pennsylvania are worth 5 shillings in Canada, how many shillings in Canada are worth 50 shillings in Massachusetts? Ans. $41\frac{2}{3}$.

36. If 6 men can build 120 rods of fencing in 4 days, how many days would seven men require to build 210 rods?

### SOLUTION.

If 6 men can build 120 rods of fencing in 4 days, one man could do $\frac{1}{6}$ of 120 rods in the same time; and $\frac{1}{6}$ of 120 rods is 20 rods. Now, if one man can build 20 rods in 4 days, in one day he would build $\frac{1}{4}$ of 20 rods, and $\frac{1}{4}$ of 20 rods is 5 rods. Now, if one man can build 5 rods in one day, 7 men would build 7 times 5 rods in one day, and 7 times 5 rods=35 rods. Lastly, if 7 men can build 35 rods in one day, it would take them as many days to build 210 rods as 35 is contained in 210, which is 6; therefore, if 6 men can build 120 rods of fencing in 4 days, 7 men would require 6 days to build 210 rods.

37. If 12 men, in 36 days, of 10 hours each, build a wall 24 feet long, 16 feet high, and 3 feet thick; in how many days, of 8

hours each, would the same lot of men build a wall 20 feet long, 12 feet high, and $2\frac{1}{2}$ feet thick? Ans. $23\frac{7}{16}$.

38. If 5 men can perform a piece of work in 12 days of 10 hours each; how many men will perform a piece of work four times as large, in a fifth part of the time, if they work the same number of hours in a day, supposing that 2 of the second set can do as much work in an hour as 3 of the first set? Ans. $66\frac{2}{3}$ men.

NOTE.—Such questions as this, where the answer involves a fraction, may frequently occur, and it may be asked how $\frac{2}{3}$ of a man can do any work. The answer is simply this, that it requires 66 men to do the work, and one man to continue on working $\frac{2}{3}$ of a day more.

39. Suppose that a wolf was observed to devour a sheep in $\frac{7}{8}$ of an hour, and a bear in $\frac{3}{4}$ of an hour; how long would it take them together to eat what remained of a sheep after the wolf had been eating $\frac{1}{2}$ an hour? Ans. $10\frac{5}{13}$ min.

40. Find the fortunes of A, B, C, D, E, and F, by knowing that A is worth $20, which is $\frac{1}{4}$ as much as B and C are worth, and that C is worth $\frac{1}{3}$ as much as A and B, and also that if 19 times the sum of A, B and C's fortune was divided in the proportion of $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{3}$, it would respectively give $\frac{3}{4}$ of D's, $\frac{1}{2}$ of E's, and $\frac{1}{3}$ of F's fortune.
Ans. A, 20; B, 55; C, 25; and D, E and F, 1200 each.

41. A and B set out from the same place, and in the same direction. A travels uniformly 18 miles per day, and after 9 days turns and goes back as far as B has travelled during those 9 days; he then turns again, and pursuing his journey, overtakes B $22\frac{1}{2}$ days after the time they first set out. It is required to find the rate at which B uniformly travelled. Ans. 10 miles per day.

42. A hare starts 40 yards before a greyhound, and is not perceived by him until she has been running 40 seconds, she scuds away at the rate of 10 miles an hour, and the dog pursues her at the rate of 18 miles an hour; how long will the chase last, and what distance will the hare have run? Ans. $60\frac{5}{22}$ sec.; 490 yards.

43. A can do a certain piece of work in 9 days, and B can do the same in 12 days; they work together for 3 days, when A is taken sick and leaves, B continues on working alone, and after 2 days he is joined by C, and they finish it together in $1\frac{1}{2}$ days; how long would C be doing it alone? Ans. 12 days.

44. A, in a scuffle, seized on $\frac{2}{3}$ of a parcel of sugar plums; B caught $\frac{3}{8}$ of it out of his hands, and C laid hold on $\frac{3}{10}$ more; D ran off with all A had left, except $\frac{1}{7}$ which E afterwards secured slyly for himself; then A and C jointly set upon B, who, in the conflict, let

fall $\frac{1}{2}$ he had, which were equally picked up by D and E, who lay perdu. B then kicked down C's hat, and to work they all went anew for what it contained ; of which A got $\frac{1}{4}$, B $\frac{1}{3}$, D $\frac{2}{7}$, and C and E equal shares of what was left of that stock. D then struck $\frac{3}{4}$ of what A and B last acquired, out of their hands ; they, with some difficulty, recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ a piece of the same. Upon this, they called a truce, and agreed that the $\frac{1}{3}$ of the whole left by A at first, should be equally divided among them ; how many plums, after this distribution, had each of the competitors?

Ans. A had 2863 ; B, 6335 ; C, 10,294, and E, 4950.

## PRACTICE.

The rule which is called Practice is nothing else than a particular case of simple proportion, viz., when the first term is unity. Thus : if it is required to find the price of 28 tons of coal, at $7 a ton—as a question in proportion, it would be, if 1 ton of coal costs $7, what will 28 tons cost? and the statement would be 1 : 28 : : 7 : F. P. Here the first term being 1, the question becomes one of simple multiplication, but the answer, $196, is really the fourth term of an analogy. Again, to find the price of 36 cwt. flour, at £1 10s. 0d. per cwt.—here again, the question stated at length would be, if 1 cwt. of flour cost £1 10s. 0d., what will 36 cwt. cost? and the statement would be 1 : 36 : : £1 10s. 0d. : F. P., which thus becomes a question of multiplication, because the first term is unity, and, dividing by 1, would not alter the product of the other two terms. There are various methods of performing the work, which we shall now illustrate by the example given above.

| (1.) | (2.) | (3.) |
|---|---|---|
| | 36 | £36 |
| £ 1 10 0 | 30 | 1 |
| 36 | | 10s.=½ of £1)36 |
| | 20)108(0 | 18 |
| £54. 0. 0 | | |
| | £54 0 0 | £54 0 0 |

1st method—multiply £1 10s. 0d. by 36, as in compound multiplication, and the result, £54, is the answer, because it is the product of the second and third terms of the analogy. In the second method the £1 10s. 0d. is reduced to shillings, and 36 multiplied by it, which gives the answer in shillings, and dividing by 20 gives 54, the answer in pounds. By the third method, we note that 36 cwt., at £1 per cwt., will be £36, and since 10s. is the half of a pound, the price at 10s. will be the half of the price at £1, and we write in the usual place for a divisor 10s.=½ of £1, and we take the half of the price at £1, namely, £18, and then by adding together the price at £1 and the price at 10s., viz., £36 and £18, we get the price at £1 10s. 0d. This is in reality nothing else than multiplying by 1½, as in the margin, which is obviously correct, for £1 10s. 0d. is *one* pound and *a-half.* We have chosen an exceedingly simple example for the purpose of illustration, and we now remark that the advantage of any one of these methods above the others will not be apparent till more difficult questions are proposed. The first method is generally the most convenient in calculating dollars and cents, because, as we have already shown, the multiplication of all quantities expressed decimally can be performed in precisely the same manner as in the case of whole numbers, care being taken to give the decimal point its proper place. The second and third methods, and especially the third, are the best adapted for calculations in pounds, shillings and pence, although the first may often be used with great advantage. The third is called the method of aliquot parts, *i. e.*, a part that is contained in the next higher denomination without remainder; thus, 10s. above is an aliquot part of £1, viz., 10s. is ½ of £1.

```
 36
  1½
 ───
 36
 16
 ───
£54
```

TABLE OF ALIQUOT PARTS.

| *Parts of $1. | | Parts of £1. | | Parts of 1s. | | Parts of a month. | |
|---|---|---|---|---|---|---|---|
| 50 cents | $=\frac{1}{2}$ | 10s. | $=\frac{1}{2}$ | 6d. | $=\frac{1}{2}$ | 15 days | $=\frac{1}{2}$ |
| 33⅓ | $=\frac{1}{3}$ | 6s. 8d. | $=\frac{1}{3}$ | 4d. | $=\frac{1}{3}$ | 10 | $=\frac{1}{3}$ |
| 25 | $=\frac{1}{4}$ | 5s. | $=\frac{1}{4}$ | 3d. | $=\frac{1}{4}$ | 7½ | $=\frac{1}{4}$ |
| 20 | $=\frac{1}{5}$ | 4s. | $=\frac{1}{5}$ | 2d. | $=\frac{1}{6}$ | 5 | $=\frac{1}{6}$ |
| 16⅔ | $=\frac{1}{6}$ | 3s. 4d. | $=\frac{1}{6}$ | 1½d. | $=\frac{1}{8}$ | 3 | $=\frac{1}{10}$ |
| 12½ | $=\frac{1}{8}$ | 2s. 6d. | $=\frac{1}{8}$ | 1d. | $=\frac{1}{12}$ | 2 | $=\frac{1}{15}$ |
| 8⅓ | $=\frac{1}{12}$ | 2s. | $=\frac{1}{10}$ | | | 1 | $=\frac{1}{30}$ |
| 6¼ | $=\frac{1}{16}$ | 1s. 8d. | $=\frac{1}{12}$ | | | | |
| 5 | $=\frac{1}{20}$ | 1s. 4d. | $=\frac{1}{15}$ | | | | |
| 4 | $=\frac{1}{25}$ | 1s. 3d. | $=\frac{1}{16}$ | | | | |
| 2 | $=\frac{1}{50}$ | 1s. | $=\frac{1}{20}$ | | | | |

---

* In Canada, the hundred weight being 100 lbs., the aliquot parts of the cwt. will be the same as the aliquot parts of the dollar. In Britain, the hundred weight is 112 lbs.

## EXERCISES.

Find the prices of the following numbers of articles at the given prices:

(1.)

187 cwt., at $5.37½:

187
5.37½
1309
561
935
93½
$1005.12½

(2.)

1857 lbs., at $3.87½

3.87½
928½=½ of 1857
12999
14856
5571
$7195.87½

(3.)

4796 tons, at $4.50

4.50
239800
19184
$21582.00

(4.)

What is the price of 29 score of sheep at $7.62½ each.

29
20
580
7.62½
290
1160
3480
4060
$4422.50

(5.)

Sold to a cattle dealer 196 head at $18.75 each.

18.75
196
11250
16875
1875
$3675.00

(6.)

Sold to a dealer 97 head at $16.12½ on an average.

16.12½
97
48½
11284
14508
$1564.12½

7. To find the price of 347 cwt. of coffee at £7.11.6 per cwt.

cwt. 347×1=£347=price at £1.
7=No. of pounds
£2429=price at £7, obtained by multiplying 347 by 7.

| | | |
|---|---|---|
| 10s. =½ of £1. | 2 | 173.10.0=price at 10s. obtained by dividing 347 by 2. |
| 1s. 3d.=⅛ of 10s. | 8 | 21.13.9=price at 1s. 3d. obtained by dividing 173.10.0 by 8. |
| 3d. =⅕ of 1s. 3d. | | 4. 6.9=price at 3d. obtained by dividing 21.13.9 by 5. |

£2628.10.6=price at £7.11.6, obtained by adding the four parts.

Here it is evident that if each cwt. cost *one* pound, the whole must be repeated as often as there are units in 7, *i. e.*, 7 times, and hence we multiply 347 by 7, and obtain 2429, which being the *price* of the hundred weights is pounds, *i. e.*, £2429. Again, since 10s. is the $\frac{1}{2}$ of £1, the price at 10s. will be the $\frac{1}{2}$ of the price at £1, *i. e.*, the $\frac{1}{2}$ of £347, which therefore we divide by 2, and place the quotient £173.10.0, under the £2429, and the divisor 2, by which it was obtained, opposite to it. Again, since 1s. 3d. is the $\frac{1}{8}$ of 10s., the price at 1s. 3d, will be the $\frac{1}{8}$ of the price at 10s., we therefore divide £173.10.0 by 8 and place the quotient £21.13.9 below it, and the 8 opposite to it. In the same way we see that as 3d. is the $\frac{1}{5}$ of 1s. 3d., that the price at 3d. will be the $\frac{1}{5}$ of £21.13.9, the price at 1s. 3d., we therefore divide £21.13.9 by 5, and obtain £4.6.9, the price at 3d, and place it under the £21.13.9, and the 5 opposite to it. We have now the price at 7. 0.0, all which make the whole price as here indicated. Having thus obtained the partial results, we add them, and the sum is the price of 347 cwt. at £7.11.6. To keep before us the aliquot parts used, we place a memorandum of this on the left, as exhibited in the examples. We have written this first example in an expanded form, in order to show every step of the process. The annexed will show in how much smaller space it may be performed :

0.10.0
0. 1.3
0. 0.3
£7.11.6

| | | |
|---|---|---|
| | | 347 @ £7.11.6 |
| | | 7 |
| 10s. | $\frac{1}{2}$ | 2429 |
| 1s. 3d. | $\frac{1}{8}$ | 173.10.0 |
| 3d. | $\frac{1}{5}$ | 21.13.9 |
| | | 4. 6.9 |
| | | £2628.10.6 |

otherwise :

| | | |
|---|---|---|
| | | 347 @ £7.11.6 |
| | | 7 |
| | | 2429 |
| 10s. | $\frac{1}{2}$ | 173.10.0 |
| 1s. | $\frac{1}{10}$ | 17. 7.0 |
| 6d. | $\frac{1}{2}$ | 8.13.6 |
| | | £2628.10.6 |

We have taken for granted that those who have learned compound division, do not need to be told that when there was £1 over in dividing £347 by 2, it was to be reduced to shillings, (20) and then divided by 2, giving 10s., and so on. There are other modes which will be illustrated by other examples.

8. If a man has an income of $12.50 per week; how much has he per year? Ans. $650.00.

9. If a clerk has \$2.12½ salary for every working day in the year; what is his yearly income? Ans. \$665.12½

10. If a tradesman earn 8s. 8d. a day; how much will he earn in the working days of the year? Ans. £135.12.8.

11. An Ensign's pay in the British army is 5s. 6d. a day; how much is that in Leap-year? Ans. £100.13.0.

12. If an officer's pay is a guinea and a half a day; how much has he in a common year? Ans. £574.17.6.

13. What will 479 cwt. of sugar come to at £4.9.6 per cwt.?

| | | |
|---|---|---|
| | | 479 |
| | | 4 |
| | | 1916 |
| 5s. | $\frac{1}{4}$ | 119.15.0 |
| 4s. | $\frac{1}{5}$ | 95.16.0 |
| 6d. | $\frac{1}{10}$ | 11.19.6 |
| | | £2143.10.6 |

In this example, as before, we multiply 479 by 4, to get the price at £4; then as 9s. 6d. is 5s.+4s.+6d., we resolve the shillings and pence into this form, and as 5s. is $\frac{1}{4}$ of £1, we divide 479, the price at £1, by 4, and obtain £119.15.0, the price at 5s. Again, as 4s. is $\frac{1}{5}$ of £1, we divide 479 by 5, and get £95.16.0, the price at 4s.; lastly, as 6d. is $\frac{1}{10}$ of 5s. we divide £119.15.0 by 10, and get £11.19.6, the price at 6d. We might here have taken 6d. as the $\frac{1}{8}$ of 4s., but the division by 10 is easier than the division by 8. We now take the sum of all the partial results.

We would call the learner's special attention to the following directions, as the neglect of it is a fertile source of error. Whenever you take any quantity as an aliquot part of a higher to find the price of the former, *be sure you divide the line which is the price at the rate of that higher denomination.* Thus, in the last example, 4s. is $\frac{1}{5}$ of £1, so we must divide £479, and *not* the £119,15.0, for that would imply that 4s. was $\frac{1}{5}$ of 5s. So also, since 6d. is the $\frac{1}{10}$ of 5s., we divide the £119 15s. 0d. by 10, but, had we taken 6d. as the $\frac{1}{8}$ of 4s., we should then have divided £95 16s. 0d., because that is the price of 4s; the result would have been the same, however, for £119 15s. 0d.÷10=£11 19s. 6d., and £95 16s. 0d.÷8= £11 19s. 6d. also.

### METHOD OF COMPLEMENTS.

14.—879 cwt., @ 19s.

| | | | |
|---|---|---|---|
| 1s.=$\frac{1}{20}$) | 879 | 0 | 0 |
| | 43 | 19 | 0 |
| | £835 | 1 | 0 |

15.—1793 lbs., @ 18s

| | | | |
|---|---|---|---|
| 10) | 1793 | 0 | 0 |
| | 179 | 6 | 0 |
| | £1613 | 14 | 0 |

| | |
|---|---|
| 16.—2781 tons, @ 17s. 6d. | 17.—987 lbs., @ 16s. |
| 8)2781 0 0 | 5)987 0 0 |
| 347 12 6 | 197 8 0 |
| £2433 7 6 | £789 12 0 |

The principle of these operations may be illustrated by Ex. 14. We observe that if the price were £1, the answer would be £879, but the price is 1s. less than a £1, and as 1s. is $\frac{1}{20}$ of £1, we find £43 19s. 0d., the price at 1s., and subtract it from £879, the price at £1, and we have £835 1s. 0d., the price at 19s. In the same manner in 15, we subtract $\frac{1}{10}$, because 18s. wants 2s. of £1, and 2s. is $\frac{1}{10}$ of £1. So in 16, we subtract $\frac{1}{8}$, and in 17 we subtract $\frac{1}{5}$. This mode is useful in many other operations. It is not of so much service, however, in calculating by dollars and cents as by pounds, shillings and pence. We shall give one example, worked both ways, and let the learner judge for himself.

(18.)

| | |
|---|---|
| 2479 @ \$3.90 | $\frac{1}{10}$ of 2479 @ \$3.90 |
| 3.90 | 4 |
| 2231.10 | 9916.00 |
| 7437 | 247.90 |
| \$9668.10 | \$9668.10 |

Here \$4 will be seen to be too large a multiplier by 10 cents, and therefore we subtract $\frac{1}{10}$ of \$2479, viz., \$247.90, and find the same answer as before.

19. To find the rent of 189 acres, 2 roods, 32 rods at \$4.20 per acre. Since the rent of 1 acre is \$4.20, the half of it, \$2.10, will be the rent of 2 roods, the rent of 20 rods will be .525, the $\frac{1}{4}$ of the rent of 2 roods, the half of that, .2625, will be the rent of 10 rods, and, lastly, .0525 will be the rent of 2 rods, which is the $\frac{1}{5}$ of 10 rods. We then multiply by 189, and set the figures of the

| | |
|---|---|
| 2 roods=$\frac{1}{2}$ of 1 acre, | 4.20 |
| | 189 |
| 20 rods=$\frac{1}{4}$ of 2 roods, | 210 |
| | 525 |
| 10 rods=$\frac{1}{2}$ of 20 rods, | 2625 |
| | 525 |
| 2 rods=$\frac{1}{5}$ of 10 rods, | 3780 |
| | 3360 |
| | 420 |
| | \$796.74 |

product in the usual order, so that the first figure of the product by 9 shall be under the units of cents, &c., and then adding all the partial results, we find the final answer, \$796.74, the rent of 189 acres, 2 roods and 32 perches.

20. What is the price of 118 acres, 3 roods and 20 rods of cleared land, at \$36.75 per acre? Ans. \$4368.66.

21. What is the price of 286 acres, 1 rood, 24 rods of uncleared land, at \$7.25 per acre? Ans. 2076.40.

22. A has 84 acres, 2 roods, 36 rods of cleared land, worth \$24.60 an acre; B has 248 acres, 3 roods, 24 rods of uncleared land, worth \$4.40 an acre—they exchange, the difference of value to be paid in cash; which has to pay, and how much?

Ans. B \$989.08.

23. What is the price of 675½ cwt. of beef, at 10.86⅔ per cwt.?

Ans. 7340.43.

24. What is the value of 483 cwt. of coffee, at \$23.33⅓ per cwt.?

Ans. \$11270.00.

25. What are 195 lbs. of raisins worth, at 30 cts. per lb.?

Ans. 58.50.

26. What is the value of 514 gallons of oil, at 43½ cts. per gallon? Ans. \$223.59.

27. What will be the price of 576 yards of broadcloth, at \$3.75 per yard? Ans. \$2160.00.

28. Find the price of 1625 yards of muslin, at 54⅙cts. per yard.

Ans. \$880.21.

29. Find the price of 4265 cwt. of flour, at \$6.82½ per cwt.

Ans. \$29108.62½.

30. What is the price of 7913 cwt. of pearl ashes, at 11.375 per cwt.? Ans. \$90010.37½.

## BILLS OF PARCELS.

A Bill of Parcels is simply a statement rendered by the seller to the purchaser, showing the articles bought, and the prices of each.

31. HAMILTON, Jan. 4th, 1866.

MR. ELIAS G. CONKLIN,

*Bought of* J. BUNTIN & Co.,

| | | | |
|---|---|---|---|
| 12 | reams of foolscap paper | @ | $3.25 |
| 15 | dozen School Books | @ | 4.50 |
| 23 | “ Slates | @ | 1.30 |
| 7 | “ Photogragh Albums | @ | 15.00 |
| 3 | “ Bullions Grammar | @ | 7.00 |
| 8 | “ Fifth Reader | @ | 3.50 |
| 5 | gallons of Black Ink | @ | 1.10 |

$295.90

Received payment,

J. BUNTIN & Co.

---

32 TORONTO, Jan. 12th, 1866.

MR. JAMES H. BURRITT,

*Bought of* MORRISON, TAYLOR & Co.,

| | | | |
|---|---|---|---|
| 15 | cwt. of cheese | @ | $9.00 |
| 4 | cwt. of flour | @ | 4.25 |
| 120 | pounds of Bacon | @ | 0.14 |
| 7 | bushels of corn meal | @ | 0.75 |
| 12 | firkins of butter | @ | 13.50 |
| 20 | bushels of dried apples | @ | 2.25 |
| 13 | “ “ peaches | @ | 4.00 |
| 11 | cwt. of buck-wheat flour | @ | 5.50 |
| 15 | cwt. maple sugar | @ | 8.00 |
| 25 | bags of common salt | @ | 1.15 |
| 57 | barrels of mess pork | @ | 13.00 |
| 68 | “ beef | @ | 9.75 |
| 13 | bushels of clover seed, | @ | 7.50 |

$2143.80

Received payment by note at 30 days.

FOR MORRISON, TAYLOR & Co.,

A. C. HENRY.

33 HAMILTON, January 2nd, 1866.

MR. M. MCCULLOCH,

*To* JOSEPH LIGHT, Stationer, *Dr.*

| | | | | |
|---|---|---|---|---|
| For | 500 French envelopes | @ | $3.00 per thousand | |
| “ | 12 doz. British American copy books | @ | 1.15 | |
| “ | 6 “ B. B. lead pencils | @ | .50 | |
| “ | 5 gross mourning envelopes | @ | 1.05 | |
| “ | 2 reams mourning note paper | @ | 3.15 | |
| “ | 4 “ tinted note paper | @ | 3.15 | |
| “ | 2½ “ Foreign note paper | @ | 1.40 | |
| “ | 3 “ “ letter paper | @ | 3.00 | |
| “ | 1 doz. First Books | @ | .15 | |
| “ | 5 boxes Gillott's No. 303 pens | @ | .90 | |
| “ | 5 doz. Third Books | @ | 1.62½ | |
| “ | 10 quires blank books, half bound | @ | .35 | |
| “ | 2 packs visiting cards | @ | .37½ | |
| | | | | $71.98 |

NOTE.—Bills should not be signed until settled.

---

34. BROCKVILLE, Jan. 5th, 1866.

N. D. GALBREAITH,

*To* R. FITZSIMMONS & Co., *Dr.*

| | | | | |
|---|---|---|---|---|
| For | 24 lbs. Mackerel | @ | 05½c. | |
| “ | 3 gallons Molasses | @ | 45 | |
| “ | 13 lbs. Young Hyson Tea | @ | 87½ | |
| “ | 13 lbs. brown Sugar | @ | 11 | |
| “ | 15 bushels of Potatoes | @ | 45 | |
| | | | | $22.23 |

CR.

| | | | | |
|---|---|---|---|---|
| For | 10 lbs. Butter | @ | 17c. | |
| “ | 5 doz. Eggs | @ | 12½ | |
| “ | 3 gallons Maple Molasses | @ | 95 | |
| “ | Note at 20 days, to balance | | | 17.05 |
| | | | | $22.23 |

R. FITZSIMMONS & Co.

NOTE.—Such a Bill as this would be termed a Barter Bill.

35 KINGSTON, Jan. 2nd, 1866.

JAMES THOMPSON, ESQ.,

*To* A. JARDINE & CO., *Dr.*

| | | |
|---|---|---|
| For 3 doz. Buttons | @ $0.12 | |
| " 5½ yards of black Broadcloth | @ 5.50 | |
| " 20 yards Sheeting | @ .15 | |
| " 1 chest Y. H. Tea, 83 lbs. | @ .95 | |
| " 18 yards French Print | @ .20 | |
| " 2 skeins of Silk Thread | @ .09 | |
| " 5 yards black Silk Velvet | @ 3.50 | |
| " 20 lbs. Loaf Sugar | @ .18 | |
| " 2 gallons Molasses | @ .40 | |
| " 1 bag of common Salt | @ 1.15 | |
| " 25 lbs. Rice | @ .09 | |
| " 3 sacks Coffee, 70 lbs. each | @ .12 | |
| CR. | | $166.74 |
| By Cash | | 50.00 |
| Balance due | | $116.74 |

36 ALGONQUIN, Jan. 15th, 1865.

W. FLEMING & CO.,

*Bought of* J. & A. WRIGHT,

| | | |
|---|---|---|
| 1500 lbs. Canadian Cheese | @ $ .09 | |
| 300 bushels Fall Wheat | @ 1.25 | |
| 9 brls Pot Ash, net 7056 lbs | @ 5.75 per cwt. | |
| 150 bushels Spring Wheat | @ 1.15 | |
| 200 " Potatoes | @ .45 | |
| 600 " Oats | @ .37½ | |
| 150 " Pease | @ .65 | |
| 50 " Indian Corn | @ .50 | |
| 60 " Apples | @ .60 | |
| 3 kegs Butter, 110 lbs. each | @ .18 | |
| 50 bushels Rye | @ .70 | |
| 40 " Barley | @ .80 | |
| | | $1688.12 |

Received payment,

J. & A. WRIGHT.

# PERCENTAGE.

**18.**—**PERCENTAGE** is an allowance, or reduction, or estimate of a certain portion of each 100 of the units that enter into any given calculation. The term is a contraction of the Latin expression for one hundred, and means literally *by the hundred.* In calculating dollars and cents, 6 per cent. means 6 dollars for every 100 dollars, or 6 cents for every $1, or 100 cents. If we are estimating the rate of yearly increase of the population of a rising village, and find that at the end of a certain year it was 100, and at the end of the next it was 106, we say it has increased 6 per cent. *i. e.*, 6 persons have been added to the 100. So, also, if a large city has a population of 100,000 at the end of a certain year, and it is found that it has 106,000 at the end of the following year, we say it has increased 6 per cent., which means that if we count the population by hundreds we shall find that for every 100 at the end of the one year, there are 106 at the end of the next; because one hundred thousands is the same as one thousand hundreds, and we have supposed the increase in every 100 to be 6, the total increase will be one thousand *sixes* or 6,000, giving a total population of 106,000 as above, or an increase at the rate of 6 per cent. A decrease would be estimated in the same manner. Thus, a falling off in the population of 6 persons in the hundred would be denoted by 100—6=94, as an increase of 6 in the hundred would be denoted by 100+6=106. So, also, in our first example, a deduction of $6 in $100 would be $100—6=$94, and a gain would be $100+$6=$106.

The portion of 100 so allowed or estimated, is called the *rate per cent.*, as in the examples given, 6 denotes the *rate per cent.*, or the allowance or estimate on every 100. Should the sum on which the estimate is made not reach 100, we can, nevertheless, estimate what is to be allowed on it at the same rate. Thus, if 6 is to be allowed for 100, then 3 must be allowed for 50, and 1½ for 25, &c.

The number on which the percentage is estimated is called the *basis.* Thus, in the example given regarding the population of a city, 100,000 is the basis.

When the basis and percentage are combined into *one*, the result is called the *amount.* If the rate per cent. be an *increase* or *gain*, it is to be added to the basis to get the amount, and if it is a *decrease*, or *loss*, it is to be subtracted from the basis to get the amount. This latter result is sometimes called the *remainder.*

From what has been said, it is plain that percentage is nothing else than taking 100 as a standard unit of measure—(See Art. 1)—and making the rate a fraction of that unit, so that 6 per cent. is $\frac{6}{100}$=(Art. 15, V.) .06. We may obtain the same result by the rule of proportion. Thus, in our illustrative example of an increase of 6 persons for every 100 on a population of 100,000, the analogy will be 100 persons : 100,000 persons : : 6 (the increase on 100) : 6,000, the increase on 100,000. It is manifest that the same result will be obtained whether we multiply the third by the second, and divide by the first, or whether we divide the third by the first, and multiply the result by the second; or, which is the same thing, multiply the second by the result. Now, we already found that $6 \div 100 = \frac{6}{100}$=.06, the same as before. So also, 7 per cent. of any loss is seven one-hundredths of it, *i. e.*, $\frac{7}{100}$=.07. It should be carefully observed that such decimals represent, *not the rate per cent.*, *but the rate per unit.*

Though this is easily comprehended, yet we know by experience that learners are constantly liable to commit errors by neglecting to place the decimal point correctly. We would therefore direct particular attention to the above caution, which, with the rule already laid down, under the head of decimal fractions, should be sufficient to guide any one who takes even moderate pains.

### EXERCISES ON FINDING THE RATE PER UNIT.

| | |
|---|---|
| At $\frac{1}{4}$ per cent., what is the rate per unit? | Ans. .00$\frac{1}{4}$. |
| At $\frac{1}{2}$ per cent., what is the rate per unit? | Ans. .00$\frac{1}{2}$. |
| At 1 per cent., what is the rate per unit? | Ans. .01. |
| At 2 per cent., what is the rate per unit? | Ans. .02. |
| At 4 per cent., what is the rate per unit? | Ans. .04. |
| At 7$\frac{1}{4}$ per cent., what is the rate per unit? | Ans. .07$\frac{1}{4}$. |
| At 10 per cent., what is the rate per unit? | Ans. .10. |
| At 12$\frac{1}{2}$ per cent., what is the rate per unit? | Ans. .12$\frac{1}{2}$. |
| At 17 per cent., what is the rate per unit? | Ans. .17. |
| At 25 per cent., what is the rate per unit? | Ans. .25. |
| At 33$\frac{1}{3}$ per cent., what is the rate per unit? | Ans. .33$\frac{1}{3}$. |
| At 66$\frac{2}{3}$ per cent., what is the rate per unit? | Ans. .66$\frac{2}{3}$. |
| At 75 per cent., what is the rate per unit? | Ans. .75. |
| At 100 per cent., what is the rate per unit? | Ans. 1.00. |
| At 112$\frac{1}{2}$ per cent., what is the rate per unit? | Ans. 1.12$\frac{1}{2}$. |
| At 150 per cent., what is the rate per unit? | Ans. 1.50. |
| At 200 per cent., what is the rate per unit? | Ans. 2.00. |

I. To find the percentage on any given quantity at a given rate :

On the principles of proportion, we have as 100 : given quantity : : rate : percentage, and as the third term, divided by the first, gives the rate per unit, we have the simple

### RULE:

*Multiply the given quantity by the rate per unit, and the product will be the percentage.*

### EXAMPLES.

To find how much 6 per cent. is on 720 bushels of wheat, we have $6 \div 100 = .06$, the rate per unit, and $720 \times .06 = 43\frac{1}{5}$ bushels, the percentage.

To find 8 per cent. of $7963-75, in like manner, we have .08, the rate per unit, and $7963.75 × .08 gives $637.10, the percentage.

Instead of *per cent* the mark (%) is now commonly used.

### EXERCISES ON THE RULE.

1. What does 6 per cent. of 450 tons of hay amount to? Ans. 27.

2. What is 10 per cent. of $879.62½? Ans. $87.96.

3. If 12 per cent. of an army of 47,800 men be lost in killed and wounded; how many remain? Ans. 42,064.

4. What is 5 per cent. of 187 bushels of potatoes? Ans. 9.35.

5. What is 2½ per cent. of a note for $870? Ans. 21.75.

6. Find 12½ per cent. of 97 hogsheads? Ans. 12.12½.

II. To find what rate per cent. one number is of another given number :—Let us take as an example, to find what per cent. 24 is of 96. Here the basis is 96, and we take 100 as a standard basis, and these are magnitudes of the same kind, and 24 is a certain rate on 96, and we wish to find what rate it is on 100, and by the rule of proportion, we have the statement $96 : 100 :: 24 : \text{F. P.} = \frac{2400}{96} = 25$. Therefore 24 is 25 per cent. of 96.

From this we can deduce the simple

### RULE.

*Annex two ciphers to the given percentage, and divide that by the basis, the quotient will be the rate per cent.*

7. What per cent. of 150 is 15? Ans. 10.

8. What per cent. of 240 is 36? Ans. 15.

9. What per cent. of 18 is 2? Ans. $11\frac{1}{9}$.

10. What per cent. of 72 is 48? Ans. $66\frac{2}{3}$.

11. What per cent. of 576 is 18? Ans. $3\frac{1}{8}$.

12. What per cent. is 12 of 480? Ans. $2\frac{1}{2}$.

13. Bought a block of buildings in King street for $1719, and sold it at a gain of 18 per cent.; what was the gain? Ans. $309.42.

14. Vested $325 in an oil well speculation, and lost 8 per cent.; what was the loss? Ans. $26.00.

15. In 1841 the population of Toronto was about 15,000, it is now about 50,000; what is the rate of increase? Ans. $233\frac{1}{3}$.

16. An estate worth $4,500 was sold; A bought 30 per cent. of it; B, 25 per cent.; C, 20 per cent.; and D purchased the remainder; what per cent. of the whole was D's share? Ans. 25.

17. If a man walk at the rate of 4 miles an hour; what per cent. is that of a journey of 32 miles? Ans. $12\frac{1}{2}$.

18. What is the percentage on $1370 at $2\frac{3}{4}$ per cent.? Ans. $37.67\frac{1}{2}$.

III. Given, a number, and the rate per cent. which it is of another number, to find that other number, 400 is 40 per cent. of a certain number, to find that number. As 40 : 100 : : 400 : F. P. $=\frac{400\times100}{40}=1{,}000$. Hence we derive the

### RULE.

*Annex two ciphers to the given number, and divide by the rate per cent.*

### EXERCISES.

1. A bankrupt can pay $2600, which is 80 per cent of his debts; how much does he owe? Ans. $3250.

2. A clerk pays $8 a month for rent, which is 16 per cent. of his salary; what is his yearly salary? Ans. $600.

3. In a manufacturing district in England, 40,000 persons died of cholera in 1832, this was 25 per cent. of the population; what was the population? Ans. 160,000.

4. Bought a certain number of bags of flour, and sold 124 of them, which is $12\frac{1}{2}$ per cent. of the whole. Required, the number of bags purchased. Ans. 992.

5. In a shipwreck 480 tons are lost, and this amount is 15 per cent. of the whole cargo. Find the cargo. Ans. 3200 tons.

6. A firm lost $1770 by the failure of another firm; the loss was 30 per cent. of their capital; what was their capital? Ans. $5900.

IV. To find the basis when the amount and rate are given:— Suppose a man buys a piece of land for a certain sum, and by selling it for $300, gains 25 per cent.; what did he pay for it at first?— Here it is plain that for every dollar of the cost, 25 cents are gained by the sale, *i. e.*, 125 cents for every 100, which gives us the analogy, 125 : 100 : : 300 : F. P.; or, dividing the two terms by 100, 1.25 : 1.00 : : 300 : F. P., which by the rules for the multiplication and division of decimals, gives $\frac{30000}{125}$=$240, the original cost.

Again, suppose the farm had been sold at a loss of 25 per cent. This being a loss, we subtract 25 from 100, and say, as 75 : 100 : : 300 : F. P.=$\frac{30000}{75}$=$400, the prime cost in this case.

Hence we derive the

### RULE.

*Divide the given amount by one increased or diminished by the given rate per unit, according as the question implies increase or decrease, gain or loss.*

### EXERCISES.

1. Given the amount $198, and the rate of increase 20 per cent. to find the number yielding that percentage. Ans. $165.

2. A field yields 840 bushels of wheat, which is 250 per cent. on the seed; how many bushels of seed were sown? Ans. 240 bushels.

3. At 5 per cent. gain; what is the basis if the amount be $126? Ans. $120.

4. At 10 per cent. loss; what is the basis, the amount being $328.5? Ans. $365.

5. A ship is sold for $12045, which is a gain of $\frac{3}{8}$ per cent. on the sum originally paid for it; for how much was it bought at first? Ans. $12000.

6. A gambler lost 10 per cent. of his money by a venture, and had $279 left; how much had he at first, and how much did he lose? Ans. He lost $31, and had $310 at first.

7. A grocer bought a lot of flour, and having lost 20 per cent. of the whole, had 160 bags remaining; how many bags did he buy? Ans. 200.

8. A merchant lost 12 per cent. of his capital by a bankruptcy, and had still $2200 left; what was his whole capital? Ans. $2500.

9. Sold a sheep for $5, and gained 25 per cent.; what did I pay for it? Ans. $4.

10. Lost $12000 on an investment, which was 30 per cent. of the whole; what was the investment? Ans. $40000.

## INTEREST.

From a transition common in language, the word interest has been inappropriately applied to the *sum paid* for the use of money, but its original and true meaning is simply the *use* of money. To illustrate this, we will suppose that A borrows of B $100 for one year, and at the end of the year, when A wishes to settle the account, he gives B $107. Were we to ask the question of almost any person except an accountant, whether A or B received the interest, we should undoubtedly receive for an answer that B received it. But such is not the case. A having had the *use* of that money for one year, paid B $7 for that *use* or *interest*; hence A received the *interterest* or *use* of that money, and B received $7 in cash for the same. It is only by considering this subject in its true light that accountants are able to determine upon the proper debits and credits that arise from a transaction where interest is involved. If an individual borrows money, he *receives* the use of that money, and when he pays for that use or interest, he places the sum so paid to that side of his "interest account" which represents interest received, and if he lends money, he *has parted with* the use of that money, and when he receives value for that use or interest, he places the sum so received to that side of his "interest account" which represents interest delivered.

We think that this explanation is sufficiently clear to illustrate the difference between *interest* and the *value received* or *paid* for it.

It will also be noticed that we have given many of the exercises in the usual form, *e. g.*, we say what is the interest on $100 for one year, instead of saying *what must be paid for the interest* of $100 for one year, but we have done this more in accordance with custom than from any intention to deviate from the true meaning of the word interest.

Interest is reckoned on a scale of so many units on every $100 for one year, and hence it is called so much *per cent. per annum*, from the Latin *per centum*, by the hundred, and *per annum*, by the year. Thus, $6 a year for every $100, is called *six per cent. per* *annum.* The term is also extended to designate the return accruing from any investment, such as shares in a joint stock company.

To show the object and use of such transactions, we may suppose a case or two.

A person feels himself cramped or embarrassed in his circumstances and operations, and he applies to some friendly party that lends him $100 for a year, on the condition that at the expiration of the year he is to receive $106, that is, the $100 lent, and $6 more as a return for the use of the $100; or, if the borrower gets $600, he pays at the end of the stipulated time not only the $600, but also $36 ($6 for each $100) in return for the use of the $600. By this means the borrower gets clear of his difficulty, and maintains his credit at a small sacrifice. Again, a merchant may find that there is an opportunity for a speculation by which a good sum may be realized, but he has not capital sufficient, and accordingly he borrows a sum sufficient for the purpose, and pays, say 6 per cent. for it. We shall now suppose that the speculation yields him 24 per cent., then it is plain that after paying 6 per cent. for the money borrowed, he is still a gainer of 18 per cent. on that money; that is, for every $100 that he borrowed he *clears* $18 We shall cite one case more of common occurrence. A mercantile house fails; another house is in danger of being involved in the disaster by having extensive transactions with the former, but by effecting a loan to meet present emergencies, maintains its credit, and goes on with the business. In such a case, a small sacrifice in the shape of interest is of no account compared with the damage of a failure, and so in numberless other cases.

The sum on which interest is paid is called the *principal.*

The sum paid for the use of money is called the *interest.*

The sum paid on each $100 is called the *rate.*

The sum of the principal and interest is called the *amount.*

When interest is charged on the principal only, it is called *simple interest.*

When interest is charged on the amount, it is called *compound interest.*

## SIMPLE INTEREST.

As simple interest, when calculated for one year, differs in no way from a percentage on a given sum, we have only four things to consider, viz., the principal, the rate, (100 being the basis,) the interest, and the time, any three of which being known, the fourth can be found. The finding of the interest includes by far the greatest number of cases.

We shall first show the general principle, and from it deduce an easy practical rule.

Let it be required to find the interest on $468 for one year, at 6 per cent.

As 100 is taken as the *basis principal* in relation to which all calculations are made, it is plain that 100 will have the same ratio to any given principal that the rate, which is the interest on 100, has to the interest on the given principal. Hence, in the question proposed, we have as $100 : $468 : : $6 : interest=$\$468\times\frac{6}{100}=$ $468×.06=$28.08. Now .06 is the rate *per unit*, and from this we can deduce rules for all cases.

### CASE I.

To find the interest of any sum of money for one year, at any given rate per cent.

### RULE.

*Multiply the principal by the rate per unit.*

### EXERCISES.

1. What is the interest on $15, for 1 year, at 3 per cent.?
Ans. $0.45.
2. What is the interest on $35, for 1 year, at 5 per cent.?
Ans. $1.75.
3. What is the interest on $100, for 1 year, at 7 per cent.?
Ans. $7.00.
4. What is the interest on $2.25, fer 1 year, at 8 per cent.?
Ans. $0.18.
5. What is the interest on $6.40, for 1 year, at 8½ per cent.?
Ans. $0.54.
6. What is the interest on $250, for 1 year, at 9½ per cent.?
Ans. $23.75.
7. What is the interest on $760.40, for 1 year, at 7½ per cent.?
Ans. $57.03.
8. What is the interest on $964.50, for 1 year, at 6½ per cent.?
Ans. $62.69.
9. What is the interest on $568.75, for 1 year, at 7¼ per cent.?
Ans. $41.23.

### CASE II.

To find the interest of any sum of money, for any number of years, at a given rate per cent.

### RULE.

*Find the interest for one year, and multiply by the number of years.*

### EXERCISES.

10. What is the interest of $4.60, for 3 years, at 6 per cent.? Ans. $0.83.

11. What is the interest of $570, for 5 years, at 7½ per cent.? Ans. $213.75.

12. What is the interest of $460.50, for 3 years, at 6¼ per cent.? Ans. $86.34.

13. What is the interest of $17.40, for 3 years, at 8⅓ per cent.? Ans. $4.35.

14. What is the interest of $321.05, for 8 years, at 5¾ per cent.? Ans. $147.68.

15. What is the interest of $1650.45, for 2 years, at 9 per cent.? Ans. $297.08.

16. What is the interest of $964.75, for 4 years, at 10 per cent.? Ans. $385.90.

17. What is the interest of $1674.50, for 3 years, at 10½ per cent.? Ans. $527.47.

18. What is the interest of $640.80, for 5 years, at 4¾ per cent.? Ans. $152.19.

19. What is the interest of $965.50, for 7 years, at 5½ per cent.? Ans. $371.72.

20. What is the interest of $2460.20, for 4 years, at 7 per cent.? Ans. $688.86.

### CASE III.

To find the interest on any sum of money for any number of months, at a given rate per cent.

### RULE.

*Find the interest for one year, and take aliquot parts for the months; or,*

*Find the interest for one year, divide by 12, and multiply by the number of months.*

### EXERCISES.

21. What is the interest on $684.20, for 4 months, at 6 per cent.? Ans. $13.68.

22. What is the interest on $760.50, for 5 months, at 7 per cent.? Ans. $22.18.

23. What is the interest on $899.99, for 2 months, at 8 per cent.? Ans. $12.00.

24. What is the interest on $964.50, for 4 months, at 9 per cent.? Ans. $28.94.

25. What is the interest on $1500, for 7 months, at 10 per cent.? Ans. $87.50.

26. What is the interest on $1560, for 11 months, at 7½ per cent.? Ans. $107.25.

27. What is the interest on $1575.54, for 8 months, at 6¼ per cent.? Ans. $65.65.

28. What is the interest on $1728.28, for 9 months, at 8½ per cent.? Ans. $110.18.

29. What is the interest on $268.25, for 13 months, at 7 per cent.? Ans. $20.34.

30. What is the interest on $1569.45, for 1 year, 3 months, at 8 per cent.? Ans. $156.95.

31. What is the interest on $642.99, for 1 year, 5 months, at 10 per cent.? Ans. $91.09.

32. What is the interest on $560.45, for 1 year, 6 months, at 9½ per cent.? Ans. $79.86.

33. What is the interest on $48.50, for 3 years, 9 months, at 10½ per cent.? Ans. $19.10.

34. What is the interest on $560.80, for 2 years, 8 months, at 11¾ per cent.? Ans. $175.72.

35. What is the interest on $2360.40, for 19 months, at 12 per cent.? Ans. $448.48.

### CASE IV.

To find the interest on any sum of money, for any number of months and days, at a given rate per cent.

### RULE.

*Find the interest for the months, and take aliquot parts for the days, reckoning the month as consisting of* 30 *days.*

### EXAMPLE.

36. What is the interest on $875.50, for 8 months, 18 days, at 11 per cent.?

### SOLUTION.

| | |
|---|---|
| Principal | $875.50 |
| Rate per unit | .11 |
| Interest for 1 year | 96.3050 |
| Interest for 6 months; or, $\frac{1}{2}$ of interest for 1 year | 48.1525 |
| Interest for 2 months; or, $\frac{1}{3}$ of interest for 6 months | 16.0508 |
| Interest for 15 days; or, $\frac{1}{4}$ of interest for 2 months | 4.0127 |
| Interest for 3 days; or, $\frac{1}{5}$ of interest for 15 days | .8025 |
| Interest for 8 months, 18 days | $69.0185 |

We find the interest for 1 year to be $96.305, and as 6 months are the $\frac{1}{2}$ of 1 year, the interest for 6 months will be the $\frac{1}{2}$ of the interest for 1 year; likewise the interest for 2 months will be the $\frac{1}{3}$ of the interest for 6 months, and as 15 days are the $\frac{1}{4}$ of 2 months or 60 days, the interest for 15 days will be the $\frac{1}{4}$ of the interest for 2 months, and likewise the interest for 3 days, will be the $\frac{1}{5}$ of the interest for 15 days. Adding the interest for the *months* and *days* together, we obtain $69.02, the sum to be paid for the use of $875.50, for 8 months, 18 days, at 11 per cent.

### EXERCISES.

37. What is the interest on $468.75, for 4 months, 15 days, at 7 per cent.? Ans. $12.30.

38. What is the interest on $1654.40, for 3 months, 8 days, at 5 per cent.? Ans. $22.52.

39. What is the interest on $345.65, for 11 months, 25 days, at 6 per cent.? Ans. $20.45.

40. What is the interest on $74.85, for 5 months, 22 days, at 9 per cent.? Ans. $3.22.

41. What is the interest on $673.75, for 8 months, 19 days, at 7½ per cent.? Ans. $36.35.

42. What is the interest on $57.45, for 1 year, 2 months, 12 days, at 6 per cent.? Ans. $4.14.

43. What is the interest on $2647, for 1 year, 5 months, 18 days, at 6¼ per cent.? Ans. $242.64.

44. What is the interest on $268.40, for 2 years, 1 month, 1 day, at 8 per cent.? Ans. $44.79.

45. What is the interest on $2345.50, for 3 years, 7 months, 20 days, at 10 per cent.? Ans. $853.50.

46. What is the interest on $4268.45, for 4 years, 11 months, 11 days, at 11¾ per cent.? Ans. $2481.24.

47. What is the interest of $642.20, for 2 years, 7 months, 24 days, at 12 per cent.? Ans. $201.65.

48. What is the interest of $64.50, for 2 years, 11 months, 2 days, at 7 per cent.? Ans. $13.19.

49. What is the amount of $746.25, for 1 year, 10 months, 12 days, at 5 per cent.? Ans. $815.90.

50. What is the interest of $680, for 4 years, 1 month, 15 days, at 6 per cent.? Ans. $168.30.

### CASE V.

To find the interest on any sum of money, for any number of days, at a given rate per cent.*

### RULE.

*Find the interest for one year, and say, as one year* (365 *days,*) *is to the given number of days, so is the interest for one year to the interest required; or,*

*Having found the interest for one year, multiply it by the given number of days, and divide by* 365.

### EXERCISES.

51. What is the interest on $464, for 15 days, at 6 per cent.? Ans. $1.14.

52. What is the interest on $364, for 12 days, at 7 per cent.? Ans. 84 cents.

53. What is the interest on $56.82, for 14 days, at 8 per cent.? Ans. 17 cents.

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* To find how many years elapse between any two dates, we have only to subtract the earlier from the later date. Thus, the number of years from 1814 to 1865 is 51 years. To find months, we must reckon from the given date in the first named month, to the same date in each successive month. Thus, five months from the 10th of March brings us on to the 10th of August. To find days, we require to count how many days each month contains, for to consider every month as consisting of 30 days, in the calculation of interest, is not strictly correct, although for portions of a single month it causes no serious error. Thus, the correct time from March 2nd to June 14th, would be 104 days, viz., 29 for March, 30 for April, 31 for May, and 14 for June. A very convenient plan for reckoning time between two given dates is to count the number of months and odd days that intervene. Thus, from June 14th to November 20th, would be 5 months and 6 days.

54. What is the interest on $75.50, for 18 days, at $8\frac{1}{2}$ per cent.?
Ans. 32 cents.

55. What is the interest on $125.25, for 20 days, at 5 per cent.?
Ans. 34 cents.

56. What is the interest on $150.40, for 33 days, at 6 per cent.?
Ans. 82 cents.

57. What is the interest on $56.48, for 45 days, at $6\frac{1}{2}$ per cent.?
Ans. 45 cents.

58. What is the interest on $75.75, for 65 days, at 7 per cent.?
Ans. 94 cents.

59. What is the interest on $268.40, for 70 days, at $7\frac{1}{2}$ per cent.?
Ans. $3.86.

60. What is the interest on $464.45. for 80 days, at 8 per cent.?
Ans. $8.14.

61. What is the interest on $15.84, for 120 days, at 9 per cent.?
Ans. 47 cents.

62. What is the interest on $240, for 135 days, at $9\frac{1}{2}$ per cent.?
Ans. $8.43.

63. What is the interest on $2460, for 145 days, at 10 per cent.?
Ans. $97.73.

64. What is the interest on $1568, for 170 days, at 11 per cent.?
Ans. $80.33.

65. What is the interest on $2688, for 235 days, at $11\frac{3}{4}$ per cent.?
Ans. $203.35.

66. What is the amount of $364.80, for 320 days, at $11\frac{1}{2}$ per cent.?
Ans. $401.58.

### CASE VI.

To find the interest on any sum of money, for any time, at 6 per cent.

Since .06 would be the rate per unit, or the interest of $1 for 1 year, it follows that the interest for *one month* would be the $\frac{1}{12}$ of .06, or $\frac{6}{12}$ of a cent, equal to $\frac{1}{2}$ cent or .005, and for 2 months it would equal $\frac{1}{2}$ cent, or $.005 \times 2 = .01$. Therefore, when interest is at the rate of 6 per cent., the interest of $1, for every 2 months, is *one cent.* Again, if the interest of $1, for *one month*, or 30 days, is $\frac{1}{2}$ cent or .005, it follows that the interest for 6 days will be the $\frac{1}{5}$ of .005 or .001. Therefore, when interest is at the rate of 6 per cent., the interest of $1 for every 6 days is *one mill.* Hence the

### RULE.

*Find the interest of* $1 *for the given time by reckoning* 6 *cents for every year,* 1 *cent for every* 2 *months, and* 1 *mill for every* 6 *days; then multiply the given principal by the number denoting that interest, and the product will be the interest required.*

NOTE.—This method can be adopted for any rate per cent. by first finding the interest at 6 per cent., then adding to, or subtracting from the interest so found, such a part or parts of it, as the given rate exceeds, or is less than 6 per cent.

This method, although adopted by some, is not exactly correct as the year is considered as consisting of 360 days, instead of 365 ; so that the interest, obtained in this manner, is too large by $\frac{5}{365}$ or $\frac{1}{73}$, which for every $73 interest, is $1 too much, and must therefore be subtracted if the exact amount be required.

### EXAMPLE.

67. What is the interest of $24, for 4 months, 8 days, at 6 per cent. ?

### SOLUTION.

The interest of $1, for 4 months, is.............................. .02
The interest of $1, for 8 days, is.................................. .001$\frac{1}{3}$

Hence the interest of $1, for 4 months, 8 days, is.......... .021$\frac{1}{3}$

Now, if the interest of $1, for the given time, is .021$\frac{1}{3}$, the interest of $24 will be 24 times .021$\frac{1}{3}$, which is $.512.

### EXERCISES.

68. What is the interest on $171, for 24 days, at 6 per cent. ?
Ans. 68 cents.

69. What is the interest on $112, for 118 days, at 6 per cent. ?
Ans. $2.20.

70. What is the interest on $11, for 112 days, at 6 per cent. ?
Ans. 21 cents.

71. What is the interest on 50 cents, for 360 days, at 6 per cent. ?
Ans. 3 cents.

72. What is the interest on $75.00, for 236 days, at 6 per cent. ?
Ans. $2.95.

73. What is the interest on $111.50, for 54 days, at 6 per cent. ?
Ans. $1.00.

74. What is the interest on $15.50, for 314 days, at 6 per cent. ?
Ans. 81 cents.

75. What is the interest on $174.25, for 42 days, at 6 per cent.? Ans. $1.22.

76. What is the interest on $10, for 1 month, 18 days, at 6 per cent. Ans. 8 cents.

77. What is the interest on $154, for 3 months, at 6 per cent.? Ans. $2.31.

78. What is the interest on $172, for 2 months, 15 days, at 6 per cent.? Ans. $2.15.

79. What is the interest on $25, for 4 months, at 6 per cent.? Ans. 50 cents.

80. What is the interest on $36, for 1 year, 3 months, 11 days, at 7 per cent.? Ans. $3.23.

81. What is the interest on $500, for 160 days, at 6 per cent.? Ans. $13.33.

82. What is the interest on $92.30, for 78 days, at 5 per cent.? Ans. $1.00.

83. What is the interest on $125, for 3 years, 5 months, 15 days, at 10 per cent. Ans. $43.23.

84. What is the amount of $200, for 9 months, 27 days, at 6 per cent.? Ans. $209.90.

85. What is the interest on $125.75, for 5 months, 17 days, at 7 per cent.? Ans. $4.08.

86. What is the interest on $84.50, for 1 month, 20 days, at 5 per cent.? Ans. 59 cents.

87. What is the amount of $45, for 1 year, 1 month, 1 day, at 8 per cent.? Ans. $48.91.

88. What is the interest on $175, for 7 months, 6 days, at 5½ per cent.? Ans. $5.78.

89. What is the interest on $225, for 3 months, 3 days, at 9 per cent.? Ans. $5.23.

90. What is the interest on $212.60, for 9 months, 8 days, at 8½ per cent.? Ans. $13.95.

### CASE VII.

**To find the interest on any sum of money, in pounds, shillings, and pence, for any time, at a given rate per cent.**

### RULE.

*Multiply the principal by the rate per cent., and divide by* 100.

### EXAMPLE.

91. What is the interest of £47 15s. 9d., for 1 year, 9 months, 15 days, at 6 per cent.?

### SOLUTION.

| | £ | s. | d. |
|---|---|---|---|
| Interest for 1 year...... | 2 | 17 | 4 |
| Interest for 6 mos., or ½ of int. for 1 year, | 1 | 8 | 8 |
| Interest for 3 mos., or ½ of int. for 6 mos., | 0 | 14 | 4 |
| Interest for 15 days, or ⅙ of int. for 3 mos., | 0 | 2 | 4½ |
| Interest for 1 year, 9 months, 15 days....£5 | | 2 | 8½ |

| £ | s. | d. |
|---|---|---|
| 47 | 15 | 9 |
| | | 6 |
| 2)86 | 14 | 6 |
| 20 | | |
| 17)34 | | |
| 12 | | |
| 4)14 | | |

92. What is the interest of £25, for 1 year, 9 months, at 5 per cent.? Ans. £2 3s. 9d.

93. What is the interest of £75 12s. 6d., for 7 months, 12 days, at 8 per cent.? Ans. £3 14s. 7½d.

94. What is the amount of £64 10s. 3d., for 3 months, 3 days, at 7 per cent.? Ans. £65 13s. 7d.

95. What is the interest of £35 4s. 8d., for 6 months, at 10 per cent.? Ans. £1 15s. 2¾d.

96. What is the amount of £18 12s., for 10 months and 3 days, at 6 per cent.? Ans. £19 10s. 9¾d.

### CASE VIII.

To find the PRINCIPAL, the interest, the time, and the rate per cent. being given.

### EXAMPLE.

97. What principal will produce $4.50 interest in 1 year, 3 months, at 6 per cent.?

### SOLUTION.

If a principal of $1 is put on interest for 1 year, 3 months, at 6 per cent., it will produce .075 interest. Now, if in this example, .075 be the interest on $1, the number of dollars required to produce $4.50, will be represented by the number of times that .075 is contained in $4.50, which is 60 times. Therefore, $60 will produce $4.50 interest in 1 year, 3 months, at 6 per cent. Hence the

### RULE.

*Divide the given interest by the interest of* $1 *for the given time, at the given rate per cent.*

### EXERCISES.

98. What principal will produce 77 cents interest in 3 months, 9 days, at 7 per cent.? Ans. $40.

99. What principal will produce $10.71 interest in 8 months, 12 days, at 7½ per cent.? Ans. $204.

100. What principal will produce $31.50 interest in 4 years, at 3½ per cent.? Ans. $225.

101. What sum of money will produce $79.30 interest in 2 years, 6 months, 15 days, at 6½ per cent.? Ans. $480.

102. What sum of money is sufficient to produce $290 interest in 2 years and 6 months, at 7¼ per cent.? Ans. $1600.

## CASE IX.

To find the RATE PER CENT., the principal, the interest, and the time being given.

### EXAMPLE.

103: If $3 be the interest of $60 for 1 year, what is the rate per cent.?

### SOLUTION.

If the interest of $60 for 1 year, at 1 per cent, is .60, the required *rate per cent.* will be represented by the number of times that .60 is contained in 3.00, which is 5 times. Therefore, if $3 is the interest of $60 for 1 year, the rate per cent. is 5. Hence the

### RULE.

*Divide the given interest by the interest of the given principal at* 1 *per cent. for the given time.*

### EXERCISES.

104. If the interest of $40, for 2 years, 9 months, 12 days, is $13.36; what is the rate per cent.? Ans. 12.

105. If I borrow $75 for 2 months, and pay $1 interest; what is the rate per cent.? Ans. 8.

106. If I give $2.25 for the use of $30 for 9 months; what rate per cent. am I paying? Ans. 10.

107. At what rate per cent. will $150 amount to $165.75, in 1 year, 4 months, 24 days? Ans. 7½.

108. At what rate per cent. must $1, or any sum of money, be on interest to double itself in 12 years? Ans. Ans. 8⅓.

109. At what rate per cent. must $425 be lent to gain $11.73 in 3 months, 18 days? Ans. 9⅕.

110. At what rate per cent. will any sum of money amount to three times itself in 25 years? Ans. 8.

111. If I give $14 for the interest of $125 for 1 year, 7 months, 6 days; what rate per cent am I paying? Ans. 7.

### CASE X.

To find the TIME, the principal, the interest, and the rate per cent. being given.

### EXAMPLE.

112. How long must $75 be at interest, at 8 per cent., to gain $12?

### SOLUTION.

The interest for $75, for 1 year, at 8 per cent., is $6. Now, if $75 require to be on interest for 1 year to produce $6, it is evident that the number of years required to produce $12 interest, will be represented by the number of times that 6 is contained in 12, which is 2. Therefore, $75 will have to be at interest for 2 *years* to gain $12. Hence the

### RULE.

*Divide the given interest by the interest of the principal for one year, at the given rate per cent.*

### EXERCISES.

113. In what time will $12 produce $2.88 interest, at 8 per cent? Ans. 3 years.

114. In what time will $25 produce 50 cents interest, at 6 per cent.? Ans. 4 months.

115. In what time will $40 produce 75 cents interest, at 6¼ per cent.? Ans. 3 months, 18 days.

116. In what time will any sum of money double itself, at 6 per cent.? Ans. 16 years, 8 months.

117. In what time will any sum of money quadruple itself, at 9 per cent.? Ans. 33 years, 4 months.

118. In what time will $125 amount to $138.75, at 8 per cent.? Ans. 1 year, 4 months, 15 days.

119. Borrowed, January 1, 1865, $60, at 6 per cent, to be paid as soon as the interest amounted to one-half the principal. When is it due? Ans. May 1, 1873.

120. A merchant borrowed a certain sum of money on January 2, 1856, at 9 per cent., agreeing to settle the account when the interest equalled the principal. When should he pay the same? Ans. Feb. 11, 1867.

### MERCHANTS' TABLE

*For showing in what time any sum of money will double itself, at any rate per cent., from one to twenty, simple interest.*

| Per cent. | Years. | Per cent. | Years. | Per cent. | Years. | Per cent. | Years. |
|---|---|---|---|---|---|---|---|
| 1 | 100 | 6 | $16\frac{2}{3}$ | 11 | $9\frac{1}{11}$ | 16 | $6\frac{1}{4}$ |
| 2 | 50 | 7 | $14\frac{2}{7}$ | 12 | $7\frac{1}{2}$ | 17 | $5\frac{15}{17}$ |
| 3 | $33\frac{1}{3}$ | 8 | $12\frac{1}{2}$ | 13 | $7\frac{9}{13}$ | 18 | $5\frac{5}{9}$ |
| 4 | 25 | 9 | $11\frac{1}{9}$ | 14 | $7\frac{1}{7}$ | 19 | $5\frac{5}{19}$ |
| 5 | 20 | 10 | 10 | 15 | $6\frac{2}{3}$ | 20 | 5 |

### MIXED EXERCISES.

121. What is the interest on $64.25 for 3 years, at 7 per cent.? Ans. $13.49.

122. What is the interest on $125.40 for 6 months, at 6 per cent.?* Ans. 3.76.

123. What is the amount of $369.29 for 2 years, 3 months, 1 day, at 9 per cent.? Ans. $444.16.

124. What must be paid for the use of 75 cents for 6 years, 9 months, 3 days, at 10 per cent.? Ans. 51 cents.

125. What will $54 amount to in 254 days, at 10 per cent.?* Ans. $57.81.

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* This and the following exercises (marked with a *) are to be worked by Case VI.

126. What must be paid for the interest of $45 for 72 days, at 9 per cent.?* Ans. 81 cents.

127. What is the interest of $240 from January 1, 1866, to June 4, 1866, at 7 per cent.? Ans. $7.14.

128. What will $140.40 amount to from August 29, 1865, to November 29, 1866, at 6½ per cent.? Ans. $151.83.

129. What principal will give $4.40 interest in 1 year, 4 months, 15 days, at 8 per cent.? Ans. $40.

130. In what time will $40 amount to $44..40, at 8 per cent.? Ans., 1 yr., 4 mos., 15 days.

131. At what rate per cent. will $40 produce in 1 yr., 4 mos., 15 days, $4.40 interest? Ans. 8.

132. What must be paid for the interest of $145.50 for 240 days, at 9½ per cent.?* Ans. $9.22.

133. What will $160 amount to in 175 days, at 6 per cent.?* Ans. $164.67.

134. At what rate per cent. must any sum of money be on interest to quadruple itself in 33 years and 4 months? Ans. 9.

135. In what time will any sum of money double itself, at 10 per cent.? Ans. 10 years.

136. What is the interest of $30 for 30 days, at 6 per cent.?* Ans. 15 cents.

137. What is the interest on $460 from January 2, 1866, to November 15, 1866, at 7½ per cent.?* Ans. $30.

138. What will $25 amount to from December 24, 1865, to January 1, 1867, at 6 per cent.? Ans. $26.53.

139. What sum of money will amount to $6400 in 4 years and 8 months, at 6 per cent.? Ans. $5000.

140. In what time will $480, at 4½ per cent., produce $81.60 interest? Ans. 3 years, 9 months, 10 days.

## NEGOTIABLE INSTRUMENTS.

Bills of Exchange and Promissory Notes constitute, in their different shapes, the commercial medium for the payment of money.

A *Bill of Exchange* is a written order for the payment of a certain sum of money unconditionally.

A *Promissory Note* is a written promise to pay a certain sum of money unconditionally.

*Cheques* are only a description of Bills of Exchange. A Cheque is a written order addressed to a bank, or banker, and directing them to pay on presentment, to a person named in the cheque (or bearer, or order), a certain sum of money.

*Bank Notes* are simple promissory notes, payable on demand.

### FORM AND REQUISITES.

A bill or note is called *inland*, when both made and payable in one country; and *foreign*, when made or payable abroad.

The following is a form of an inland bill:

$500. TORONTO, January 1, 1866.

*At sight, (or on demand, or at ——— days after sight, or at ——— days after date), pay to Messrs. A. B. Smith & Co., or order, (or bearer), Five Hundred Dollars.*

MUSGROVE & WRIGHT.

To Messrs. JONES & BROWN, Montreal.

The following may be taken as an example of a promissory note:

$500. TORONTO, January 1, 1866.

*Three months after date we promise to pay to Messrs. A. B. Smith & Co., or order, Five Hundred Dollars, value received.*

MUSGROVE & WRIGHT.

A bill of exchange amounts to an undertaking on the part of the drawer to the payee, and every subsequent holder, that the drawee will accept the bill when requested, and pay it when it becomes due.

The maker of a promissory note undertakes to the payee, and every subsequent holder, that he will pay the note when due.

All bills and notes made, drawn or accepted in Canada, are subject to a duty, collected by means of stamps. On every such instrument, if the amount do not exceed $25, a duty of *one cent* is imposed. From $25 up to $50, the duty is *two cents;* from $50 to $100 it is *three cents;* and after that, it is *three cents* for every hundred dollars, or fraction of a hundred dollars. In the case of foreign bills, in sets of two, the duty is *two cents* for each of the set, on the $100; in sets of three or more, the duty is only *one cent* for each of the set on the $100.

Foreign bills are usually drawn in *sets*; that is, copies of the bills are made on separate pieces of paper, each part containing a condition that it shall continue payable only so long as the others remain unpaid. The object of this is to diminish the chances of losing a bill; for if one part should fail to reach its destination, one of the others would be likely to do so.

### PARTIES TO BILLS AND NOTES.

The person who draws a bill of exchange is called the *drawer;* he, to whom it is addressed, the *drawee*, and, when he accepts it, the *acceptor*; he in whose favor it is made, the *payee.*

The person who signs a promissory note is called the *maker;* the person to whom the promise is made, the *payee.*

The moment a promissory note is *indorsed* by the payee, by his writing his name on the back of it, he is called the *indorser.* The person to whom it is indorsed is the *indorsee.*

When a bill is accepted, or a note is made by several persons who are not in Partnership, the question whether they are bound jointly, or jointly and separately, depends upon the wording of the document. If a note begin thus:—" I promise," and be signed by several persons, it is several, as well as joint.

The name of the maker or drawer must be inserted or subscribed by himself, or his agent. There must be no uncertainty about the maker or drawer. For example, a note may not be signed "John Smith, *or else* Robert Jones."

### TRANSFER OF BILLS AND NOTES.

A bill or note may be payable to a particular individual, or to a particular individual or his *order*, or generally to *bearer.* When a bill or note is made payable simply to an individual, it is not negotiable. If it be payable to an individual or *order*, he may transfer his right to another, by endorsing his name upon it. If it be payable to an individual or *bearer*, it may be transferred by mere delivery, without any indorsement. If a blank be left for the payee's name, any *bona fide* holder may insert his own.

An indorsement is said to be *in blank* when it does not mention the name of the party in whose favor it is made. A bill or note, when indorsed in blank, is transferable by delivery.

A full or special indorsement is one which mentions the name of the party in whose favor it is made; and it has to be endorsed in blank by the latter before it can be rendered negotiable.

*A restrictive* indorsement puts an end entirely to any further negotiability of a note or bill. For instance, if a note be indorsed thus:—" Pay the contents to John Smith only,"—it would be no longer transferable.

Negotiable paper in the hands of an innocent holder, without notice of anything wrong, is good, although the person from whom he obtained it may have come by it as a thief or finder. There is an exception to this when a person takes a bill or note *after* it is due. He then is in no better position than the person from whom it was received, and could not recover if it had been fraudulently obtained by the latter.

## PRESENTMENT AND ACCEPTANCE.

A bill should be always presented for acceptance; and bills and notes for payment, when payable at or after sight, or at some particular place. A note payable on demand need not be presented in order to charge the maker. Presentment for payment must be made at a reasonable hour of the day upon which an instrument becomes payable. An acceptance of a bill may be made without any particular form of words, and even without the signature of the acceptor. An acceptance may be conditional, and in that case is not of any force until the condition is performed. When a bill or note specifies a particular place for payment, it should be presented there; although the maker or acceptor would not be discharged by a failure to present there, unless the words "and not otherwise or elsewhere" had been added.

Presentment for acceptance must be made within a reasonable time. No delay upon the part of the holder should extend over any period not rendered clearly necessary by the circumstances of the case.

## PROTEST AND NOTICE.

When a bill or note is dishonoured, either by non-acceptance or non-payment, it should be protested by a notary public. Notice of such protest should be sent to every party on the instrument to whom the holder desires to have recourse. Notice means more than mere knowledge, and must be a formal communication of the dishonour of the bill. This notice of protest may be given by being deposited in the post office nearest to the place of presentment during the day the protest is made, or the next lawful day. From the date of protest interest commences to run upon the instrument, although there are no words in it about interest. Besides interest, and the the expenses of protest and notice, the holder is entitled, when foreign bills are protested, to damages varying from four to ten per cent. of the principal amount specified in the bill.

### DAYS OF GRACE.

The time limited by the words of a bill or note for its payment is extended in this country by *days of grace;* so that a bill is not really payable till three days after the day upon which it purports to fall due. When a bill is made payable so many days after the the happening of a particular event, for instance, after sight, the day on which that event happens is excluded. Accordingly, a bill payable ten days after sight, and presented on the 1st August, would purport to be due on the 11th, but adding the days of grace, would in reality not be payable till the 14th.

## PARTIAL PAYMENTS.

Partial payments, as the term indicates, are the part payments of promissory notes, bonds, or other obligations.

When these payments are made the creditor specifies in writing, on the *back* of the note, or other instrument, the sum paid, and the time when it is paid, and acknowledges it by signing his name.

The method we generally adopt in Canada for casting interest upon bonds, notes, or other obligations, upon which partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the time when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal.

### RULE:

*Find the amount of the principal to the time of the first payment; subtract the payment from the amount, and then find the amount of the remainder to the time of the second payment; deduct the payment as before; and so on to the time of settlement.*

*But if any payment is less than the interest then due, find the amount of the sum due to the time when the payments, added together, shall be equal, at least, to the interest already due; then find the balance, and proceed as before.*

### EXAMPLE.

1. On the 4th of January, 1865, a note was given for $800, payable on demand, with interest at 6 per cent. The following payments were receipted on the back of the note:

| | | |
|---|---|---|
| February 7th, 1865, | received......... | $150 |
| April 16th, " | " ......... | 100 |
| Sept., 30th, " | " ......... | 180 |
| January 4th, 1866. | " ...... .. | 170 |
| March 24th, " | " ......... | 100 |
| June 12th, " | " ......... | 50 |

Settled July 1st, 1867. How much was due?

### SOLUTION:

| | |
|---|---|
| Face of the note, or principal | $800.00 |
| Interest on the same to February 7th, 1865 (1 month, 3 days) | 4.40 |
| Amount due at time of 1st payment | 804.40 |
| First payment to be taken from this amount | 150.00 |
| Balance remaining due February 7th, 1865 | 654.40 |
| Interest on the same from February 7th, 1865, to April 16th, 1865 | 7.525 |
| Amount due at time of 2nd payment | 661.925 |
| Second payment to be taken from this amount | 100.000 |
| Balance remaining due April 16th, 1865 | 561.925 |
| Interest on the same from April 16th, 1865, to September 30th, 1865 | 15.359 |
| Amount due at time of 3rd payment | 577.284 |
| Third payment to be taken from this amount | 180.000 |
| Balance remaining due Sept. 30th, 1865 | 397.284 |
| Interest on the same from Sept. 30th, 1865, to January 4th, 1866 | 6.290 |
| Amount due at time of 4th payment | 403.574 |
| Fourth payment to be taken from this amount | 170.000 |
| Balance remaining due January 4th, 1866 | 233.574 |

| | |
|---|---|
| Interest on the same from Jan. 4th, 1866, to March 24th, 1866 | 3.114 |
| Amount due at time of 5th payment | 236.688 |
| Fifth payment to be taken from this amount | 100.000 |
| Balance remaining due, March 24th, 1866 | 136.688 |
| Interest on the same from March 24th, 1866, to June 12th, 1866 | 1.799 |
| Amount due at time of 6th payment | 138.487 |
| Sixth payment to be taken from this amount | 50.000 |
| Balance remaining due June 12th, 1866 | 88.487 |
| Interest on the same from June 12th, 1866, to July 1st, 1867 | 5.589 |
| Amount due on settlement | 94.076 |

2. $1600. PERTH, C. W., February 16th, 1865.

*On demand, I promise to pay Jacob Anderson, or order, one thousand six hundred dollars, with interest, at 7 per cent.*

JOHN FORTUNE JR.

There was paid on this note,

| | |
|---|---|
| April 19th, 1865 | $460 |
| July 22nd " | 150 |
| August 25th, 1866 | 50 |
| Sept. 12th, " | 100 |
| Dec. 24th. " | 700 |

How much was due December 31st, 1866 ?

### SOLUTION.

| | |
|---|---|
| Face of the note or principal | $1600.00 |
| Interest on the same from Feb. 16th, 1865, to April 19th, 1865 | 19.60 |
| Amount due at time of 1st payment | 1619.60 |
| First payment to be taken from this amount | 460.00 |
| Balance remaining due, April 19th. 1865 | 1159.60 |

| | |
|---|---|
| Interest on the same from April 19th, 1865, to July 22nd, 1865 | 20.969 |
| Amount due at time of 2nd payment | 1180.569 |
| Second payment to be taken from this amount | 150.000 |
| Balance remaining due, July 22nd, 1865 | 1030.569 |
| Interest on the same from July 22nd, 1865, to Aug. 25th, 1866, less than 3rd payment,[*] | |
| Interest on the same from July 22nd, 1865, to Sept. 12th, 1866 | 82.359 |
| Amount due at time of 4th payment | 1112.928 |
| Third and fourth payments to be taken from this amount, | 150.000 |
| Balance remaining due Sept. 12th, 1866 | 962.928 |
| Interest on the same from Sept. 12th, 1866, to Dec. 24th, 1866 | 19.098 |
| Amount due at time of last payment | 982.026 |
| Last payment to be taken from this amount | 700.000 |
| Balance remaining due Dec. 24th, 1866 | 282.026 |
| Interest on the same from Dec. 24th, 1866, to Dec. 31st, 1866 | .382 |
| Amount due at time of settlement, Dec. 31st, 1866 | $282.408 |

3. $350. Ottawa, May 1st, 1864.

*On demand I promise to pay William Brown, or order, three hundred and fifty dollars, with interest, at 6 per cent.*

James Weston.

There was paid on this note,

| | |
|---|---|
| December 25th, 1864 | $50 |
| June 30th, 1865 | 5 |

---

[*] The interest on $1030.569, from July 22nd, 1865, to August 25th, 1866, is $78.752, and the payment made at this date, is only $50, not enough to pay the interest, so if we proceeded, as in the former case, to add the interest to the principal, and subtract the payment from the amount obtained, we would be taking interest, until the next payment, on the excess of the interest, $78.752, over the payment, $50, which would be in effect interest upon interest. or compound interest which the law does not allow.

| | |
|---|---|
| August 22nd, 1866 | 15 |
| June 4th, 1867 | 100 |

How much was due April 5th, 1868? Ans. $251.67.

4. $609.65. Brantford, June 8th, 1861.

*Six months after date, we jointly and severally promise to pay John Anderson, or order, six hundred and nine $\frac{65}{100}$ dollars, at the Royal Canadian Bank in Toronto, with interest at 6 per cent. after maturity.*

Samuel Graham.
T. B. Bearman.

There was paid on this note,

| | |
|---|---|
| October 4th, 1862 | $25.00 |
| March 15th, 1863 | 16.25 |
| August 24th, 1864 | 36.56 |

What was due December 19th, 1865? Ans. 679.27.

5. $874.95. Kingston, May 9th, 1863.

*Three months after date, I promise to pay Harmon Cummings, or order, eight hundred and seventy-four $\frac{95}{100}$ dollars, with interest after maturity at 6 per cent.*

Thomas Goodpay.

There was paid on this note,

| | |
|---|---|
| April 12th, 1864 | $56.30 |
| July 14th, 1865 | 24.80 |
| Sept. 18th, 1866 | 240.60 |

What was due February 9th, 1868? Ans. $773.07.

When the interest accruing on a note is to be paid annually adopt the following

### RULE.[*]

*Compute the interest on the principal to the time of settlement, and on each year's interest after it is due, then add the sum of the*

---

[*] When notes, bonds, or other obligations, are given, "with interest payable annually," the interest is due at the end of each year, and may be collected, but if not collected at that time, the interest due draws only *simple interest*, and the original principal must not be increased by any addition of yearly interest. If nothing has been paid until maturity on a note drawing annual interest, the amount due consists of the principal, the total annual interest, or the simple interest, and the simple interest on each item of annual interest from the time it became due until paid.

*interests on the annual interests to the amount of the principal, and from this amount take the payments, and the interest on each, from the time they were paid to the time of settlement, the remainder will be the amount due.*

6. $500. PRESCOTT, May 1st, 1864.

*One year after date, for value received, I promise to pay Musgrove & Wright, or order, Five Hundred Dollars, at their office, in the city of Toronto, with interest at 6 per cent., payable annually.*

JAMES MANNING.

There was paid on this note:

| | |
|---|---|
| May 4th, 1865 | $150 |
| Dec. 18th, " | 300 |

How much was due June 1st, 1866?

SOLUTION.

| | | | |
|---|---|---|---|
| Face of note, or principal | | | $500.00 |
| Interest on the same from May 1st 1864, to June 1st, 1866 | | | 62.50 |
| Amount of the principal at time of settlement | | | 562.50 |
| First year's interest on principal | $30 | | |
| Interest on the same from May 1st, 1865, to June 1st, 1866 | | $1.95 | |
| Second year's interest on principal | $30 | | |
| Interest on the same from May 1st, 1866, to June 1st, 1866 | | .15 | |
| Amount of interest upon *annual interest* | | | 2.10 |
| Total amount of principal | | | $564.60 |
| First payment, May 4th, 1865 | | $150.00 | |
| Interest on the same from May 4th, 1865, to June 1st, 1866 | | 9.70 | |
| Second payment, December 18th, 1865 | | 300.00 | |
| Interest on the same from December 18th, 1865, to June 1st, 1866 | | 8.20 | |
| Payments and interest on the same | | | 467.90 |
| Amount due June 1st, 1866 | | | $96.70 |

7. $700. BELLEVILLE, January 2nd, 1863.

*Eighteen months after date, I promise to pay to the order of H. C. Wright, Seven Hundred Dollars, for value received, with interest at 6 per cent., payable annually.*

THOS. A. BRYCE.

There was paid on this note:

| | |
|---|---|
| January 15th, 1864.................... | $350 |
| July 2nd, 1864........................... | 300 |

What amount was due January 2nd, 1865? Ans. $107.22.

8. $950. DUNDAS, C. W., Jan. 3rd, 1863.

*Two years after date, I promise to pay T. C. Musgrove, or order, Nine Hundred and Fifty Dollars, with interest at 9 per cent., payable annually, value received.*

JAMES S. PARMENTER.

The following payments were receipted on the back of this note:

| | |
|---|---|
| February 1st, 1864, received......... | $500 |
| May 14th, " " ........ | 100 |
| January 12th, 1865, " ......... | 300 |

What was due May 6th, 1865? Ans. 188.94.

9. $250. OSHAWA, January 2nd, 1863.

*Three years from date, for value received, I promise to pay Michael Wright, or order, Two Hundred and Fifty Dollars, with interest, payable annually, at 6 per cent.*

CALVIN W. PEARSONS,

*At Bank of Montreal, Brockville.*

What was the amount of this note at maturity? Ans. $297.70.

### CONNECTICUT RULE.

The Supreme Court of the State of Connecticut has adopted the following

### RULE.

*Compute the interest on the principal to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above, and in like manner from one payment to another, till all the* *payments are absorbed, provided the time between one payment and another be one year or more.*

*If any payments be made before one year's interest has accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest, added as above.*

*If any payments be made, of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.*

NOTE.—If a year extends beyond the time of *settlement*, find the amount of the remaining principal to the time of *settlement*; find also the amount of the payment or payments, if any, from the time they were paid to the time of settlement, and subtract their sum from the amount of the principal.

### EXAMPLE.

10. $900. TORONTO, June 1st, 1862.

*On demand we promise to pay J. R. Smith & Co., or order, nine hundred dollars, for value received, with interest from date, at 6 per cent.*

MUSGROVE & WRIGHT.

On the back of this note were receipted the following payments:

| | | |
|---|---|---|
| June 16th, 1863, | received | $200 |
| August 1st, 1864, | “ | 160 |
| Nov. 16th, 1864, | “ | 75 |
| Feby. 1st, 1866 | “ | 220 |

What amount was due August 1st, 1866?

### SOLUTION.

| | |
|---|---|
| Face of note or principal | $900.00 |
| Interest on the same from June 1st, 1862, to June 16th, 1863 | 56.25 |
| Amount of principal and interest, June 16th, 1863 | 956.25 |
| First payment to be taken from this amount | 200.00 |
| Balance due | 756.25 |
| Interest on the same from June 16th, 1863, to August 1st, 1864 | 51.046 |
| Amount due August 1st, 1864 | 807.296 |

| | |
|---|---|
| Second payment to be taken from this amount............ | 160.000 |
| Balance due............................................ | 647.296 |
| Interest on the same for *one year*.......................... | 38.837 |
| Amount due August 1st, 1865 ........................... | 686.133 |
| Amount of 3rd payment from Nov. 16th, 1864, to August 1st, 1865..................................... | 78.187 |
| Balance due............................................ | 607.946 |
| Interest on the same from August 1st, 1865, to August 1st, 1866.................................... | 36.476 |
| Amount due August 1st, 1866............................ | 644.422 |
| Amount of 4th payment from February 1st, 1866, to August 1st, 1866............................. | 226.600 |
| Balance due August 1st, 1866............................. | $417.822 |

### MERCHANTS' RULE.

It is customary among merchants and others, when partial payments of notes or other debts are made, when the note or debt is settled within a year after becoming due, to adopt the following

### RULE.

*Find the amount of the principal from the time it became due until the time of settlement. Then find the amount of each payment from the time it was paid until settlement, and subtract their sum from the amount of the principal.*

### EXAMPLE.

11. $400. MAITLAND, January 1st, 1865.

*For value received, I promise to pay J. B. Smith & Co., or order, on demand, four hundred dollars, with interest at* 6 *per cent.*

A. R. CASSELS.

The following payments were receipted on the back of this note:

| | | | |
|---|---|---|---|
| February 4th, 1865, | received | ........ | $100 |
| May 16th, | " | " ......... | 75 |
| August 28th | " | " ......... | 100 |
| November 25th, | " | " ......... | 80 |

What was due at time of settlement, which was December 28th, 1865?

SOLUTION.

| | | |
|---|---|---|
| Principal or face of note | | $400.00 |
| Interest on the same from Jan. 1st, 1865, to Dec. 28th, 1865 | | 23.80 |
| Amount of principal at settlement | | $423.80 |
| First payment | $100.00 | |
| Interest on the same from Feb. 4th, 1865, to Dec. 28th, 1865 | 5.40 | |
| Second payment | 75.00 | |
| Interest on the same from May 16th, 1865, to Dec. 28th, 1865 | 2.77½ | |
| Third payment | 100.00 | |
| Interest on the same from August 28th, 1865, to Dec. 25th, 1865 | 2.00 | |
| Fourth payment | 80.00 | |
| Interest on the same from Nov. 25th, 1865, to Dec. 28th, 1865 | .44 | |
| Amount of payments to be taken from *amount* of principal | | 365.61½ |
| Balance due, December 28th, 1865 | | $58.18½ |

12. $500. HAMILTON, January 1st, 1865.

*Three months after date, I promise to pay James Manning, or order, five hundred dollars, for value received, at the Royal Canadian Bank in Toronto.* CYRUS KING.

Mr. King paid on this note, July 1st, 1865, $200.

What was due April 1st, 1866, the rate of interest being 7 per cent.? Ans. $324.50.

13. $240. SMITH'S FALLS, May 4th, 1865.

*On demand, I promise to pay A. K. Frost & Co., or order, two hundred and forty dollars, for value received, with interest at 6 per cent.* DAVID FLOOK.

The following payments were receipted on the back of this note:

| | |
|---|---|
| September 10th, 1865, received | $60 |
| January 16th, 1866, " | 90 |

What was due at the time of settlement, which was May 4th, 1866? Ans. $100.44.

14. $340. NEWMARKET, June 16th, 1864.

*Three months after date, I promise to pay Thomas Culverwell, or order, three hundred and forty dollars, with interest, at* 6 *per cent.* WILLIAM MUSGROVE.

On this note were receipted the following payments:

| | |
|---|---|
| October 14th, 1864, received......... | $86 |
| February, 12th, 1865, " ....... | 40 |

What was due at time of settlement, Aug. 10, 1865? Ans. $232.06.

## COMPOUND INTEREST.

When interest is unpaid at the end of a year, it may, by special agreement, be added to the principal, and in its turn bear interest, and so on from year to year. When added to the principal in this way, it is said to be *compounded.*

It is not against the law in Canada to take compound interest; but it can never be collected unless it has been specially agreed upon beforehand, or unless it is the custom of a house, and known to the customer to that effect.

### EXAMPLE.

1. What is the compound interest of $60, for 4 years, at 7 per cent.?

### SOLUTION.

| | |
|---|---|
| Principal.......... | $60.00 |
| Interest on the same for one year.......... | 4.20 |
| New principal for 2nd year.......... | 64.20 |
| Interest on the same for one year.......... | 4.494 |
| New principal for 3rd year.......... | 68.694 |
| Interest on the same for one year.......... | 4.808 |
| New principal for 4th year.......... | 73.502 |
| Interest on the same for one year.......... | 5.145 |
| Amount for 4 years.......... | 78.647 |
| Principal to be taken from same.......... | 60.000 |
| Compound interest for 4 years.......... | $18.647 |

The method of finding compound interest is usually much shortened by the following table, which shows the amount of $1 or £1 for any number of years not exceeding 50, at 3, 3½, 4, 5, 6 and 7 per cent. The amount of $1 or £1 thus obtained, being multiplied by the given principal, will give the required amount, from which, if the principal be taken, the remainder will be the compound interest:

# TABLE,

### SHOWING THE AMOUNT OF ONE DOLLAR AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

| No. | 3 per cent. | 3½ per cent. | 4 per cent. | 5 per cent. | 6 per cent. | 7 per cent. |
|---|---|---|---|---|---|---|
| 1 | 1.030 000 | 1.035 000 | 1.040 000 | 1.050 000 | 1.060 000 | 1.070 000 |
| 2 | 1.060 900 | 1.071 225 | 1.081 600 | 1.102 500 | 1.123 600 | 1.144 900 |
| 3 | 1.092 727 | 1.108 718 | 1.124 864 | 1.157 625 | 1.191 016 | 1.225 043 |
| 4 | 1.125 509 | 1.147 523 | 1.169 859 | 1.215 506 | 1.262 477 | 1.310 796 |
| 5 | 1.159 274 | 1.187 686 | 1.216 653 | 1.276 282 | 1.338 226 | 1.402 552 |
| 6 | 1.194 052 | 1.229 255 | 1.265 319 | 1.340 096 | 1.418 519 | 1.500 730 |
| 7 | 1.229 874 | 1.272 279 | 1.315 932 | 1.407 100 | 1.503 630 | 1.605 781 |
| 8 | 1.266 770 | 1.316 809 | 1.368 569 | 1.477 455 | 1.593 848 | 1.718 186 |
| 9 | 1.304 773 | 1.362 897 | 1.423 312 | 1.551 328 | 1.689 479 | 1.838 459 |
| 10 | 1.343 916 | 1.410 599 | 1.480 244 | 1.628 895 | 1.790 848 | 1.967 151 |
| 11 | 1.384 234 | 1.459 970 | 1.539 454 | 1.710 339 | 1.898 299 | 2.104 852 |
| 12 | 1.425 761 | 1.511 069 | 1.601 032 | 1.795 856 | 2.012 196 | 2.252 192 |
| 13 | 1.468 534 | 1.563 956 | 1.665 074 | 1.885 649 | 2.132 928 | 2.409 845 |
| 14 | 1.512 590 | 1.618 694 | 1.731 676 | 1.979 932 | 2.260 904 | 2.578 534 |
| 15 | 1.557 967 | 1.675 349 | 1.800 944 | 2.078 928 | 2.396 558 | 2.759 032 |
| 16 | 1.604 706 | 1.733 986 | 1.872 981 | 2.182 875 | 2.540 352 | 2.952 164 |
| 17 | 1.652 848 | 1.794 675 | 1.947 901 | 2.292 018 | 2.692 773 | 3.158 815 |
| 18 | 1.702 433 | 1.857 489 | 2.025 817 | 2.406 619 | 2.854 339 | 3.379 932 |
| 19 | 1.753 506 | 1.922 501 | 2.106 849 | 2.526 950 | 3.025 600 | 3.616 526 |
| 20 | 1.806 111 | 1.989 789 | 2.191 123 | 2.653 298 | 3.207 135 | 3.869 684 |
| 21 | 1.860 295 | 2.059 431 | 2.278 768 | 2.785 963 | 3.399 564 | 4.140 562 |
| 22 | 1.916 103 | 2.131 512 | 2.369 919 | 2.925 261 | 3.603 537 | 4.430 402 |
| 23 | 1.973 587 | 2.206 114 | 2.464 716 | 3.071 524 | 3.819 750 | 4.740 530 |
| 24 | 2.032 794 | 2.283 328 | 2.563 304 | 3.225 100 | 4.048 935 | 5.072 367 |
| 25 | 2.093 778 | 2.363 245 | 2.665 836 | 3.386 355 | 4.291 871 | 5.427 433 |
| 26 | 2.156 591 | 2.445 959 | 2.772 470 | 3.555 673 | 4.549 383 | 5.807 353 |
| 27 | 2.221 289 | 2.531 567 | 2.883 369 | 3.733 456 | 4.822 346 | 6.213 868 |
| 28 | 2.287 928 | 2.620 177 | 2.998 703 | 3.920 129 | 5.111 687 | 6.648 838 |
| 29 | 2.356 566 | 2.711 878 | 3.118 651 | 4.116 136 | 5.418 388 | 7.114 257 |
| 30 | 2.427 262 | 2.806 794 | 3.243 398 | 4.321 942 | 5.743 491 | 7.612 255 |
| 31 | 2.500 080 | 2.905 031 | 3.373 133 | 4.538 039 | 6.088 101 | 8.145 113 |
| 32 | 2.575 083 | 3.006 708 | 3.508 059 | 4.764 941 | 6.453 387 | 8.715 271 |
| 33 | 2.652 335 | 3.111 942 | 3.648 381 | 5.003 189 | 6.840 590 | 9.325 340 |
| 34 | 2.731 905 | 3.220 860 | 3.794 316 | 5.253 348 | 7.251 025 | 9.978 114 |
| 35 | 2.813 862 | 3.333 590 | 3.946 089 | 5.516 015 | 7.686 087 | 10.676 581 |
| 36 | 2.890 278 | 3.450 266 | 4.103 933 | 5.791 816 | 8.147 252 | 11.423 942 |
| 37 | 2.985 227 | 3.571 025 | 4.268 090 | 6.081 407 | 8.636 087 | 12.223 618 |
| 38 | 3.074 783 | 3.696 011 | 4.438 813 | 6.385 477 | 9.154 252 | 13.079 271 |
| 39 | 3.167 027 | 3.825 372 | 4.616 366 | 6.704 751 | 9.703 507 | 13.994 820 |
| 40 | 3.262 038 | 3.959 260 | 4.801 021 | 7.039 989 | 10.285 718 | 14.974 458 |
| 41 | 3.359 899 | 4.097 834 | 4.993 061 | 7.391 988 | 10.902 861 | 16.022 670 |
| 42 | 3.460 696 | 4.241 258 | 5.192 784 | 7.761 588 | 11.557 033 | 17.144 257 |
| 43 | 3.564 517 | 4.389 702 | 5.400 495 | 8.149 667 | 12.250 455 | 18.344 355 |
| 44 | 3.671 452 | 4.543 342 | 5.616 515 | 8.557 150 | 12.985 482 | 19.628 460 |
| 45 | 3.781 596 | 4.702 358 | 5.841 176 | 8.985 003 | 13.764 611 | 21.002 452 |
| 46 | 3.895 044 | 4.866 941 | 6.074 823 | 9.434 258 | 14.590 487 | 22.472 623 |
| 47 | 4.011 895 | 5.037 284 | 6.317 816 | 9.905 971 | 15.465 917 | 24.045 707 |
| 48 | 4.132 252 | 5.213 589 | 6.570 528 | 10.401 270 | 16.393 872 | 25.728 907 |
| 49 | 4.256 219 | 5.396 065 | 6.833 349 | 10.921 333 | 17.377 504 | 27.529 930 |
| 50 | 4.383 906 | 5.584 927 | 7.106 683 | 11.467 400 | 18.420 154 | 29.457 025 |

**NOTE.**—If each of the numbers in the table be diminished by 1, the remainder will denote the interest of $1, instead of its amount.

### EXERCISES.

2. What is the compound interest on $75, for 2 years, at 7 per cent.? Ans. $10.87.

3. What will $50 amount to in 3 years, at 6 per cent., compound interest? Ans. $59.55.

4. What is the compound interest on $600, for 2 years, at 6 per cent., payable half-yearly? Ans. $75.31.

5. What will $320 amount to in $2\frac{1}{2}$ years, at 7 per cent., compound interest? Ans. $379.19.

6. What is the compound interest of $150, for 3 years, at 9 per cent.? Ans. $44.25.

7. What is the compound interest on $1,000, for 2 years, at $3\frac{1}{2}$ per cent, payable quarterly? Ans. $72.18.

8. What will $460 amount to in 3 years, 4 months, 10 days, at 6 per cent., compound interest? Ans. $559.74.

9. What is the compound interest on $1860, for 8 years, at 7 per cent.? Ans. $1335.83.

10. What will be the compound interest on $75.20, for 20 years, at $3\frac{1}{2}$ per cent.? Ans. $74.43.

11. How much more will $500 amount to at compound than simple interest, for 20 years, 3 months, 15 days, at 7 per cent.? Ans. $764.14.

12. What sum will $50, deposited in a savings bank, amount to at compound interest, for 21 years, at 3 per cent, payable half-yearly? Ans. $173.03.

13. If a note of $60.60, dated October 25th, 1856, with the interest payable yearly, at 6 per cent., be paid October 25th, 1860; what will it amount to at compound interest? Ans. $76.51.

14. What remains due on the following note, April 1st, 1863, at 7 per cent. compound interest?

$1,000. PERTH, C. W., January 1, 1858.

*For value received, I promise to pay A. B. Smith & Co., or order, one thousand dollars on demand, with interest at 7 per cent.*

J. D. FOSTER.

On the back of this note were receipted the following payments:

| | | |
|---|---|---|
| June 10, 1858, | received | $70 |
| Sept. 25, 1859, | " | 80 |
| July 4, 1860, | " | 100 |

Nov. 11, 1861, " .............. 30
June 5, 1862, " ................ 50

Ans. $1022.34.

## DISCOUNT AND PRESENT WORTH.

Discount being of the same nature as interest, is, strictly speaking, the use of money before it is due. The term is applied, however, to a deduction of so much per cent. from the face of a bill, or the deducting of interest from a note before any interest has accrued. This is the practice followed in our Banks, and is therefore called Bank discount, in order to distinguish it from true discount.

The method of computing bank discount differs in no way from that of computing simple interest, but the method of finding *true discount* is quite different, *e. g.*, a debt of $107, due one year hence, is considered to be worth $100 now, for the reason that $100 let out at interest now, at 7 per cent., would amount to $107 at the end of a year.

In calculating interest, the sum on which interest is to be paid is known, but in computing discount we have to find *what sum* must be placed at interest so that *that sum*, together with its interest, will amount to the given principal. The sum thus found is called the "Present Worth."

We have already seen that $1.00 is the present worth of $1.07 due one year hence, at 7 per cent., therefore, to get the present worth of any sum due one year hence, at 7 per cent., it is only necessary to find how many times $1.07 is contained in the given sum, and we have the present worth; hence

To find the present worth of any sum, and the discount for any time, at any rate per cent., we have the following

### RULE.

*Divide the given sum by the amount of* $1 *for the given time and rate, and the quotient will be the present worth.*

*From the given sum subtract the present worth, and the remainder will be the discount.*

### EXERCISES.

1. What is the present worth of $224, due 2 years hence, at 6 per cent.? Ans. $200.

2. What is the discount on $670, due 1 year and 8 months hence, at 7 per cent.? Ans. $70.

3. What is the discount on $501, due 1 year and 5 months hence, at 8 per cent.? Ans. $51.

4. What is the present value of a debt of $678.75, due 3 years and 7 months hence, at 7½ per cent.? Ans. $534.97½.

5. What is the discount on $88.16, due 1 year, 8 months, and 12 days hence, at 6 per cent.? Ans. $8.16.

6. If the discount on $1060, for 1 year, at 6 per cent., is $60; what is the discount on the same sum for one-half the time? Ans. $30.87.

7. How much cash will discharge a debt of $145.50, due 2 years, 6 months and 12 days hence, at 6 per cent.? Ans. $126.30.

8. If I am offered a certain quantity of goods for $2500 cash, or for $2821.50, on 9 months credit; which is the best offer, and by how much? Ans. Cash by $200.

9. What is the difference between the interest and discount of $46.16, due at the end of 2 years, 6 months, and 24 days, at 6 per cent.? Ans. 95 cents.

10. A merchant sold goods to the amount of $1500, one-half to be paid in 6 months, and the balance in 9 months; how much cash ought he to receive for them after deducting 1½ per cent. a month? Ans. $1331.25.

11. Suppose a merchant contracts a debt of $24000, to be paid in four instalments, as follows: one-fifth in 4 months; one-fourth in 9 months; one-sixth in 1 year and 2 months, and the rest in 1 year and 7 months; how much cash must he give at once to discharge the debt, money being worth 6 per cent.? Ans. 22587.65.

12. Bought goods to the amount of $840, on 9 months credit; how much money would discharge the debt at the time of purchasing the goods, interest being 8 per cent.? Ans. $792.45.

13. A bookseller marks two prices in a book, one for ready money, and the other for one year's credit, allowing discount at 5 per cent. If the credit price be marked $9.80; what ought to be the price marked for cash? Ans. $9.33.

14. A man having a horse for sale, offered it for $225, cash; or, $230 at 9 months credit; the buyer chose the latter; did the seller lose or make by his bargain, and how much, supposing money to be worth 7 per cent.? Ans. He lost $6.47.

15. A. B. Smith owes John Manning as follows:—$365.87, to

be paid December 19th, 1863; $161.15, to be paid July 16th, 1864; $112.50, to be paid June 23rd, 1862; $96.81, to be paid April 19th, 1866, allowing discount at 6 per cent.; how much cash should Manning receive as an equivalent, January 1st, 1862 ?
Ans. $653.40.

16. I buy a bill of goods amounting to $2500 on 6 months' credit, and can get 5 per cent. off by paying cash; how much would I gain by paying the bill now, provided I have to borrow the money, and pay 6 per cent. a year for it? Ans. $53.75.

## BANKING.

Money has a tendency to concentrate. We thus hear of the money centres of a country, which are the cities where the largest amount of available capital has accumulated. The money which represents this capital is to be found in largest quantities in the Banks of these places. There are many reasons why this should be the case. In the first place, let us consider these institutions as

### BANKS OF DEPOSIT.

One strong inducement to place money in a bank is, that it is likely to be more safely kept for the owner than in any private hands. Then the funds are more easily available by means of cheques upon the banker, than in the shape of coin. When cheques are drawn payable to the order of a person, they serve, after indorsement, as receipts. It would be a serious loss of time, as well as a fertile source of mistake, to be always counting out, in small currency, the large amounts that a business man receives and pays. In countries where silver and baser metals constitute the currency, payments and exchanges are very much facilitated by the system of bank deposits and cheques.

In keeping a deposit at a particular bank, it is clear that the depositer makes a continual loan to the institution. For this, business men generally expect an equivalent in the shape of temporary loans; or, " accommodation," as they call it. This is only given by

### BANKS OF DISCOUNT,

Which are combined, in Canada, with banks of deposit, issue and exchange. These temporary loans are usually effected by means of endorsed notes or bills, which are cashed by the banks, and the interest upon them is paid in advance by being discounted, or counted off the face of the paper. A wholesale merchant when he makes a sale, very often takes a note at three or four months for part of his money. He is willing to pay a trifle to get this paper cashed, so he hands it to his banker, who places the amount to his credit, less the interest in advance for the time it has to run.

### BANKS OF ISSUE.

All the Canadian banks, with the exception of Savings banks, belong to this class. They are at liberty to issue their notes payable in specie on demand, at the place where they bear date. The amount of notes in circulation at any one time must not exceed the aggregate amount of the paid-up capital stock of the bank, and the specie and government securities on hand. A bank cannot, any more than a private individual, make or issue a note for any less sum than one dollar; and there is generally a restriction in the charter against issuing more than a certain proportion of notes of the lower denominations.

It will be evident that a bank can have a much larger circulation or capital in use, than the original capital as paid in by stockholders. Indeed, part of the notes that they circulate represent government securities, which are paying them interest all the time. To those who do not understand this, it is a matter of surprise that banks can lend their money on notes that they discount at 7 per cent., and still pay working expenses, and declare dividends of 8 per cent. or more. The reason is, that they can lend so much more than they have to pay dividends upon. Besides the large circulation, the deposits of customers are a source of profit, for while interest is seldom paid upon deposits, the money is not allowed to remain idle by the banks, but a portion, at any rate, is loaned out and made productive.

Another source of profit arises from our banks being

### BANKS OF EXCHANGE.

In this capacity they are the medium of making payments at distant places through drafts or bills of exchange, and without the actual transmission of money. Under the head of "Exchange," in another place, this branch of the working of banking institutions is fully explained.

### PASS BOOKS.

A book, called a *pass book,* is delivered by the bankers to the customer, in which the bankers are described as the debtors, and the customer as the creditor, in the account. On the debtor side are entered all sums paid to, or received by the bankers on account of the customer, and on the credit side, all sums paid to him, or on his account. For the purpose of having the book made up by the bankers from their own books of account, the customer returns it to them from time to time; and the proper entries being made by them, they hand it again to the customer, who then examines it, and if there appears any error or omission, it is his business to send it back to be rectified; if he does not, his silence is regarded as an admission that the entries are correct.

### BANK CORRESPONDENTS.

In the ordinary course of a banking business, it is often necessary to engage the services of other banking houses carrying on business at a distance, who are called *correspondents.* The principal legal point to be noticed in this connection is, that when a bank employs a correspondent, and a loss ensues from the conduct of such agent, the bank is directly responsible to the customer for the loss. This does not prevent the bank having recourse against their correspondent to make good the loss.

## BANK DISCOUNT.

The Bank Discount of a note is the simple interest on the sum for which it is given from the time it is *discounted* to the time it becomes due, including three days of grace.

Suppose, for example, in getting a note of $200 discounted at a bank I am charged $12 for discount, which being deducted, I receive but $188, so that I pay interest on $12 which I did not receive. From this it is clear that I am paying a higher rate of interest in discounting a note at a bank, than I would pay were I to borrow money at the same rate. As bank discount is the same as interest, we derive the following

### RULE.

*Find the interest on the sum specified in the note at the given rate, and for the given time, including three days of grace, and this will be the* BANK DISCOUNT.

*Subtract the discount from the face of the note, and the remainder will be the* PROCEEDS OR PRESENT WORTH.

### EXERCISES.

1. What is the bank discount on a note, given for 60 days, for $350, at 6 per cent ?* Ans. $3.62.

2. What is the bank discount on a note of $495, for 2 months, at 5 per cent. ? Ans. $4.33.

3. What is the present value of a note of $7840 discounted at a bank, for 4 months and 15 days, at 6 per cent. ? Ans, $7659.68.

4. How much money should be received on a note for $125, payable at the end of 1 year, 3 months, and 15 days, if discounted at a bank at 8 per cent. ? Ans. $112.

5. A note, dated December 3rd, 1860, for $160.40, and having 6 months to run, was discounted at a bank, April 3rd, 1861, at 6 per cent. ; how long had it to run, and what were the proceeds?
Ans. 64 days; proceeds $158.71.

6. On the first day of January, 1866, I received a note for $240, at 60 days, and on the 12th of the same month had it discounted at a bank at 7 per cent; how much did I realize upon it.
Ans. $237.61.

7. A merchant sold 240 bales of cotton, each weighing 280 pounds, for 12½ cents per pound, which cost him, the same day, 10 cents per pound; he received in payment a good note, for 4 months' time, which he discounted immediately at a bank at 7 per cent.; what will be his profits ? Ans. $1479.10.

It is sometimes necessary to know the amount for which a note must be given, in order that it shall produce a given sum when discounted at a bank.

### EXAMPLE.

1. Suppose we require to obtain $236.22 from a bank, and that we are to give our note, due in two months; for what amount must we draw the note, supposing that money is worth 9 per cent. ?

### SOLUTION.

From the nature of this example, we can readily perceive that such a sum must be put on the face of the note, that when dis-

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* Throughout all the exercises, unless otherwise specified, the year is to be considered as consisting of 365 days. Since it is customary in business when a fraction of a cent occurs in any result to reject it, if less than half a cent, and if not less, to call it a cent, we have adopted this principle throughout the book.

counted the proceeds will be exactly $236.22. If we were to take a *one dollar note* and discount it at a bank for the given time, and at the given rate, the proceeds would be .98425. Hence, for *every dollar* we put upon the face of the note we receive .98425, and to receive $236.22 we would have to put as many dollars on the face of the note as are represented by the number of times that .98425 is contained in $236.22, which is 240. Therefore, we must put $240 on the face of a note due at the end of two months to produce $236.22 when discounted at a bank at 9 per cent. From this we deduce the following

### RULE.

*Deduct the bank discount on* $1, *for the given time and rate, from* $1, *and divide the desired amount by the remainder. The quotient will be the face of the note required.*

2. For what sum must a note be given, having 4 months to run, that shall produce $1950, if discounted at a bank at 7 per cent.? Ans. $1997.78.

3. What must be the face of a note, so that when discounted for 5 months and 21 days, at 7 per cent., it will produce $57.97, cash? Ans. $60.

4. Suppose your note for 6 months is discounted at a bank at 6 per cent., and $484.75 placed to your credit, what must have been the face of the note? Ans. $500.

5. A merchant bought a quantity of goods for $600. For what sum must he write his note, to be discounted at a bank for 6 months, at 6 per cent.? Ans. $618.88.

6. A farmer bought a farm for $5000 cash, and having only one-half of the sum on hand, he wishes to obtain the balance from the bank. For what sum must he give his note, to be discounted for 9 months, at 6 per cent.? Ans. $2619.17.

7. If a merchant wishes to obtain $550 of a bank, for what sum must he give his note, payable in 60 days, allowing it to be discounted at $\frac{1}{2}$ per cent. per month? Ans. $555.75.

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From the many dealings business men have, in regard to discount and interest, it is frequently required to know what rate of interest corresponds to a given rate of bank discount.

### EXAMPLE.

1. What rate of interest is paid when a note, payable in 362 days, is discounted at 10 per cent.?

### SOLUTION.

If we discount $1 for the given time, and at the given rate, the proceeds will be .90, or 90 cents. Hence, the discount being 10 cents, we are paying 10 cents for the use of 90 cents. Now, if we pay 10 cents for the use of 90, for the use of 1 cent we must pay $\frac{1}{90}$ of 10 cents, or $\frac{1}{9}$ of a cent, and for $1, or 100 cents, we must pay 100 times $\frac{1}{9}$ of a cent, or $\frac{100}{9}$=.11$\frac{1}{9}$, and for $100, $11$\frac{1}{9}$, or 11$\frac{1}{9}$ per cent. Therefore, to find the rate of interest corresponding to a given rate of bank discount, we deduce the following

### RULE.

*Divide the given rate per cent., expressed decimally, or the rate per unit, by the number denoting the proceeds of* $1 *for the given time and rate. The quotient will be the rate of interest required.*

### EXERCISES.

2. What rate of interest is paid when a note, payable in 60 days, is discounted at 7 per cent.? Ans. $7\frac{21}{247}$.

3. What rate of interest is paid when a note, payable in 3 months, is discounted at 6 per cent.? Ans. $6\frac{186}{1969}$.

4. A note, payable in 6 months, is discounted at 1 per cent. a month; what rate of interest is paid? Ans. $12\frac{244}{313}$.

5. What rate of interest is paid, when a note of $200, payable in 70 days, is discounted at $\frac{3}{4}$ per cent. a month? Ans. $9\frac{81}{491}$.

6. When a note of $45, payable in 65 days, is discounted at 7 per cent., to what rate of interest does the bank discount correspond? Ans. $7\frac{833}{9006}$.

7. A bank, by discounting a note at 6 per cent., receives for its money a discount equivalent to 6$\frac{1}{2}$ per cent. interest; how long must the note have been discounted before it was due? Ans. 1 yr., 3 mos., 12d.

## COMMISSION.

**COMMISSION** is the term applied to money paid to an agent to remunerate him for his trouble in buying, selling, valuing, or for forwarding merchandise or other property.

The goods sent to a commission merchant or agent, to be sold on account and risk of another, are termed a *consignment*.

The person to whom these goods are consigned is called the *consignee* or *correspondent.*

The term shipment is sometimes used instead of consignment.

### EXAMPLE.

A commission merchant sells for me goods worth \$1200, and charges 4 per cent.; what have I to pay him?

### SOLUTION.

4 per cent. of \$1200 is equal to $\$1200 \times .04 = \$48$. Hence I would have to pay \$48, and from this we deduce the following

### RULE.

*Find the percentage on the given sum at the given rate, which will be the commission.*

### EXERCISES.

1. Consigned to A. K. Boomer, Esq., Toronto, by the Montreal foundry, agricultural implements which are sold for \$1875.75; what is the agent's commission at $2\frac{1}{2}$ per cent.? Ans, \$46.89.

2. Bought in Montreal 12 chests of tea, containing on an average 64 lbs. each, the price was $\$1.12\frac{1}{2}$ per lb., and my employer allowed me $1\frac{3}{4}$ per cent.; how much did I make? Ans. \$15.12.

3. My London correspondent has bought for me 2768 lbs. of bacon, at $12\frac{1}{2}$ cts. a pound; what is his commission at $3\frac{1}{4}$ per cent.? Ans. \$11.25.

4. Bought a carriage and pair of horses for W. K., Esq., Lachine Rapids; paid for the horses \$240, and charged $4\frac{1}{2}$ per cent., and paid for the carriage \$160, and charged $1\frac{1}{2}$ per cent.; how much did I earn? Ans. \$13.20.

5. A commission agent in a Southern State bought cotton worth \$2284 for an English manufacturer, and charged $5\frac{1}{2}$ per cent.; what is his commission? Ans. \$125.62.

6. On another occasion the manufacturer gave the commission merchant \$165.78, for purchasing for him cotton worth \$3684; what was the rate per cent.? Ans. $4\frac{1}{2}$.

7. An English commission merchant buys for a Quebec house, £576 10s. 0d. worth of provisions, and charges $4\frac{1}{2}$ per cent.; what is his commission? Ans. £25 18s. $10\frac{1}{5}$d.

8. A Toronto provision merchant instructs a Belfast (Ireland) commission merchant to purchase for him £534 4s. 0d. worth of

bacon and hams, and offers him 7¼ per cent.; what does the agent get? Ans. £38 14s. 7d.

9. A book agent in Port Hope, sells $487.50 worth of books for Day & Co., of Montreal, and receives $72.05 for his trouble; at what rate per cent. was he paid? Ans. 15 nearly.

10. An agent sells 84 sewing machines at $25 each, and his commission amounts to $262.60; what is the rate? Ans. 12½.

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When a sum has to be sent to a commission agent, such that it will be equal both to the sum to be invested, and the agent's commission, it is plain, as already noted, that this is merely a case of percentage. It is the same as the first part of case IV., and we will have the corresponding

### RULE.

*Divide the given amount by* 1, *increased by the given rate per unit, and the quotient will be the sum to be invested; subtract this from the given amount, and the remainder will be the commission.*

### EXAMPLE.

If I send $1890 to a commission merchant, and instruct him to buy merchandise with what is left after his commission at 5 per cent. is deducted; what will be the sum invested, and the agent's commission?

### SOLUTION.

It is plain that for every dollar of the proposed investment I must remit 105 cents, 100 towards the investment, and 5 towards the commission, and hence the number of dollars which can be invested from the sum remitted will be the same as the number of times that 1.05 is contained in 1890. Now, $1890÷1.05 gives $1800, the sum to be invested, and this subtracted from $1890, leaves $90, the commission to which the agent is entitled.

### EXERCISES.

1. Remitted to A. B., Montreal, $988 to purchase flour for me with the balance that remains after deducting his commission at 4 per cent.; required the purchase money and percentage?
Ans. $950 and $38.

2. Received a commission to buy wheat with $779, less by my commission at 2½ per cent.; required the price of the wheat and my commission. Ans. $760, and $19.

3. Remitted to my correspondent to Ottawa $266.76, to pay for lumber which he purchased for me, and to pay his own commission at 4 per cent.; what was the price of the lumber, and what the commission? Ans. $256.50, and $10.26.

4. John Jones, Newmarket, commissions W. Orr, Port Hope, to procure for him a quantity of fine flour, and remits $917.61; how much flour can he have, after allowing 4¾ per cent., and what will the commission amount to? Ans. $876, and $41.61.

5. John Stalker, London, commissions J. Fleming, Toronto, to purchase for him as much butter as he can procure for the balance between $779.52, and his own commission at 1½ per cent.; how many pounds butter did he get at 25 cents per lb.; what the whole price, and what was the commission? Ans. 3072 lbs., $768, and $11.52.

6. Dr. Gallipot is about to remove to England, and sends to a London cabinet maker $4005.45 towards getting his house furnished, he is charged 3½ per cent. over and above the price of the furniture, for time and labour; what does the furniture cost? Ans. $3870.

7. Graham Bros., of Londonderry, send to R. White, Kingston, bacon and hams worth $1560, they charge 5½ per cent. commission, and the charge for lading is $75.15; how much does R. White owe them? Ans. $1720.95.

8. P. Robson, commission merchant, Montreal, buys for T. Black & Co., London, C. W., groceries, the price of which, together with their commission at 4 per cent., comes to $475.02; what was the price of the goods, and what was the amount of the commission? Ans. $456.75, and $18.27.

## BROKERAGE.

**Brokerage** is a percentage paid to an agent for negociating bills, collecting accounts, exchanging money, buying and selling shares and stocks, and all similar transactions. Such an agent is called a Broker. A smaller percentage is usually allowed to a broker than to a commission merchant, because the work he has to do requires less time and labour. Like commission, brokerage is merely a particular case of percentage, and hence the

### RULE.

*To find the brokerage on any sum, find the percentage on the given sum at the given rate, which will be the brokerage.*

### EXERCISES.

1. A broker in Hamilton has bought for me $1275 worth of R. R. stock; what will be the brokerage at $2\frac{1}{8}$ per cent.? Ans. $27.09.

2. I pay a collector of accounts 2 per cent. for cellecting $118.50; how much does it cost me? Ans. $2.37.

3. I pay a broker $1\frac{7}{8}$ per cent. for selling $2716.75 government stock; how much do I give him? Ans. $50.94 nearly.

4. Advised R. P., broker, to collect two bills amounting to $897, he has collected $\frac{2}{3}$ of it, and I have given him $1\frac{1}{2}$ per cent. on the amount collected; how much have I paid him? Ans. $8.97.

5. A. B. sent me $756 to purchase flour for him. I have charged $2\frac{1}{4}$ per cent. commission on the whole sum, and purchased flour with the remainder; what is my commission, and how much do I vest in flour for A. B. Ans. $738.99, and $17.01.

6. The school taxes on all the sections of a county amount to $1180, and collectors get $2\frac{5}{8}$ per cent.; how much remains available for school purposes? Ans. $1149.03.

7. Remitted to a broker in London, $798 to buy G. W. R. shares, deducting his charge for brokerage at $\frac{7}{8}$ per cent.; how much did he invest for me? Ans. $791.02.

8. I am charged $\frac{1}{4}$ per cent by a broker in Chatham, for negociating a draft for $750; what are the proceeds coming to me? Ans. $$748.12\frac{1}{2}$.

9. Bought G. W. R. shares to the amount of $578, and paid my broker $2\frac{1}{4}$ per cent.; how much did I give him? Ans. $13.01.

10. Gave D. F. $3\frac{1}{3}$ per cent. for collecting accounts for me to the amount of $639; how much did I give him? Ans. $21.30.

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To find the sum that can be invested when the given amount includes both the brokerage and the investment.

For example, if I wish a broker to invest for me $700, and his charge is 2 per cent., I must obviously remit to him $714, as $14 is 2 per cent. on $700; conversely if I send him $714, and instruct him to invest for me that sum, *minus* his own percentage, he will have to calculate how much he will have remaining to invest after deducting his own charge. Now, since his percentage is $2 on every $100, he should get from me $102 for every $100 he is to invest, and therefore the sum he can invest will be the 102nd part of what I remit, *i. e.*, $714÷1.02=$700. Hence the

### RULE.

*Divide the given amount by one, increased by the given rate per unit of brokerage, and the quotient will be the sum to be invested; subtract this from the given amount, and the remainder will be the brokerage.*

### EXERCISES.

1. A broker receives $574, with instructions to invest what remains after deducting brokerage at $2\frac{1}{2}$ per cent., in R. R. shares; how much has he to invest? Ans. $560.

2. The assessment on a certain district, together with the percentage for collection at $2\frac{1}{4}$ per cent., is $1717.80; what is the amount of the assessment, and what the expense of collection?
Ans. $1680, and $37.80.

3. A tax amounting to $3276.52, including collector's fees at 4 per cent., is levied on a certain town; what is the amount of the tax, and how much is the collector entitled to?
Ans. $3150.50, and $126.02.

4. A gentleman once invested in U. S. government bonds, a certain sum which, with the broker's fee at $1\frac{3}{4}$ per cent., amounted to $18,315; what was the amount of the investment? Ans. $18,000.

5. A Montreal broker negociates a draft for $1218 for a Hamilton merchant, at $1\frac{1}{2}$ per cent.; what are the proceeds?
Ans. $1199.73.

6. A broker, after deducting his charge at $1\frac{1}{4}$ per cent., invests the balance of $2450.25 for his employer in bank stock; how much does he invest? Ans. $2420.

7. My broker invests for me in oil well shares, at $83 each, what remains after deducting his fee at $\frac{1}{2}$ per cent. from $8341.50; how much does he invest. and how many shares does he purchase?
Ans. $8300, and 100 shares.

8. A broker's charge is $285, at $1\frac{1}{2}$ per cent., on a certain sum invested; what is the sum?—(See Percentage, Case II.)
Ans. $19000.

9. A broker sells stocks for me, and the sum which is realized, together with the brokerage at 4 per cent., amounts to $910; what is the sum procured, and what the brokerage? Ans. $875 and $35.

### MIXED EXERCISES ON COMMISSION AND BROKERAGE.

1. My Liverpool correspondent charges me $1\frac{1}{2}$ per cent. commission on goods which he has purchased for me worth \$32,000, and $\frac{3}{8}$ per cent. for harbour dues paid by him; how much have I to remit to him? Ans. \$32,600.

2. A broker in Montreal has negotiated bills for me to the amount of \$1287.50, and charges $1\frac{5}{8}$ per cent.; how much do I owe him? Ans. \$20.92.

3. A collector receives \$20 for collecting \$900; at what per cent. is he paid? Ans. $2\frac{2}{9}$.

4. One afternoon, the train being late, and the banks closed before I reached Toronto, I wanted gold for Canadian bills to the amount of \$256; I applied to a broker, who charged me $\frac{1}{4}$ per cent.; how much did the lateness of the cars cost me? Ans. 64 cents.

5. A broker invests for me \$1750, and I pay him for his trouble \$43.75, at what rate per cent. do I pay him? Ans. $2\frac{1}{2}$.

6. An Auctioneer valued the furniture of a deceased gentleman, and charging 4 per cent., he was paid \$53.86; what was the value of the furniture? Ans. \$1346.50.

7. Attorney Screw was instructed to pay the widow Flaunt \$1500 yearly, he was to have \$48.75 for himself; what was the rate per cent. of his remuneration? Ans. $3\frac{1}{4}$.

8. A certain district pays \$800 school taxes, the collector gets \$38 for collecting; what percentage does he get? Ans. $4\frac{3}{4}$.

9. Sold by my broker at Portland, my shares in the G. T. R. for \$1780; what brokerage must I pay at $1\frac{3}{4}$ per cent.? Ans. \$31.15.

10. An accountant is entrusted to make schedules of the debts and assets of a bankrupt; he charges only $2\frac{1}{2}$ per cent. on the debts, on the principle that he will have little trouble in getting the accounts due by the bankrupt sent in; but as he knows very well that he will have trouble in getting correct statements sent in of accounts due to the bankrupt, he stipulates for $5\frac{1}{2}$ per cent. on these; how much does he get altogether, the debts being \$2786, and the assets \$618? Ans. \$103.64.

# INSURANCE.

**Insurance** is an engagement by which one party is bound, in consideration of receiving a certain sum, to indemnify another for something in case it should in any way be lost. The party undertaking the risk is seldom, if ever, an individual, but a joint stock company, represented by an agent or agents, and doing business under the title of an "*Insurance Company,*" or "*Assurance Company,*" such as the "Royal Insurance Company," the "Mutual Insurance Company."

Some companies are formed on the principle that each individual shareholder is insured, and shares in the profits, and bears his portion of the losses. Such a company is usually called a *Mutual Insurance Company.*

The sum paid to the party taking the risk is called the *Premium of Insurance,* or simply the *Premium.*

The document binding the parties to the contract, is called the *Policy of Insurance,* or simply the *Policy.*

The party that undertakes to indemnify is called the *Insurer,* or *underwriter* after he has written his name at the foot of the policy.

The person or party guaranteed is called the *Insured.*

As there are many different kinds of things that may be at stake or risked, so there are different kinds of insurance which may be classified under three heads.

*Fire Insurance,* including all cases on land where property is exposed to the risk of being destroyed by fire, such as dwelling houses, stores and factories.

*Marine Insurance.*—This includes all insurances on ships and cargoes. Such an insurance may be made on the ship alone, and in that case it is sometimes called *hull insurance,* and sometimes *bottomry,* the ship's bottom representing the whole ship, just as we say fifty sail for fifty ships. The insurance may be made on the cargo alone, and is then usually called *Cargo Insurance.* It may be made on both ship and cargo, in which case the general term *Marine Insurance* will be applicable. This kind, as the name implies, insures against all accidents by sea.

*Life Insurance.*—This is an agreement between two parties, that in case the one insured should die within a certain stated time, the other shall, in consideration of having received a stipulated sum annually, pay to the lawful heir of the deceased, or some one mentioned in his will, or some other party entitled thereto, the amount recorded in the policy.

For instance, a man may, on the occasion of his marriage, insure his life for a certain sum, so that should he die within a certain time, his widow or children shall be paid that sum by the other party. Again, a father may insure the life of his child, so that in case of the child's death within a specified time, he shall be paid the sum agreed upon, or that the child, if it lives to a certain age, shall be entitled to that sum. One person may insure the life of another. Supposing that A owes B a certain sum, there is the risk that A may die before he is able to pay B; another party engages, for a certain yearly sum, to pay B in case A should fail to do so during his life time.

In some instances, insurances are effected to gain a support in case of sickness. Such a contract is called a *Health Insurance.* Insurances are now also effected for compensation in case of railway accidents. These we may call *Railway Accident Insurances.*

A policy is often transferred from one party to another, especially as collateral security for debt or some analogous obligation. If the payments agreed upon are not regularly kept up, the policy lapses, that is, becomes null and void, so that the holder of it forfeits not only his claim to the sum insured, but also the instalments previously paid. In many companies a person can insure in such a way as to be entitled to have a share of the profits.

The date at which the system of insurance began cannot be clearly ascertained; but, whatever its date, its origin seems to have been protection against the perils of the sea. We know that it was practised, in a certain way, by the ancient Greeks and Romans. If a Roman merchant sent a cargo to a distant port, he made a contract with some one engaged in such business, that he would advance a certain sum, to be repaid with interest, if the vessel reached her destination in safety, but should the vessel or cargo, or both be lost, the lender was to bear the loss. This was termed *respondentia,* (a respondence) a term corresponding pretty nearly to the English word repayment. It was lawful to charge interest in such cases, above the legal interest in ordinary cases, on account of the greatness of the risk. The lender of the money usually sent an agent of his own on board the vessel to look after the cargo, and receive the repayment on the safe delivery of the goods. This agent corresponded pretty nearly to our more modern *supercargo.* As the art of navigation advanced, and the securities afforded by law became more stringent, and also facilities of communication increased, this system gradually gave way, and has eventually been supplanted by communications by post, and telegraphic messages to agents at the ports of destination.

With regard to the equitableness of insurances, and their utility in promoting commercial exterprise, we may remark that they make the interest of every merchant, the interest of every other. To show this, we may compare an *insurance office* to a *club*. Suppose the merchants of Canada to form a club, and establish a fund, out of which every member, if a loser, was to be indemnified, it is plain that no loss would fall on the individual, except his share as a member of the club. Even so the insurance system causes that each speculator, by insuring his own stake, contributes so much to the funds of a company, which is bound to indemnify each loser. On the other hand, the insurer or insuring company, gains in this way, that the profits accruing from cases where no loss is sustained, far exceed the cases where loss is sustained, and the trifling expense of insuring is of no moment to the insured, in comparison with the damage of a disastrous voyage, or consuming conflagration. By the insurance system, loss is virtually distributed over a large community, and therefore falls heavily on no individual, from which we draw our conclusion, that it is equivalent to a *mutual mercantile indemnification club.*

We must now show the rules of the *club*, and principles on which its calculations are made.

The principal thing to be taken into account, in all insurances, is the amount of risk. For example, a store, where nothing but iron is kept, would be considered *safe;* a factory, where fire is used, would be accounted *hazardous*, and one where inflammable substances are used would be designated *extra hazardous*, and the rates would be higher in proportion to the increased risks. As, however, the degrees of risk are so very varied, only a rough scale can be made, and hence the estimate is nothing more than a calculation of probabilities. In life insurances, the rates are regulated chiefly by the age, and general health of the individual, and also by the general health of the family relations. Connected with this is the calculation of the average length of human life.

Almost all the calculations in insurance come under two heads. FIRST, to find the premium of insurance on a given amount, and at a given rate; and, SECONDLY, to find how much must be insured at a given rate, so that in case of loss, both the principal and premium may be recovered.

As the premium is reckoned as so much by the hundred, insurance is merely a particular case of percentage. Hence to find the premium of insurance on any given amount, at a given rate per cent., we deduce the following

### RULE.

*Multiply the given amount by the rate per unit.**

### EXAMPLES.

1. To find the cost of insuring a block of buildings valued at \$2688, at 6 per cent.? Here we have .06 for the rate per unit, and \$2688×.06=\$161.28, the answer.

2. What will be the cost of insuring a cargo worth \$3679, at 3 per cent.? The rate per unit is .03, and \$3679×.03=\$110.37, the answer.

3. A gentleman employed a broker to insure his residence and outhouses, valued at \$2760, the rate being 8 per cent., and the broker's charge 1½ per cent,; how much had he to pay? The cost of insurance is \$2760×.08=\$220.80, and the brokerage \$41.40, which added to \$220.80, will give \$262.20, the answer.

### EXERCISES.

What will be the premium of insurance on goods worth \$1280, at 5½ per cent.? Ans. \$70.40.

2. A ship and cargo, valued at \$85,000, is insured at 2¼ per cent.; what is the premium? Ans. \$1912.50.

3. A ship worth \$35,000, is insured at 1½ per cent., and her cargo, worth \$55,000, at 2½ per cent.; what is the whole cost? Ans. \$1900.00.

4. What will be the cost of insuring a building valued at \$58,000, at 2½ per cent.? Ans. \$1450.00.

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* It is plain that the rate can be found, if the amount and premium are given, and the amount can be found if the rate and premium are given. In the case of insuring property, a professional surveyor is often employed to value it, and likewise in the case of life insurance, a medical certificate is required, and in each case the fee must be paid by the person insured. As 100, the basis of percentage, is a constant quantity, when any two of the other quantities are given, the third can be found.

5. What must I pay to insure a house valued at $898.50, at $\frac{3}{4}$ per cent.? Ans. $673.88.

6. A village store was valued at $1180; the proprietor insured it for six years; the rate for the first year was $3\frac{1}{4}$ per cent., with a reduction of $\frac{1}{4}$ each succeeding year; the stock maintained an average value of $1568, and was insured each of the six years, at $2\frac{1}{4}$ per cent.; how much did the proprietor pay for insurance during the six years? Ans. $397.53.

7. A store and yard were valued at $1280, and insured at $1\frac{1}{8}$ per cent.; the policy and surveyor's fee came to $2.25; what was the whole cost of insuring? Ans. $16.65.

8. W. Smith, Port Hope, requests R. Tomlinson, Toronto, to insure for him a building valued at $976; R. Tomlinson effects the insurance at $4\frac{3}{8}$ per cent., and charges $\frac{3}{8}$ per cent commission; how much has W. Smith to remit to R. Tomlinson, the latter having paid the premium? Ans. $46.36.

9. The cost of insuring a factory, valued at $25,000, is $125; what is the rate per cent.? Ans. $\frac{1}{2}$.

10. A $1\frac{1}{4}$ per cent. insuring my dwelling house cost me $50; what is the value of the house? Ans. $4000.00.

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To find how much must be insured for, so that in case of loss, both principal and premium may be recovered.

Here it is obvious that the sum insured for must exceed the value of the property in the same ratio that 100 exceeds the rate.

### EXAMPLE.

To find what sum must be insured for on property worth $600, at 4 per cent., to secure both property and premium, we have as $100—4=$96 : $100 : : $600 : F. P.$=\frac{600\times100}{96}=$$625, the sum required. Taking the rate per unit we find $\frac{100-4}{100}=\frac{96}{100}=.96$. This gives the

### RULE.

*Divide the value of the property by* 1, *diminished by the rate per unit, and the quotient will be the sum required.*

### EXAMPLES.

1. A foundry is valued at $874: for what sum at 8 per cent. must it be insured to secure both the value of the property and the premium? One *minus* the rate or 1.00—.08=.92, and $874÷.92 =$950, the answer.

The premises of a gunsmith, who sells gunpowder, are valued at $2618.85: for how much, at 15 per cent. must they be insured in order to recover the value of the property and also the premium of insurance? Subtract .15, the rate per unit, from 1, and the remainder is .85 and $2618.85÷.85 gives $3081, the sum required.

### EXERCISES.

1. A chemist's laboratory and appurtenances are valued at $26,250, for what sum should he insure them at 6¼ per cent., to secure both property and premium? Ans. $28,000.

2. A Liverpool merchant sent goods worth $1,186, by the steamer *Hibernian* to Quebec. He insured them from Liverpool to Londonderry at 1¼ per cent., and from Londonderry to Quebec at 2½ per cent., and in both cases so as to secure the premium as well as the cargo, how much did the whole cost him?

Ans. $45.42.

3. A person owned a flour mill, valued at $1846.05, which he insured at 1¼ per cent. He also owned a flax mill, valued at $846.30, which he insured at 2½ per cent., and in both cases at such a sum as to secure both property and premium. Which cost him most, and how much more?

Ans. The flour mill cost him $1.67. more than the other.

4. Tidman & Co., Montreal, order a quantity of pork from T. S. Coates & Son, Belfast, which amounts to $2423.10. They insure it to Liverpool at ½ per cent., and from Liverpool to Portland at 3 per cent., and in all cases so as to secure the price and the premium both. How much does the whole insurance come to?

Ans. $87.12.

5. In order to secure both the value of goods shipped and the premium, at 1¾ per cent., an insurance is effected on $1526.72. What is the value of the goods? Ans. $1500.00.

6. The Mechanics' Institute is valued at $18,000: it is insured at 1¼ per cent., so that in case of fire, the property and premium may both be recovered. For how much is it insured?

Ans. $18,227.85.

7. How much must be insured on a cargo worth $40,000, at ½ per cent., to secure both the value of the cargo and the cost of insurance? Ans. $40,201.00.

8. The Rossin House, King-street, Toronto, is valued at, say, $150,000, and is insured at 1¾ per cent, so that in case of another conflagration, both the value of the property and the premium of insurance may be recovered. For how much must it be insured?
Ans. $152,671.76, nearly.

9. A jail and court-house, adjoining chemical works, and therefore deemed hazardous, will not be insured under 2½ per cent. How much will secure both property and premium, the valuation being $17,550.00? Ans. $18,000.00.

10. A cotton mill is insured for $12,000, at 4 per cent., to secure both premium and property. What is the value of the property?
Ans. $12,500.00.

11. What sum must be insured on a vessel and cargo valued at $40,000, at 5½ per cent., in order to secure both the premium and property? Ans. $42,328.04.

12. How much must be insured on property worth $70,000, at 4½ per cent., to secure both premium and property, a commission of ¾ per cent. having been charged? Ans. $73,848.17.

## LIFE INSURANCE.

A **Life Insurance** may be effected either for a term of years or for the whole period of life. The former is called a *Temporary Insurance*, and binds the insurer to pay the amount to the legal heir or legatee or creditor, if the insured should die within the specified time. The latter is called a *Life Insurance*, because it is demandable at death, no matter how long the insured may live.

The rate per annum that the insured is to pay is reckoned from tables constructed on a calculation of the average duration of life beyond different ages. This calculation is made from statistical returns called **Bills of Mortality**, and the result is called **The Expectation of Life**.

The annual premium is fixed at such a rate as would, at the end of the expectation of life, amount to the sum insured. From tables of the expectation of life other tables are constructed, showing the premium on $100 for one year, calculated on the supposition that it is to be paid annually in advance.

## LIFE INSURANCE TABLE.

| Age next Birthday. | 1 year. | 7 years. | For Life. | Age next Birthday. | 1 year. | 7 years. | For Life. |
|---|---|---|---|---|---|---|---|
| 15 | .83 | .85 | 1.44 | 38 | 1.19 | 1.28 | 2.75 |
| 16 | .84 | .86 | 1.47 | 39 | 1.22 | 1.31 | 2.85 |
| 17 | .85 | .87 | 1.51 | 40 | 1.24 | 1.36 | 2.95 |
| 18 | .86 | .88 | 1.54 | 41 | 1.27 | 1.41 | 3.07 |
| 19 | .87 | .90 | 1.58 | 42 | 1.31 | 1.47 | 3.19 |
| 20 | .88 | .91 | 1.62 | 43 | 1.35 | 1.54 | 3.32 |
| 21 | .89 | .92 | 1.66 | 44 | 1.40 | 1.62 | 3.45 |
| 22 | .90 | .93 | 1.70 | 45 | 1.47 | 1.71 | 3.60 |
| 23 | .91 | .95 | 1.74 | 46 | 1.54 | 1.80 | 3.75 |
| 24 | .92 | .96 | 1.79 | 47 | 1.62 | 1.90 | 3.92 |
| 25 | .93 | .98 | 1.84 | 48 | 1.71 | 2.02 | 4.09 |
| 26 | .95 | .99 | 1.89 | 49 | 1.81 | 2.14 | 4.27 |
| 27 | .96 | 1.01 | 1.94 | 50 | 1.91 | 2.28 | 4.46 |
| 28 | .98 | 1.03 | 2.00 | 51 | 2.03 | 2.42 | 4.67 |
| 29 | .99 | 1.05 | 2.06 | 52 | 2.15 | 2.59 | 4.89 |
| 30 | 1.01 | 1.07 | 2.12 | 53 | 2.29 | 2.76 | 5.12 |
| 31 | 1.03 | 1.09 | 2.18 | 54 | 2.44 | 2.95 | 5.36 |
| 32 | 1.05 | 1.11 | 2.25 | 55 | 2.60 | 3.15 | 5.62 |
| 33 | 1.07 | 1.14 | 2.32 | 56 | 2.78 | 3.38 | 5.89 |
| 34 | 1.09 | 1.16 | 2.40 | 57 | 2.96 | 3.62 | 6.19 |
| 35 | 1.11 | 1.19 | 2.48 | 58 | 3.17 | 3.87 | 6.50 |
| 36 | 1.14 | 1.21 | 2.56 | 59 | 3.39 | 4.17 | 6.83 |
| 37 | 1.16 | 1.24 | 2.65 | 60 | 3.64 | 4.50 | 7.18 |

### EXAMPLES.

Supposing a young man, on coming of age, wishes to effect an insurance for $3000 *for the whole period of his life.* To find the annual premium which he must pay, we look for 21 in the left hand column, and opposite that, in the column headed FOR LIFE, we find the number 1.66, which is the premium for one year on $100, and $\frac{1.66}{100}$=.0166 is the premium on $1 for 1 year, and hence $3000× .0166=$49.80, is the whole annual premium.

If the insurance is to last *for seven years only*, we find under that heading .92, and $\frac{.92}{100}$=.092, and $3000×.092=$27.60, the annual premium.

If the insurance is to be *for one year only*, we find .89 under that head, and $3000×.089=$26.70, the premium.

From these explanations we can now derive a rule for finding the annual premium, when the age of the individual and the sum to be insured for are known.

### RULE.

*Find the age in the left hand column of the table, and opposite this in the vertical column for the given period will be found the premium on* $100 *for one year, and this divided by* 100 *will give the premium on* $1 *for one year, and the given sum multiplied by this will be the whole annual premium.*

### EXERCISES.

1. What will be the annual premium for insuring a person's life, who is 18 years old, for $1000 for 7 years? Ans. $8.80.

2. What amount of annual premium must be paid by A. B. Smith, who wishes to insure his life for 7 years for $2000, his age being 25 years? Ans. $19.60.

3. John Jones, 35 years of age, wishes to effect an insurance for life for $1500. What amount of annual premium must he pay? Ans. $37.20.

4. A gentleman in Toronto, 32 years of age, being about to start for Australia, and wishing to provide for his family in case of his death, obtains an insurance for seven years for $3000. What amount of annual premium must he pay? Ans. $33.30.

5. Amos Fairplay, 48 years of age, being bound on a dangerous voyage, and wishing to provide for the support of his widowed mother, in case of accident to himself, insures his life for 1 year for $2500. What amount of premium must he pay? Ans. $42.75

6. A gentleman, 50 years of age, gets his life insured for $3000, by paying an annual premium of $4.46 on each $100 insured; if he should die at the age of 75 years, how much less will be the amount of insurance than the payments, allowing the latter to be without interest? Ans. $345

7. A gentleman, 45 years of age, gets his life insured for $5000, for which he pays an annual premium of $180, and dies at the age of 50 years. Suppose we reckon simple interest at 7 per cent. on his payments, what is gained by the insurance? Ans. $3911.

## PROFIT AND LOSS.

In the language of arithmetic, the expression *Profit and Loss* is usually applied to something gained or something lost in mercantile transactions, and the most important rule relating to it directs how to find at what increased rate above the cost price goods must be sold to produce a fair remuneration for time, labour and expenditure; or, in case of loss by unforeseen circumstanees, to estimate the amount of that loss as a guide in future transactions.

There are other cases, however, which we shall consider in detail.

### CASE I.

When the prime cost and selling price are known, to find the gain or loss.

### RULE.

*Find, by the rule of practice, the price at the difference between the prime cost and selling price, which will be the gain or loss according as the selling price is greater or less than the prime cost; or,*

*Find the price at each rate, and take the difference.*

### EXAMPLES.

To find what is gained by selling 4 cwt. of sugar, which cost 12½ cents per lb., at 15 cents per lb.

Here the difference between the two prices is 2½ cents per lb., and 400 lbs., at 2½ cents per lb., will give $10. Also, 400 lbs. at 15 cents per lb.=$60, and at 12½ cents=$50, and $60—$50=$10.

Again, if 120 lbs. of tobacco be bought at 92 cts. per lb., and, being damaged, is sold at 75 cents per lb., the loss will be a loss of 17 cents in the pound, and 120 lbs., at 17 cents per lb., is $20.40; or, 120 lbs., at 92 cents, will come to $110.40, and at 75 cents, to $90, and $110.40—$90=$20.40.

### EXERCISES.

1. If 224 lbs. of tea be bought at 60 cents per lb., and sold at 95 cents per lb.; how much is gained? Ans. $78.40.

2. A grocer bought 24 barrels of flour, at $5.80 per barrel, and sold 12 barrels of it at $6.10 per barrel, 9 barrels at $6.20 per barrel, and the rest at $6.25; how much did he gain? Ans. $8.55.

3. If a person is obliged to sell 216 yards of flannel, which cost him $86.40, at 37½ cents per yard; how much does he lose? Ans. $5.40.

4. If a dealer buys 78 bushels of potatoes, at 62½ cents per bushel, and retails them at 87½ cents per bushel; how much does he gain? Ans. $19.50.

5. A wine merchant bought 374 gallons of wine, at $3.20 per gallon, and sold it at $3.35 per gallon; how much did he gain? Ans. $56.10.

### CASE II.

To find at what price any article must be sold, to gain a certain rate per cent., the cost price, and the gain or loss per cent. being known.

### RULE.

*Multiply the cost price by* 1 *plus the gain, or* 1 *minus the loss.*

### EXAMPLE.

If a quantity of linen be bought for 75 cents a yard; at what price must it be sold to gain 16 per cent.?

Since 16 per cent. is 16 cents for every dollar, each dollar in the cost price would bring $1.16 in the selling price, so that we have $1.16×.75=.86, or 86 cents.

### EXERCISES.

1. Railroad shares being purchased for $2500, and sold at a gain of 20 per cent.; for what amount were they sold? Ans. $3000.

2. A property having been bought for $2000 was sold at a gain of 10 per cent. For what was it sold? Ans. $2200.

3. A horse was bought for $50, but, proving lame, was sold at a loss of 15 per cent. At what price was he sold? Ans. $42.50.

4. Bought a horse for $897 and sold it at a loss of 11 per cent; for what sum was it sold? Ans. $798.33.

5. A merchant buys dry goods for $1562, and sells them at a profit of 22 per cent. For what were they sold? Ans. $1905.64.

### CASE III.

To find the cost when the selling price and the gain per cent. are known.

### RULE.

*Divide the selling price by* 1 *plus the gain, or* 1 *minus the loss.*

To find what was the first cost of a quantity of flour which produced 8 per cent. profit by being sold for $127.44.

Since the gain is 8 per cent. of the cost, it follows that each dollar laid out has brought in a return of $1.08, and therefore the cost must have been as many dollars as the number of times that 1.08 is contained in 127.44, which is 118, and therefore the first cost must have been $118.

### EXERCISES.

1. If flaxseed is sold at $17.40 per bushel, and 13 per cent. lost, what was the first cost? Ans. $20.00.

2. A dealer bought 116 hogs for $580, and sold them at a gain of 25 per cent.; at what price did he sell each on an average? Ans. $4.

3. If 13 sheep be sold for $52.65, and 25 per cent. gained on the first cost, how much was paid for each at first? Ans. $3.24.

4. If 16¾ per cent. be lost on the sale of linen at $1.25, what was the first cost? Ans. $1.50.

5. If a quantity of glass be sold for $4, and 10 per cent. gained, for what sum was it bought? Ans. $3.64, nearly.

### CASE IV.

To find the gain or loss per cent. when the first cost and selling price are known.

### RULE.

*Divide the gain or loss by the first cost.*

### EXAMPLE.

If a web of linen be bought for $20 and sold for $25, what is the gain per cent?

Here $5 are gained on $20, and $20 is ⅕ of $100, therefore $25 will be gained on $100, *i. e.*, 25 per cent.

### EXERCISES.

1. If a quantity of goods be bought for $318.50, and sold for $299.39, how much per cent. is lost? Ans. 6 per cent.

2. If two houses are bought, the one for $150 and the other for $250; and the first sold again for $100 and the latter for $350, what per cent. is gained on the whole? Ans. 12½.

A grocer buys butter at 24 cents per lb. and sells it at 30 cents per lb., what does he gain per cent? Ans. 25.

4. If a cattle dealer buys 20 cows, at an average price of $20, and pays 50 cents for the freight of each per railroad, what per cent. does he gain by selling them at $25.62½ each? Ans. 25.

5. A tobacconist bought a quantity of tobacco for $75, but a part of it being lost, he sold the remainder for $60: what per cent. did he lose? Ans. 20.

### CASE V.

Given the gain or loss per cent. resulting from the sale of goods at *one* price, to find the gain or loss per cent. by selling the same at *another* price.

### RULE.

*Find by case* III. *the first cost, and then by case* IV. *the gain or loss per cent. on that cost at the second selling price.*

### EXAMPLE.

If a farmer sells his hogs at $5 each, and realizes 25 per cent.; what per cent. would he realize by selling them at $7 each.

We find by case III., that the cost was $4, and then by case IV. what the gain per cent. would be on the second supposition, that is $3÷4=.75, or 75 per cent.

### EXERCISES.

1. If a grocer sells rum at 90 cents per bottle, and gains 20 per cent.; what per cent. would he gain by selling it at $1.00 per bottle? Ans. 33⅓.

2. If a hatter sells hats at $1.25 each, and loses 25 per cent.; what per cent. would he lose by selling them at $1.60 each? Ans. 4.

3. If a storekeeper sells cloth at $1.25, and loses 15 per cent.; would he gain or lose, and how much, by selling at $1.65? Ans. He would gain 12 per cent. nearly.

4. A milliner sold bonnets at $1.25, and thereby lost 25 per cent.; would she have gained or lost by selling them at $1.40? Ans. She would have lost 16 per cent.

5. A merchant sold a lot of goods for $480, and lost 20 per cent.; would he have gained or lost by selling them for $720, and how much? Ans. He would have gained 20 per cent.

6. A quantity of grain was sold for $90, which was 10 per cent. less than the cost; what would have been the gain per cent. if it had been sold for $150? Ans. 50.

7. A grocer sold tea at 45 cents per pound, and thereby gained $12\frac{1}{2}$ per cent.; what would he have gained per cent. if he had sold the tea at 54 cents per pound? Ans. 35.

8. A farmer sold corn at 65 cents per bushel, and gained 5 per cent.; what per cent. would he have gained if he had sold the corn at 70 cents per bushel? Ans. $13\frac{1}{13}$.

### MISCELLANEOUS EXERCISES.

1. If I buy goods amounting to $465, and sell them at a gain of 15 per cent.; what are my profits?

2. Suppose I buy $400\frac{1}{5}$ barrels of flour, at $16.75 a barrel, and sell it at an advance of $\frac{3}{8}$ per cent.; how much do I gain? Ans. $25.14.

3. If I buy 220 bushels of wheat, at $1.15 per bushel, and wish to gain 15 per cent. in selling it; what must I ask a bushel?

4. A grocer bought molasses for 24 cents a gallon, which he sold for 30 cents; what was his gain per cent.? Ans. 25.

5. A man bought a horse for $150, and a chaise for $250, and sold the chaise for $350, and the horse for 100; what was his gain per cent.? Ans. $12\frac{1}{2}$.

6. A gentleman sold a horse for $180, and thereby gained 20 per cent.; how much did the horse cost him? Ans. $150.

7. In one year the principal and interest of a certain note amounted to $810, at 8 per cent.; what was the face of the note? Ans. $750.

8. A carpenter built a house for $990, which was 10 per cent. less than what it was worth; how much should he have received for it so as to have made 40 per cent.? Ans. $1540.

9. A broker bought stocks at $96 per share, and sold them at $102 per share; what was his gain per cent.? Ans. $6\frac{1}{4}$.

10. A merchant sold sugar at $6\frac{1}{2}$ cents a pound, which was 10 per cent. less than it cost him; what was the cost price? Ans. $7\frac{2}{9}$ cents per pound.

11. A merchant sold broadcloth at $4.75 per yard, and gained $12\frac{1}{2}$ per cent.; what would he have gained per cent. if he had sold it at $5.25 per yard? Ans. $24\frac{13}{38}$.

12. I sold a horse for $75, and by so doing, I lost 25 per cent.; whereas, I ought to have gained 30 per cent.; how much was he sold for under his real value? Ans. $55.

13. A watch which cost me $30 I have sold for $35, on a credit of 8 months; what did I gain by my bargain, allowing money to be worth 6 per cent.? Ans. $3.65.

14. Bought 84 yards of broadcloth, at $5.00 per yard; what must be my asking price in order to fall 10 per cent., and still make 10 per cent. on the cost? Ans. $6.11$\frac{1}{9}$.

15. A farmer sold land at 5 cents per foot, and gained 25 per cent. more than it cost him; what would have been his gain or loss per cent. if he had sold it at 3$\frac{1}{2}$ cents per foot?

Ans. 12$\frac{1}{2}$ per cent. loss.

16. What must I ask per yard for cloth that cost $3.52, so that I may fall 8 per cent., and still make 15 per cent., allowing 12 per cent. of sales to be in bad debts? Ans. $5.

17. A merchant sold two bales of cotton at $240 each; for one he received 60 per cent. more than its cost, and for the other 60 per cent. less than its cost. Did he gain or lose by the operation, and how much? Ans. loss $270.

18. Bought 2688 yards of cloth at $2.16 per yard, and sold one-fourth of it at $2.54 per yard; one-third of it at $2.75 per yard, and the remainder at $2,90 per yard. Find the whole gain, and the gain per cent. Ans. $1612.80 and 27$\frac{883}{1121}$ per cent.

19. A flour merchant bought the following lots:—

| | | | |
|---|---|---|---|
| 118 | barrels at | $9.25 | per barrel. |
| 212 | " | 9.50 | " |
| 315 | " | 9.12$\frac{1}{2}$ | " |
| 400 | " | 10.00 | " |

The expenses amounted to $29.50, besides insurance at $\frac{1}{30}$ per cent. At what price must he sell it per barrel to gain 15 per cent?

Ans., $11.05.

20. Bought 100 sheep at $5 each; having resold them at once and received a note at six months for the amount; having got the note discounted at the Royal Canadian Bank, at six per cent., I found I had gained 20 per cent. by the transaction. What was the selling price of each sheep? Ans., $6.19.

## STORAGE.

When a charge is made for the accommodation of having goods kept in store, it is called *storage.*

Accounts of storage contain the entries showing when the goods were received and when delivered, with the number, the description of the articles, the sum charged on each for a certain time, and the total amount charged for storage, which is generally determined by an average reckoned for some specified time, usually one month (30 days).

### EXAMPLES.

1. What will be the cost of storing wheat at 3 cents per bushel per month, which was received and delivered as follows:—Received, August 3rd, 1865, 800 bushels; August 12th, 600 bushels. Delivered, August 9th, 250 bushels; September 12th, 350 bushels; September 15th, 400 bushels, and October 1st, the balance.

### SOLUTION.

| 1865. | | Bush. | | Days. | | Bush. | |
|---|---|---|---|---|---|---|---|
| August 3. | Received…… | 800 | × | 6 | = | 4800 | in store for one day. |
| " 9. | Delivered…… | 250 | | | | | |
| | Balance. …… | 550 | × | 3 | = | 1650 | in store for one day. |
| " 12. | Received…… | 600 | | | | | |
| | Balance ……. | 1150 | × | 31 | = | 35650 | in store for one day. |
| Sept. 12. | Delivered…… | 350 | | | | | |
| | Balance…….. | 800 | × | 3 | = | 2400 | in store for one day. |
| " 15. | Delivered……. | 400 | | | | | |
| | Balance…….. | 400 | × | 16 | = | 6400 | in store for one day. |
| Oct. 1. | Delivered……. | 400 | | | | | |
| | Total………………………………… | | | | | 50900 | in store for one day. |

50,900 bushels in store for *one day* would be the same as 50900÷30=1696$\frac{2}{3}$ bushels in store for *one month* of 30 *days,* and the storage of 1697 bushels for one month, at 3 cents per month, would equal 1697×.03=$50.91.

It is customary, in business, when the number of articles upon which storage is to be charged, as found, contains a fraction *less* *than a half,* to reject the fraction; but if it is *more than a half,* to regard it as an entire article.

From the solution of the foregoing example, we deduce the following

### RULE.

*Multiply the number of bushels, barrels, or other articles, by the number of days they are in store, and divide the sum of the products by* 30, *or the number of days in any term agreed upon. The quotient will give the number of bushels, barrels, or other articles on which storage is to be charged for that term.*

2. What will be the cost of storing salt at 3 cents a barrel per month, which was put in store and taken out as follows: Put in, January 2, 1866, 450 barrels; January 3, 75 barrels; January 18, 300 barrels; January 27, 200 barrels; February 2, 75 barrels. Taken out, January 10, 60 barrels; January 30, 150 barrels; February 10, 190 barrels; February 20, 300 barrels; March 1, 250 barrels; and on March 12, the balance, 150 barrels.

Ans., $39.44.

3. Received and delivered, on account of T. C. Musgrove, sundry bales of cotton, as follows: Received January 1, 1866, 2310 bales; January 16, 120 bales; February 1, 300 bales. Delivered February 12, 1000 bales; March 1, 600 bales; April 3, 400 bales; April 10, 312 bales; May 10, 200 bales Required the number of bales remaining in store on June 1, and the cost of storage up to that date, at the rate of 5 cents a bale per month.

Ans., 218 bales in store; $321.18 cost of storage.

4. W. T. Leeming & Co., Commission Merchants, Montreal, in account with A. B. Smith & Co., Toronto, for storage of salt and gunpowder, which was received and delivered as follows:

Received January 18, 1866, 400 kegs of gunpowder and 50 barrels of salt; January 25, 250 barrels of salt; February 4, 150 barrels of salt and 50 kegs of gunpowder; February 15, 100 kegs of gunpowder; March 5, 64 kegs of gunpowder; April 15, 50 kegs of gunpowder and 75 barrels of salt. Delivered, February 25, 15 kegs of gunpowder, and 40 barrels of salt; March 10, 150 kegs of gunpowder and 285 barrels of salt; April 20, 200 kegs of gunpowder; April 25, 150 barrels of salt and 200 kegs of gunpowder. Required, the number of barrels of salt and kegs of gunpowder in store May 1, and the bill of storage up to that date. The rate of storage for salt being 3 cents a barrel per month, and for gunpowder 12 cents a keg per month.

Ans. In store, 50 barrels of salt and 99 kegs of gunpowder; bill of storage, $206.01.

## GENERAL AVERAGE.

THIS is the term used to denote the contribution of all persons interested in a ship, freight, or cargo, towards the loss or damage incurred by any particular part of a ship, or cargo, for the preservation of the rest. This sacrifice of property is called *jettison*, from the goods being cast into the sea to save the vessel; although not only property destroyed in that way is the subject of general average, but also any damages or expenses voluntarily incurred for the good of all. For example, the expense of unloading the cargo that the ship may be repaired; masts or sails cut away and abandoned to save the ship.

The only articles exempt from contribution are provisions, wearing apparel of passengers, and wages of the seamen.

The owners contribute according to the clear value of the ship and freight at the end of the voyage, after deducting the wages of the crew and other expenses.

Goods that have been subject to jettison, and are lost, are valued, when the average is calculated at the place of the ship's destination, at the price they could have sold for there; but when the average is to be ascertained at the port of lading, the invoice price is the standard of value.

In making an account of the articles which are to contribute, the property lost or sacrificed must be included, and its owners must suffer the same proportionate loss as the rest. The losses to the different parties interested in the vessel, freight, and cargo, are paid by their insurers.

When repairs have to be made to a ship—new sails, masts, or rigging, for example,—one-third of the expense is deducted on account of *melioration*, or the improved condition of the ship by these repairs. When the ship is new, and on her first voyage, the full amount of the expense of repairs is allowed in computation of the loss.

### EXAMPLE.

On the 26th June, 1865, the steamer *Canada* left Quebec for Liverpool, with a cargo as follows:—Shipped by T. A. Collins, $7480; R. Evans & Co., $5365; H. C. Wright, $9218; W. Manning & Co., $11,428; E. Carpenter, $7559. In the Gulf of St. Lawrence a heavy gale was experienced, during which cargo to the value of $3498 was thrown overboard; of this $1123.40 belonged to R. Evans & Co., and the balance to E. Carpenter. The necessary repairs of the steamer cost $876, and the expenses in port, while getting repaired, were $253. The steamer was valued at $100,000; gross freight, $4310. The seamen's wages were $860. What was the loss per cent., and what was the loss of each contributory interest?

### SOLUTION.

| *Loss for general benefit.* | | *Contributory interests.* | |
|---|---|---|---|
| Cargo thrown overboard, | $3498 | Value of steamer.......... | $100,000 |
| Repairs to steamer less ⅓ | 584 | Invoice price of cargo.... | 41,050 |
| Expenses in port......... | 253 | Fr'ght less seamen's wages | 3,450 |
| Total loss........... | $4335 | Total contrib. int.. | $144,500 |

$4335÷144,500=.03 loss per unit, or 3 per cent.

$100,000×.03=$3000.00, steamer's share of loss.

7,480×.03= 224.40, T. A. Collins' share of loss.

5,365×.03= 160.95, R. Evans & Co.'s share of loss.

9,218×.03= 276.54, H. C. Wright's share of loss.

11,428×.03= 342.84, W. Manning & Co.'s share of loss.

7,559×.03= 226.77, E. Carpenter's share of loss.

3.450×.03= 103.50, Freight's share of loss.

$4335.00, Total loss.

$3000.00—837.00=$2163.00, balance payable by steamer.

1123.40—160.95= 962.45, balance receivable by R. Evans & Co.

2374.60—226.77= 2147.83, balance receivable by E. Carpenter.

NOTE.—It is evident that since the steamer lost $837, ($584 by repairs, and $253 by expenses,)-that the net amount required from the steamer will be $3000—837=$2163. R. Evans & Co. having lost by merchandise being thrown overboard $1123.46, a sum greater than their share of the general loss, so that there must be due them $1123.40—160.95=$962.45; so also the amount of E. Carpenter's share of the general loss must be deducted from his individual loss in order to find the balance due him.

### RULE.

*Find the rate per unit of loss, by which multiply the value of each contributory interest, and the product will be the share of loss to be sustained by each.*

### EXERCISES.

1. The steamship *Nova Scotian*, on her trip from Quebec to Liverpool, was crippled in a storm, in consequence of which the captain had to throw overboard a portion of the cargo, amounting in value to $4465.50, and the necessary repairs of the vessel cost $423. The contributory interests were as follows :—Vessel, $30,000; gross freight, $6225; cargo shipped by J. Jones & Co., $3650; by Henry Anderson, $6500; by George Millan, $2000; by J. Foster & Son, $550; by Brown Brothers, $5450; and by Wilson & Carter, $8500. Of the cargo thrown overboard, there belonged to Henry Anderson the value of $3000, and to Brown Brothers the remainder, $1465.50. The cost of detention in port in consequence of repairs, was $116.50; seamen's wages, $2075. How ought the loss to be shared among the contributory interests? Ans. 8 per cent.

2. The steamer *Spartan* left Toronto for Montreal, June 30th, loaded with 7210 bushels of spring wheat, shipped by J. M. Musgrove, and invoiced at 95 cents per bushel; 4815 bushels of corn, shipped by Thomas A. Bryce & Co., and invoiced at 60 cents per bushel; 2180 barrels of flour, shipped by A. B. Smith & Co., and invoiced at $5.50 per barrel. When near Montreal, the steamer ran upon a rock, and the captain found it necessary to throw overboard 1600 bushels of wheat, 1280 bushels of corn, and 720 barrels of flour. On estimating the proportionate loss, it was allowed that the wheat would have sold in Montreal at an advance of 10 per cent., the corn at an advance of 15 per cent., and the flour for $5 per barrel. The contributory interests were :—Steamer, $95,000; cargo, $ ; gross freight, $2361.20. The cost of repairs to steamer was $2198.15; cost arising from detention during repairs, $318; seamen's wages, $252.50. How much of the loss had each contributory interest to bear? Ans. 7½ per cent.

3. The propeller *Edith* left Hamilton for Kingston with 7600 bushels of wheat, valued at $1.25 per bushel, shipped by Dunn, Lloyd & Co., and insured in the Queen Insurance Company at 1¾ per cent.; 9300 bushels of corn, valued at 75 cents per bushel,

shipped by J. W. Roe, and insured in the Ætna Insurance Company at $1\frac{3}{4}$ per cent.; 14,800 bushels of oats, valued at $37\frac{1}{2}$ cents per bushel, shipped by Morris, Wright & Co., and insured in the Provincial Insurance Company at $1\frac{1}{2}$ per cent.; 1,800 barrels of flour, valued at $5.25 per barrel, shipped by Smith & Worth, and insured in the Beaver Insurance Company at $1\frac{1}{4}$ per cent. When near Kingston, a collision with the steamer *Spartan* occurred, and it was found necessary to throw overboard the flour, 4600 bushels of oats, and 3150 bushels of wheat. The propeller was valued at $45,000, and insured in the Beaver Insurance Company for $12,000, at 2 per cent., and in the Royal for $25,000, at $2\frac{1}{4}$ per cent. The gross freight was $4950; seamen's wages, $340, and repairs to the boat, $3953.75; what was the loss sustained by each of the contributory interests, the propeller being on her first trip?

## TAXES AND CUSTOMS DUTIES.

*A tax* is a money payment levied upon the subjects of a State, or the members of any community, for the support of the government.

A tax is either levied upon the property or the persons of individuals. When levied upon the person, it is called a *poll tax.*

It may be either *direct* or *indirect.* When direct, it is levied from the individuals, or the property in the hands of the ultimate owners. When indirect, it is in the nature of a *customs* or *excise duty,* which is levied upon imports, or manufactures, before they reach the consumer, although in the end they are paid by the latter.

*Customs duties* are paid by the importer of goods at the port of entry, where a *custom-house* is stationed, with government employees called *custom-house officers,* to collect these dues.

*Excise duties* are those levied upon articles manufactured in the country.

*An invoice* is a complete list of the particulars and prices of goods sent from one place to another.

*A specific* duty is a certain sum paid on a ton, hundred weight, yard, gallon, &c., without regard to the cost of the article.

An *ad valorem* duty is a percentage levied on the actual cost, or fair market value of the goods in the country from which they are imported.

*Gross weight* is the weight of goods, upon which a specific duty is to be levied, before any allowances are deducted.

*Net weight* is the weight of the goods after all allowances are deducted.

Among the allowances made are the following:—

*Breakage*—an allowance on fluids contained in bottles or breakable vessels.

*Draft*—the allowance for waste.

*Leakage*—an allowance for waste by leaking.

*Tare* and *tret* are the deductions made for the weight of the case or barrel which contains the goods.

When goods, upon which duty is payable, are exported to Canada from any foreign country, through the United States, under bonds, they are only valued for duty as if they were imported direct into Canada from the place of export. An Order in Council has extended this provision to *free* as well as dutiable goods.

As customs duties are payable in gold, or its equivalent, upon goods imported from the United States, it is evident that an injustice would be done if the amount mentioned in the American invoice were taken to be the dutiable value of the goods. Yet, as the value of American currency, when compared with the gold standard, varies almost every day, it would be a source of great confusion and irregularity to allow each custom-house officer to affix his own value to the currency. To avoid this, notices are published every week in the *Canada Gazette* as to the rate of discount to be allowed on American invoices, which is fixed in accordance with the price of gold, as represented by exchange. This method is at once fair and productive of uniformity.

### EXAMPLES.

To find the specific duty on any quantity of goods.

Suppose a Montreal Provision Merchant imports from Ireland 59 casks of butter, each weighing 68 lbs., and that 12 lbs. tare is allowed on each cask, and 2 cents per lb. duty on the net weight.

We find the gross is....................$59\times68=4012$ lbs.
" tare is....................$59\times12=708$ lbs.

Hence the net weight is..........................3304 lbs.
The duty is 2 cents per lb.......................... 2

The duty, therefore, is..........................$66.08

To find the *ad valorem* duty on any quantity of goods.

Suppose a Hamilton dry goods merchant to import from France 436 yards of silk, at $1.75 per yard, and that 35 per cent. duty is charged on them.

Here we have first the whole price by the rule of Practice to be $763, then the rest of the operation is a direct case of percentage, and therefore we multiply $763, by .35, which gives $267.05, the amount of duty on the whole.

Hence we have the following

### RULE FOR SPECIFIC DUTY.

*Subtract the tare, or other allowance, and multiply the remainder by the rate of duty per box, gallon, &c.*

### RULE FOR AD VALOREM DUTY.

*Multiply the amount of the invoice by the rate per unit.*

### EXERCISES.

1. Find the specific duty on 5120 lbs. of sugar, the tare being 14 per cent., and the duty 2¾ cents per lb. Ans. $121.09.

2. What is the ad valorem duty on a quantity of silks, the amount of the invoice being $95,800, and the duty 62½ per cent? Ans. $59,875.

3. At 30 per cent., what is the ad valorem duty on an importation of china worth $1260? Ans. $378.

4. What is the specific duty at 10 cents per lb. on 45 chests of tea, each weighing 120 lbs., the tare being 10 per cent? Ans. $486.

5. What is the ad valorem duty on a shipment of fruit, invoiced at $4560, the duty being 40 per cent? Ans. $1824.

6. What is the specific duty on 950 bags of coffee, each weighing 200 lbs., the duty being 2 cents per lb. and the tare 2 per cent? Ans. $3724.

7. What is the ad valorem duty on 20 casks of wine, each containing 75 gallons, at 18 cents a gallon? Ans. $270.

8. A. B. shipped from Montreal 24 pipes of molasses, each containing 96 gallons; 2 per cent. was deducted for leakage, and 12 cents duty per gallon charged on the remainder; how much was the duty? Ans. $270.95.

9. What is the ad valorem duty on a shipment of cutlery from Sheffield, England, to Montreal, the invoice amounting to $840, and the duty charged 25 per cent.? Ans. $210.

10. What is the duty on 11,900 lbs. of pepper at $6\frac{1}{2}$ cents per lb., the duty being charged at $3\frac{1}{2}$ per cent., and the tare being 5 per cent.? Ans. $25.72.

11. Peter Smith & Co., London, import from Cadiz, 80 baskets of port wine, at 70 francs per basket; 42 baskets of sherry wine, at 35 francs per basket; 60 casks of champagne, containing 31 gallons each, at 4 francs per gallon. The waste of the wine in the casks was reckoned at a gallon each cask, and the allowance for breakage in the baskets was 5 per cent.; what was the duty at 30 per cent., $18\frac{3}{5}$ cents being taken as equal to 1 franc? Ans. $776.54.

12. Mitchell and Graham, Glasgow, import from Quebec, per ship *Walter Scott*, 24 boxes of sugar, 400 lbs. each, at 5 cents; 40 hogsheads of molasses, containing 63 gallons each, at 30 cents per gallon; 260 boxes of oranges, at $2 per box, and 410 boxes of cigars, at $7 a box; the tare on the sugar was 10 per cent., and the leakage on the molasses 2 per cent.; the duty on the sugar and molasses was 24 per cent.; on the oranges 8 per cent., and on the cigars 30 per cent.; what was the whole duty? Ans. $1184.09.

13. What duty would a merchant in Toronto have to pay on merchandise purchased in New York city to the amount of $1834.60, American currency, the rate of duty being 25 per cent., and a discount of $31\frac{1}{2}$ per cent. being allowed at the custom-house on the invoice price? Ans. $314.18.

14. John McMaster & Co., of Collingwood, bought of A. Smith, of Buffalo, N. Y., goods invoiced at $5430.50, which should have passed through the custom-house during the first week in May, when the discount on American invoices was $43\frac{1}{4}$ per cent., but they were not passed until the fourth week in May, when the discount was $36\frac{3}{4}$ per cent. The duty in both cases being 20 per cent.; what was the loss sustained by McMaster & Co. on account of their goods being delayed? Ans. $70.60.

In the above case, at what per cent. higher would McMaster & Co. require to mark their goods, above cost, in order to make a clear gain of 25 per cent.?

## STOCKS AND BONDS.

The capital of a company is called its stock, and is usually divided into portions or shares, which are subscribed for by those who intend becoming members, or stockholders in the company. The whole sum is very seldom paid in at once on each share, but is only paid in by instalments or *calls*, according as the money is required for the purposes of the undertaking. It very often happens that the whole amount of the stock is never called in; for example, if the shares are $100, the first call may be for $20, then the next for $10, and so on, as the necessities of the company demand. It is quite possible that more money may be required after the original shares have all been paid up. To raise this sum the company frequently is compelled to dispose of *preference shares*, upon which a certain rate of interest is guaranteed out of the first profits.

The interest or profits paid upon stock are called *dividends*, because they are declared by the board to be the amount that the shareholders are entitled to have divided amongst them.

When $100 stock sells for $100 cash, it is said to be at *par;* when it can be placed on the market for more than $100 cash, it is at a *premium;* and when it will only sell for less than $100 cash, it is at a *discount.*

At the meetings of shareholders the election of officers and other questions are usually decided by vote. It is often provided that the right to vote shall not be increased in exact proportion to the number of shares held by any one person. For example, all persons holding, say four shares and under, may cast a vote for each share; those holding ten shares may give seven votes; those holding twenty may give ten votes, and so on, with a provision that no shareholder shall have more than thirty votes. The vote may generally be given by a proxy, who must be a qualified stockholder.

Governments, municipal corporations, and incorporated companies, contract loans at some fixed rate of interest, and give their *bonds* to the lenders as security for the repayment. The interest is usually payable half-yearly, and the time for payment of the principal is, in most cases, specified in the bond, although in the case of the National debt of England, the time for the payment of the principal is in the option of the government. The rate of interest upon this debt is only three per cent., and is equivalent to a perpetual annuity. These stocks are called the *Funds*, or *Consols*, a contraction for "consolidated annuities." As the interest on them is fixed, the value of the principal will rise and fall according to the abundance or scarcity of money, and its value, from time to time. When capital is abundant and there are few channels for profitable investment, the rate of interest will be low, and the funds will accordingly rise. On the other hand, when there is a demand for capital, and interest is high, the funds will fall. When the rate of interest is fixed, the value of the principal must vary, and when the amount of the principal is fixed, the rate of interest upon it must vary, in order to make the stock marketable at all times.

The interest upon bonds, or debentures, as they are sometimes called, is often expressed to be payable upon the presentment of *coupons* attached to the bonds originally, and cut off as they are required for use.

*Certificates of stock* are given to shareholders by companies, to show what stock they are entitled to upon the books.

### CASE I.

**The premium or discount being known, to find the market value of any amount of stock.**

### EXAMPLES.

If G. W. R. shares are at 7 per cent. *premium*, to find the value of 30 shares of $100.

Here it is plain that each $100 will bring $107, and that each $1 will bring $1.07, and as the par value is $3000, the advanced value will be 3000 times 1.07, which gives $3210, the market value, and $3210—$3000=$210, the gain.

Again, if the same are sold at a *discount* of 7 per cent., it is plain that each $100 would bring only $93, and therefore each $1 would bring only $0.93, and therefore as the par value is $3000, the depreciated value will be 3000 times .93, which gives $2790, and therefore the loss would be $3000—2790=210.

From this we derive the

### RULE.

*Multiply the par value by 1 plus or minus the rate per unit, according as the shares are at a premium or a discount.*

### EXERCISES.

1. What is the market value of $450 stock, at 8½ per cent. discount? Ans. $411.75.

2. What is the value of 29 shares of $50 each, when the shares are 11 per cent. below par? Ans. $1290.50.

3. A man purchased 60 shares of $5 each, from an oil well company, when the shares were at a discount of 8 per cent., and sold them when they were at a premium of 10 per cent; how much did he gain? Ans. $54.

4. A man purchased $10,000 stock when it was at an advance of 8 per cent., and sold when it was at a discount of 8 per cent.; how much did he lose? Ans. $1600.

5. If a man buys 15 shares of $100 each, when the shares are at a premium of 5 per cent., and sells when they have advanced to 12 per cent., how much does he gain? Ans. $105.

### CASE II.

To find how much stock a given sum will purchase at a given premium or discount.

Let it be required to find how much stock can be purchased for $21,600 when at a *premium* of 8 per cent.

In this case it will require $108 to purchase $100 stock, and therefore $1.08 to purchase $1 stock, and hence the amount that can be purchased for $21600 will be represented by the number of times that $1.08 is contained in 21600, which gives $20000.

Again: Let it be required to find how much stock can be purchased for $5520, when at a discount of 8 per cent. When stocks are 8 per cent. below par, $92 will purchase $100 stock, and therefore $0.92 will purchase $1, and hence the amount that can be purchased for $5520 will be represented by the number of times that .92 is contained in 5520, which gives $6000 stock.

Hence we derive the

### RULE.

*Divide the given sum by* 1 *plus or minus the rate per unit, according as the shares are at a premium or a discount.*

### EXERCISES.

6. When stocks are at a premium of 12 per cent., how much can be purchased for $8064? Ans. $7200.

7. When stocks are at a discount of 9 per cent., how much can be bought for $3640 ? Ans. $4000.

8. When G. T. R. stock is at 18 per cent. below par, how much can be bought for $42,640. Ans. $52000.

9. When G. W. R. stock is at a premium of 9 per cent., how much will $4578 purchase ? Ans. $4200.

10. When government stock is selling at 92½, what amount of stock will $28,675 purchase, and to what will it amount with brokerage at ¼ per cent. ? Ans. $31077.50.

### CASE III.

The premium or discount being known, to find the par value.

To find the par value of $1,296, when stock is at a premium of 8 per cent.

At 8 per cent. premium, each $1 brings $1.08, hence the par value will be represented by the number of times 1.08 is contained in 1296, which gives $1200 for the par value.

To find the par value of $1104, when stock is at a discount of 8 per cent.

Each $1 will bring $0.92, and therefore the par value will be represented by the number of times that .92 is contained in 1104, which gives $1200, the par value. Hence the

### RULE.

*Divide the market value by* 1 *plus or minus the rate per unit, according as the stocks are selling above or below par.*

### EXERCISES.

11. What is the par value of $24420, when stock is 11 per cent. above par ? Ans. $22000.

12. What is the par value of $10800, when stocks are at a discount of 4 per cent. ? Ans. $11250.

13. When government stocks are at 6 per cent. premium ; how much will $20246 purchase at par value ? Ans. $19100.

14. The shares in a canal company are at 15 per cent. discount ; how many shares of $100 will $11390 purchase ? Ans. 134.

15. The shares of a British gas company were selling in 1848, at a discount of 12 per cent. ; a speculator purchased a certain number of shares for £792 ; the value of the shares suddenly rose to par ; how many shares did he purchase, and how much did he gain ?

Ans. 9 shares ; £108 gain.

### CASE IV.

To find to what rate of interest a given dividend corresponds.

If a person receives a dividend of 12 per cent. on an investment made at 20 per cent. above par, the corresponding interest may be calculated thus:

As the stock was bought at 20 per cent., or .20 above par, $1.20 of market value corresponds to $1 of par value, and as every $1 of par value corresponds to 12 per cent. interest, or .12, it follows that the per cent. which was invested will be represented by the number of times that 1.20 is contained in .12, which is .10 or 10 per cent. Hence the

### RULE.

*Divide the rate per unit of dividend by* 1 *plus or minus the rate per cent. premium or discount, according as the stocks are above or below par.**

### EXERCISES.

16. If a dividend of 10 per cent. be declared on stock vested at 25 per cent. advance; what is the corresponding interest?
Ans. 8 per cent.

17. If a dividend of 4 per cent. be declared on stock invested at 12 per cent. below par, what is the corresponding interest?
Ans. $4\frac{6}{11}$.

18. If money invested at 24 per cent. yields a dividend of 15 per cent., what is the rate of interest? Ans. $12\frac{3}{31}$.

19. If railroad stock is invested at 18 per cent. above par, and a dividend of 6 per cent. be declared, what is the rate of interest?
Ans. $5\frac{5}{59}$.

20. If bank stock be invested at 15 per cent. below par, and a dividend of 10 per cent. declared, what is the rate of interest?
Ans. $11\frac{13}{17}$.

### MISCELLANEOUS EXERCISES.

1. What must be paid for 20 shares of railway stock, at 5 per cent. premium, the shares being $100 each? Ans. $2100.

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* To find at what price stock paying a given rate per cent. dividend can be purchased, so that the money invested shall produce a given rate of interest, *divide the rate per unit of dividend by the rate per unit of interest.*

2. What is the par value of bank stock worth \$8740, at a premium of 15 per cent.? Ans. \$7600.

3. Railway stock was bought at 15¾ below par, for \$1895.62½; how many shares were there, each share being \$150? Ans. 15 shares.

4. If 6 per cent. stock yields 8 per cent. on an investment, at what per cent. discount was it bought? Ans. 25.

5. If bank stock which pays 11 per cent. dividend, is 10 per cent. above par, what is the corresponding rate of interest on any investment? Ans. 10.

6. When 4 per cent. stocks were at 17⅞ discount, A bought \$1000; how much did he pay, and how much did he gain by selling when stock had risen to 86¼? Ans. \$821.25, and \$41.25.

7. What will \$850 bank stock cost at a discount of 9⅜ per cent., ⅛ per cent. being charged for brokerage? Ans. \$771.38.

8. On the data of the last example, how much would be lost by selling out at 10½ per cent.? Ans. \$10.03.

9. What income should I get by laying out \$1620 in the purchase of 3 per cent. stock at 81? Ans. \$60.

10. What sum must be invested in the 4 per cent. stocks at 84, to yield an income of \$280? Ans. \$5880.

11. What rate of interest will a person receive by investing in the 4½ per cent. stocks at 90? Ans. 5 per cent.

12. A person transfers his capital from the 3½ per cent. stocks at 77, to the 4 per cent. at 89; what is the increase or decrease per cent. in his income? Ans. Decrease 25.

13. A person sells out of the 3 per cent. stock at 96, and invests his money in railway 5 per cent. stock at par; how much per cent. is his income increased? Ans. 60.

14. What must be the market value of 5½ per cent. stock, so that after deducting an income tax of 2 cents on the dollar, it may produce 5 per cent. interest? Ans. 107⅘.

15. A gentleman invested \$7560 in the 3½ per cent. stocks at 94½, and on their rising to 95 sold out, and purchased G. T. R. 4 per cent. stock at par; what increase did he make in his annual income? Ans. \$24.

16. How much more may a person increase his annual income by lending \$3800, at 6 per cent., than by purchasing railway 5 per cent. stock at 95? Ans. \$28.

17. A person sells $4200 railway stock which pays 6 per cent. at 115, and invests one-third of the proceeds in the 3 per cent. consols at $80\frac{1}{2}$, and the balance in savings' bank stock, which pays 9 per cent. at par; what is the decrease or increase of his annual income?
Ans. Increase $97.80.

18. A person having $10,000 consols, sells $5000 at $94\frac{7}{8}$, and on their rising to $98\frac{5}{8}$ he sells $5000 more; on their again rising he buys back the whole at 96; how much does he gain? Ans. $75.

19. The sum of $4004 was laid out in purchasing 3 per cent. stocks at $89\frac{3}{8}$, and a whole year's dividend having been received upon it, it was sold out, the whole increase of capital being $302.40; at what price was it sold out? Ans. $93\frac{1}{8}$.

20. Suppose a person to have been an original subscriber for 500 shares of $50 each, in the Royal Canadian Bank, payable by instalments, as follows:—$\frac{1}{8}$ in three months, which he sold for $5\frac{1}{4}$ per cent. advance; $\frac{2}{3}$ in 6 months, which brought him $6\frac{3}{5}$ per cent. advance, and the balance in nine months, which he was compelled to sell at $8\frac{3}{4}$ per cent. discount; what did he gain by the whole transaction? Ans. $808.33.

## PARTNERSHIP.

Partnership has been defined to be the result of a contract, under which two or more persons agree to combine property, or labour, for the purpose of a common undertaking, and the acquisition of a common profit.

A *dormant*, or *sleeping* partner, is one who shares in the concern, but does not appear to the world as such.

A *nominal* partner is one who lends his name and credit to a firm, without having any real interest in the profits.

All the partners may contribute equally to the business; or the capital may be contributed by some or one, and the skill and labour by the other. Or, unequal proportions may be furnished by each.

The contract need not be in writing, but all parties to be bound must assent to it, and it is usually contained in an instrument called "*Articles of Partnership.*" A dissolution can take place at any time by mutual consent.

A partnership *at will* is one to which there is no limited time affixed for its continuance, and the whole firm may be dissolved by any of its members at a moment's notice. A document is, however, generally drawn up and signed upon a dissolution, called a *settlement*, which contains a statement of the mode of adjustment of the accounts, and the apportionment of profits or losses.

### EXAMPLE.

Two persons, A. and B., enter into partnership. A. invests $300 and B. $400. They gain during one year $210; what is each man's share of the profit?

### SOLUTION BY PROPORTION.

A.'s stock, $300
B.'s " 400

Entire stock $700 : 300 : : $210 : $90 A.'s gain.
" " 700 : 400 : : $210 : 120 B.'s "

### SOLUTION BY PERCENTAGE.

Since the entire amount invested is $700, and the gain $210, the gain on every $1 of investment will be represented by the number of times that 700 is contained in $210, which is .30 or 30 cents on the dollar. Now if each man's stock be multiplied by .30 it will represent his share of the gain thus:

$300×.30=$ 90 A.'s gain.
400×.30= 120 B.'s "

Entire stock....... 700 210 Entire gain.

Hence,—To find each partner's share of the profit or loss, when there is no reference to time, we have the following

### RULE.

*As the whole stock is to each partner's stock, so is the whole gain or loss to each partner's gain or loss; or, divide the whole gain or loss by the number denoting the entire stock, and the quotient will be the gain or loss on each dollar of stock; which multiplied by the number denoting each partner's share of the entire stock, will give his share of the entire gain or loss.*

### EXERCISES.

1. Three persons, A., B., and C., enter into partnership. A. advances $500, B. $550, and C. $600; they gain by trade $412.50. What is each partner's share of the profit?

Ans. A.'s $125; B.'s $137.50; C.'s $150.

2, A, B, C and D purchase an oil well. A pays for 6 shares, B for 5, C for 7, and D for 8. Their net profits at the end of three months have amounted to $7800; what sum ought each to receive?

Ans. A, $1800; B, $1500; C, $2100; D, $2400.

3. A and B purchased a lot of land for $4500. A paid $\frac{1}{3}$ of the price, and B the remainder; they gained by the sale of it 20 per cent.; what was each man's share of the profit?

Ans. A, $300; B, $600.

4. A captain, mate, and 12 sailors, won a prize of $2240, of which the captain took 14 shares, the mate 6, and the remainder was equally divided among the sailors; how much did each receive?

Ans. The captain, $980; the mate, $420; each sailor, $70.

5. A and B invest equal sums in trade, and clear $220, of which A is to have 8 shares on account of transacting the business, and B only 3 shares; what is each man's gain, and what allowance is made A for his time? Ans. Each man's gain $60; A $100 for his time.

6. A, B, C and D enter into partnership with a joint capital of $4000, of which A furnishes $1000; B $800; C $1300, and D the balance; at the end of nine months their net profits amount to $1700; what is each partner's share of the gain, supposing B to receive $100 for extra services?

Ans. A, $400; B, $320; C, $520; D, $360.

7. Six persons, A, B, C, D, E and F, enter into partnership, and gain $7000, which is to be divided among them in the following manner:—A to have $\frac{1}{5}$; B, $\frac{1}{7}$; C, $\frac{1}{3}$ as much as A and B, and the remainder to be divided between D, E and F, in the proportion of 2, $2\frac{1}{2}$ and $3\frac{1}{2}$; how much does each partner receive?

Ans. A, $1400; B, $1000; C, $800; D, $950; E, $1187.50; F, $1662.50.

8. A, B and C enter into partnership with a joint stock of $30,000, of which A furnished an unknown sum; B furnished $1\frac{1}{2}$, and C $1\frac{1}{4}$ times as much. At the end of six months their profits were 25 per cent. of the investment; what was each man's share of the gain? Ans. A's, $2000; B's, $3000; and C's, $2500.

9. A, B, C and D trade in company with a joint capital of $3000; on dividing the profits, it is found that A's share is $120; B's, $255; C's, $225; and D's, $300; what was each partner's stock?

Ans. A's, $400; B's, $850; C's, $750; and D's, $1000.

10. Three labouring men, A, B and C, join together to reap a certain field of wheat, for which they agree to take the sum of

$19.84; A and B calculate that they can do $\frac{4}{5}$ of the work; A and C $\frac{2}{3}$; B and C $\frac{3}{5}$ of it; how much should each receive according to these estimates? Ans. A, $8.32; B, $7.04; and C, $4.48.

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To find each partner's share of the gain or loss, when the capital is invested for different periods.

### EXAMPLE.

Two merchants, A and B, enter into partnership. A invests $700 for 15 months, and B $800 for 12 months; they gain $603; what is each man's share of the profits?

### SOLUTION.

$$
\begin{array}{l}
\$700\times15=\$10500\\
\$800\times12=\quad 9600\\
\hline
\end{array}
$$

$$20100 : 10500 :: \$603 : \$315 \text{ A's gain.}$$
$$20100 : 9600 :: \$603 : \$288 \text{ B's gain.}$$

The reason for multiplying each partner's stock by the time it was in trade, is evident from the consideration that $700 invested for 15 months would be equivalent to $700×15 equal to $10500 for one month, that is $10500 would yield, *in one month*, the same interest that $700 would *in fifteen months*. Likewise $800 invested for 12 months would be the same as $9600 for one month; hence the question becomes one of the previous case, that is, their investments are the same as if they had invested respectively $10500 and $9600 for equal times; hence the

### RULE.

*Multiply each man's stock by the time he continues it in trade; then say, as the sum of the products is to each particular product, so is the whole gain or loss to each man's share of the gain or loss.*

### EXERCISES.

11. A, B and C are associated in trade. A furnished $300 for 6 months; B, $350 for 7 months, and C, $400 for 8 months. Their profits amounted to $1490 at the time of dissolution; what was the profit belonging to each partner?

Ans. A, $360; B, $490; C, $640.

12. A, B and C contract to perform a certain piece of work; A employs 40 men for $4\frac{1}{2}$ months; B 45 men for $3\frac{1}{2}$ months, and C 50 men for $2\frac{1}{4}$ months. Their profits, after paying all expenses, are $850; how much of this belongs to each?

Ans. A, $340; B, $297.50; C, $212.50.

13. Four men, A, B, C and D, hired a pasture for $27.80; A puts in 18 sheep for 4 months; B, 24 for 3 months; C, 22 for 2 months; and D, 30 for 3 months; how much ought each to pay?

Ans. A and B each, $7.20; C, $4.40; D, $9.

14. On the first day of January A began business with a capital of $760, and on the first of February following he took in B, who invested $540; and on the first of June following they took in C, who put into the business $800. At the end of the year they found they had gained $872; how much of this was each man entitled to?

Ans. A, $384.93; B, $250.71; C, $236.36.

15. Three merchants, A, B and C, entered into partnership with a joint capital of $5875, A investing his stock for 6 months, B his for 8 months, and C his for 10 months; of the profits each partner took an equal share; how much of the capital did each invest?

Ans. A, $2500; B, $1875; C, $1500.

16. Two merchants, A and B, entered into partnership for two years; A at first furnished $800, and at the end of one year, $500 more; B furnished at first $1000, at the end of 6 months, $500 more, and after they had been in business one year, he was compelled to withdraw $600 At the expiration of the partnership their net profits were $2550; how much must A pay B who wishes to retire from the business? Ans. $2190.

17. Three persons, A, B and C, form a partnership for one year, commencing January 1st, 1865; A puts in $4000; B, $3000; and C, $2500; April 1st, A withdraws $500, and B withdraws $600; June 1st, C puts in $800 more; September 1st, A furnishes $700 more, and B $400 more. At the end of the year they find they have gained $1500; what is each partner's share of it?

Ans. A, $608.68; B, $423.31; C, $468.01.

18. John Adams commenced business January first, 1865, with a capital of $10000, and after some time formed a partnership with William Hickman, who contributed to the joint stock the sum of $2800 cash. In course of time they admitted into the firm Joseph Williams, with a stock worth $3600. On making a settlement January first, 1866, it was found that Adams had gained $2250;

Hickman, $420; and Williams, $405; how long had Hickman's and Williams' money been employed in the business, and what rate of interest per annum had each of the partners gained on their stock?

Ans. Hickman's, 8 months; Williams', 6 months. Gain, 22½ per cent. interest.

## BANKRUPTCY.

When a trader in Lower Canada, and any person in Upper Canada, is unable to meet his liabilities, he may make an *assignment* of all his property to an *Official Assignee*, to be by him distributed for the benefit of the creditors generally. Official Assignees are appointed by the different Boards of Trade for that purpose.

Creditors may also, under certain circumstances, compel a dishonest debtor to part with all his property for their benefit, and place his estate in bankruptcy.

The shares of the property, which are divided among the creditors, are called *dividends*.

The property to be divided is called *assets*.

### EXAMPLE.

A bankrupt owes A $400; B, $350, and C, $600; his net assets amount to $810 cash; how much is he able to pay on the $1, and how much will each creditor receive?

### SOLUTION:

$400+$350+$600=$1350, total liabilities. Now, if he has $1350 to pay, and only $810 to pay it with, he will only be able to pay $810÷1350=.60 or 60 cents on the $1. Therefore, A will receive $400×.60=$240; B, $350×.60=$210, and C, $600×.60 =$360. Hence the

### RULE.

*Divide the net assets by the number denoting the total amount of the debts, and the quotient will be the sum to be paid on each dollar, then multiply each man's claim by the sum paid on the dollar, and the product will be the amount he is to receive.*

### EXERCISES.

1. A becomes bankrupt. He owes B, $800; C, $500; D, $1100, and C, $600. The assets amount to $1110; how much can he pay on the dollar, and how much does each creditor receive?

Ans. He can pay 37 cents on the dollar, and B receives $296; C, $185; D, $407, and E, $222.

2. A house becomes bankrupt; its liabilities are $17940; its assets are $8970; what is the dividend, and what is the share of the chief creditor to whom $1282 are due?

Ans. The dividend is 50 cents on the dollar, and the principal creditor gets $641.

3. A shipbuilder becomes bankrupt, and his liabilities are $303000; the premises, buildings and stock are worth $220000, and he has in cash and notes $12875; the creditors allow him $3000 for maintenance of his family; the costs are $3\frac{1}{2}$ per cent. of the amount available for the creditors; what is the dividend, and how much does a creditor get to whom $1360.60 are due?

Ans. The dividend is 75 cents on the dollar, and the creditor specified gets $1020.

4. Foster & Co. fail. They owe in Toronto, $22000; in Montreal, $18000; in Hamilton, $17100; in Kingston, $16000; in London, $4400, and in Quebec, $4200. Their assets are: house property, $14000; farms, $2200; Cash in bank, $4400; railway stock, $4200; sundry sums due to them, $20135. What is the dividend, and how much goes to each city?

Ans. The dividend is 55 cents in the dollar; $12100 are paid in Toronto; $9900 are sent to Montreal; $9405 to Hamilton; $8800 to Kingston; $2420 to London, and $2310 to Quebec.

5. The firm of Reuben Ring & Nephews becomes bankrupt. It owes to Buchanan & Ramsay, $1080; to Kinneburgh & McNabb, $850; to Collier, Bros., $1720; to David Bryce & Son, $1580; to Sinclair & Boyd, $970. The assets are; house and store, valued at $848; merchandise in stock, $420; sundry debts, $220. What can the estate pay, and what is the share of each creditor?

Ans. The estate pays 24 cents on the dollar, and the payments are: to Buchanan & Ramsay, $259.20; to Kinneburgh & McNabb, $204; to Collier, Bros., $412.80; to David Bryce & Son, $379.20; to Sinclair & Boyd, $232.80.

## EQUATION OF PAYMENTS.

*Equation of Payments* is the process of finding the average or mean time at which the payment of several sums, due at different times, may all be made at one time, so that neither the debtor nor creditor shall be at any loss.

The date to be found is called the *equated time.*

The mode of finding equated time almost universally adopted is very simple, though, as we shall show in the sequel, not altogether correct. It is known as *the mercantile rule.*

Let us observe, in the first place, that the standard by which men of business reckon the advantage that accrues to them from receiving money before the time fixed for its payment, and the loss they sustain by the payment being deferred beyond the appointed time, is the interest of money for each such period. Thus, if $50 be a year overdue, the loss is $3, at 6 per cent.; and, if $50 be paid a year in advance of the time agreed upon, the gain to the payee is $3, at the same rate. In the former case, the person receiving the money charges the payer $3 interest for the inconvenience of lying out of his money, but, in the latter case, he deducts $3 from the debt, for the advantage of having the money in hand. If, on the 1st May, A gives B two notes, one for $50, at a term of three months, and the other for $80, at a term of seven months, the first will be legally due on the 1st August, and the 2nd on the 1st December; but A is not able to meet the first at August, and it is held over till the 1st November, when A finds himself in a position to pay both at once. The first is then three months over-due, and accordingly B claims interest for that time, which, at 6 per cent., is 75 cents, but as A tenders payment of the whole debt at once, and the second note will not be due for another month, A claims a deduction of one month's interest, which, at the same rate, is 40 cents, and accordingly A, in addition to the debt, pays B 35 cents.

Let us now suppose another case. A owes B $130, as before, and he gives B two notes—one for $50, on 1st May, at 3 months, and another, on the 6th May, for $80, at 8 months. The first falls due on 1st August, and the other on the 6th January, but A and B agree to settle at such a time that neither shall have interest to pay, but that A shall simply have to pay the principal. Supposing that a settlement is made on 6th November, we find that the 1st note is

3 months and 6 days over due, and the interest on it for that period is 80 cents, while the second will not be due for 2 months, and the interest on it for that period is also 80 cents; consequently, the interest that A should pay, and that which B should allow being equal, they balance each other, and the principal only has to be paid.

There are, then, three methods for the payment of several debts, or a debt to be paid by instalments. The first is to pay each instalment as it becomes due. This needs no elucidation, nor is it often practised, except in the case of small debts, due by persons of contracted means.

The second is what has been illustrated above by the first example, viz., that interest is added for overdue money, and deducted for sums paid in advance of the stipulated time.

The third has been illustrated by the second example, viz., to fix on such a time that the interests on the overdue and underdue sums shall be equal, so that the debtor has only to give the principal to the creditor. If, in this last case, the time should come out as a mixed number, the fraction must be taken as another day, or thrown off, making the payment fall due a day earlier. The principle on which all such settlements are made is, that the interest of any sum paid *in advance* of a stipulated time is equivalent to the interest of the same sum *overdue* for a like time.

With these explanations we are now ready to investigate a rule for the Equation of Payments. For this purpose let us suppose a case. R. Evans owes J. Jones $200, which he undertakes to pay by two instalments of $100 each, with interest at six per cent.; the first payment to be made at once, and the second at the expiration of two years. But the first payment is not made till the end of the first year, at which time R. E. tenders payment of the whole amount. For the accommodation of having the first payment deferred for one year he is to pay $6, *i. e.*, $106 in all, and in return for making the second payment a year before it is due, he claims a discount at the same rate, which gives $6. He has therefore, by the mercantile rule, to pay $106+94=$200, so that the $6 in the latter case balances the $6 in the former. This takes one year as the equated time, and is the mode usually adopted on account of its simplicity, though not strictly accurate.

To find the equated time when there are several payments to be made at different dates.

If A owes B \$300, payable at the end of 4 months; \$500, payable at the end of 6 months, and \$400, payable at the end of $10\frac{1}{2}$ months, to find at what time the whole may be paid, so that interest shall be chargeable to neither party. The interest of \$300 for 4 months is the same as the interest of \$1 for 1200 months; the interest of \$500 for 6 months is the same as the interest of \$1 for 3000 months, and the interest of \$400 for $10\frac{1}{2}$ months is the same as the interest of \$1 for 4200 months. The sum of all these is 8400 months, and the interest of the whole is the same as the interest of \$1 for 8400 months, and if \$1 requires 8400 months to produce a certain interest, the sum of all the principals will require only the $\frac{1}{1200}$ part of 8400 months to produce the same interest, and $8400 \div 1200 = 7$, and hence the equated time is 7 months.

### RULE.

*Multiply each payment by the time that must elapse before it becomes due, and divide the sum of these products by the sum of the payments.*

### EXAMPLE.

To find the equated time for the payment of three debts, the first for \$45, due at the end of 6 months; the second for \$70, due at the end of 11 months, and the third for \$75, due at the end of 13 months.

$$\begin{array}{r}
\$45 \times 6 = \$270 \\
70 \times 11 = 770 \\
75 \times 13 = 975 \\
\overline{190} \qquad \overline{2015}
\end{array}$$

and $2015 \div 19 = 10\frac{23}{38}$, so that the equated time will be 10 months and 18 days, the small remaining fraction being rejected.

Let us suppose that nothing is paid until the end of the 13 months, and all paid at once, then the amount to be paid will be, at 6 per cent.,

| | |
|---|---|
| For first debt overdue 7 months, \$45+1.57½, interest for 7 months | \$46.57½ |
| For second debt overdue 2 months, \$70+.70, interest for 2 months | 70.70 |
| For third debt just due, \$75, no interest | 75.00 |
| | \$192.27½ |

The work may often be somewhat shortened by counting the differences of time from the date at which the first payment becomes due, the mean time between the dates when the first and last become due being alone required.

If a person owes $1200 to be paid in four instalments, $100 in 3 months; $200 in 10 months; $300 in 15 months, and $600 in 18 months, then the excesses of time of the last three above the first are 7, 12 and 15 months, and the work will stand as below.

$100 (no time.)
200 × 7 = 1400
300 × 12 = 3600
600 × 15 = 9000

1200) 14000(11$\frac{2}{3}$

and *11$\frac{2}{3}$×3=14$\frac{2}{3}$* months. This gives the

### RULE.

*Multiply each debt, except the one first due, by the difference between its term and the term of the first; divide the sum of the products by the sum of the debts, the quotient with the term of the first added to it will be the equated time.*

Another method, which is often convenient, may be illustrated by the example already given, as the two operations will give the same result.

Interest on $300 for 4 months = $ 6.00
Interest on 500 for 6 " = 15.00
Interest on 400 for 10$\frac{1}{2}$ " = 21.00

Interest on 1200 for 1 month = 6)42.00(7 months as before.

### RULE.

*Find the interest on each instalment for the given time, and divide the sum of these by the interest of the whole debt for one month, and the quotient will be the equated time.*

As the sum of the instalments is equal to the debt, the result will be the same for any rate of interest.

| | |
|---|---|
| For the first instalment, $300, overdue 3 months, A has to pay.................................................... | $4 50 |
| For the second instalment, $500, overdue 1 month, A has to pay.................................................... | 2 50 |
| | $7 00 |

For the third instalment, \$400, not due for $3\frac{1}{2}$ months, A has to get.............................................. \$7 00

so that the amounts of interest exactly balance, and the paying of the whole, at the end of 7 months, is precisely equivalent to the paying of each instalment as it falls due. The only difference that could arise is, that it might be inconvenient for the creditor to lie out of the first instalment for the three months. In all other respects the settlement is *strictly equitable*, according to the *understanding* that exists among business men. In the first place, the difference between this and what is called "the accurate rule," is insignificantly small; and, in the second place, the "mercantile rule" saves much time, and time is equivalent to so much capital in mercantile transactions. Independently, however, of any other consideration, we may remark that when the mode of reckoning is *conventionally understood*, it becomes perfectly equitable, because every merchant knows the terms on which he can do business with any other, just as bank discount becomes perfectly equitable, because every man, before going to a bank for the discounting of a note, knows perfectly well on what terms he can have it.

Much warm discussion has been indulged in on this subject; but, as we consider the discussion more subtle than profitable, we shall dismiss the subject in a few words. We shall adopt the usual case, that A owes B \$200, one-half to be paid at the present time, and the remainder at the end of two years. It is perfectly obvious that, at the end of the first year, A should pay \$106, that is, the principal, *plus* the interest agreed upon. Regarding the settlement of the second instalment, if A proffers payment of the whole at once, he is clearly entitled to claim a reduction for the unexpired term. Now, the question is, what ought the reduction to be. By the mercantile rule he should pay \$94, but the true present worth of \$100, due at the end of the year, would be $94.33\frac{51}{53}$, so that he would have to pay \$106 on the instalment over due, and $\$94.33\frac{51}{53}$ on the one not due, making $\$200.33\frac{51}{53}$, whereas the object is to find at what time interest should be chargeable to neither party.

As a further illustration of the general rule, let us suppose that J. Smith owes R. Evans \$1300, of which \$700 are to be paid at the end of 3 months, \$100 at the end of 4 months, and the balance at the end of 8 months, to find the equated time.

We shall suppose that J. Smith agrees to pay R. Evans the whole amount at the time the debt was contracted; then J. Smith would owe R. Evans $1300, *minus* the discount for the length of time the amount was paid before it became due, viz., three months, equalling the discount on $210 for 1 month; $100, less the discount for 4 months, equalling the discount on $400 for 1 month; $500, less the discount for 8 months, equalling the discount on $4000 for 1 month. This gives a total of $2100+$400+$4000=$6500, for 1 month.

Now, it is evident that if J. Smith wished to pay the whole amount at such a time that there should be no loss to either party, he must retain this amount for such a length of time as it will take this amount to equal the discount on $6500 for 1 month, which will be $\frac{1}{1300}$ of $6500, that is, for 5 months.

To prove that 5 months must be the equated time, we have recourse to the principles laid down under the head of Interest. If a settlement is not made until the expiration of 5 months from the time the debt was contracted, then J. Smith would owe R. Evans $700, *plus* the interest of that principal during the time it remained unpaid after becoming due, viz., two months, which would give an amount of $707. So also, $100, *plus* the interest for 1 month, would be $100.50, and $500, *minus* its discount for 3 months (the length of time paid before due), would give $7.50, leaving $492.50, and $707+$100.50+$492.50=$1300.

### EXERCISES.

1. T. C. Musgrove owes H. W. Field $900, of which $300 are due in 4 months; $400 in 6 months, $200 in 9 months; what is the equated time for the payment of the whole amount? Ans. 6 months.

2. E. P. Hall & Co. have in their possession 5 notes drawn by G. W. Armstrong, all dated 1st January, 1865; the first is drawn at 4 months, for $45; the second at 8 months, for $120; the third at 10 months, for $75; the fourth at 11 months, for $60; and the fifth at 15 months, for $90; for what length of time must a single note be drawn, dated 1st May, 1865, so that it may fall due at the properly equated time? Ans. 6 months.

3. A merchant sold goods as follows, on a credit of 6 months:— May 10, a bill of $600; June 12, a bill of $450; September 20, a bill of $900; at what time will the whole become due?

Ans. January 16.

4. A merchant proposed to sell goods amounting to $4000 on 8 months' credit, but the purchaser preferred to pay $\frac{1}{2}$ in cash and $\frac{1}{4}$ in 3 months; what time should be allowed him for the payment of the remainder? Ans. 2 years, 5 months.

5. A gentlemon left his son $1500, to be paid as follows: $\frac{1}{3}$ in 3 months, $\frac{1}{3}$ in 4 months, $\frac{1}{4}$ in 6 months, and the remainder in 8 months; at what time ought the whole to be paid at once?

Ans. 4 mos., 15 days.

6. A merchant bought goods amounting to $6000. He agrees to pay $500 in cash, $600 in six months, $1500 in 9 months, and the remainder in 10 months; at what time ought he to pay the whole in one payment? Ans. $8\frac{31}{60}$ months.

7. There is due to a merchant $800, one-sixth of which is to be paid in 2 months, one-third in 3 months, and the remainder in 6 months; but the debtor agrees to pay one-half in cash; how long may he retain the other half, so that neither party may sustain loss?

Ans. $8\frac{2}{3}$ months.

8. A merchant sold to W. L. Brown, Esq., goods to the amount of $3051, on a credit of 6 months, from September 25th, 1864. October 4th Brown paid $476; November 12th, $375; December 5th, $800; January 1st, 1865, $200. When, in equity, ought the merchant to receive the balance? Ans. Oct. 8th, 1867.

9. A having sold B goods to the amount of $1200, left it optional with him either to take them on 8 month's credit, or to pay one-half in cash, one-fifth in two months, one-sixth in four months, and the remainder at an equated time, to correspond with the terms first named; what was the time? Ans. 4 years, 4 mos.

10. A grocer sold 484 barrels of rosin, as follows:

February 6th, 35 barrels @ $3.12½, on 4 months' time.
March 12th, 38 barrels @ 3.00, on 4 months' time.
March 12th, 411 barrels @ 2.62½, on 4 months' time.

What is the equated time for the payment of the whole?

Ans. July 8th.

11. Bought of A. B. Smith & Co. 1650 barrels of flour, at different times, and on various terms of credit, as by the following statement; what is the equated time for the payment of the whole?

May 6th, 150 barrels, at $4.50, on 3 months' credit.
May 20th, 400 barrels, at 4.75, on 4 months' credit.
July 10th, 500 barrels, at 5.00, on 5 months' credit.
August 4th, 600 barrels, at 4.25, on 4 months' credit.

Ans. November 6th.

12. J. B. Smith & Co. bought of A. Hamilton & Son 576 barrels of rosin, as follows:

May 3rd, 62 barrels @ $2.50, on 6 months' credit.

May 10th, 100 barrels @ 2.50, on 6 months' credit.
May 18th, 10 barrels @ 2.50, as cash.
May 26th, 50 barrels @ 2.75, on 30 days' credit.
May 26th, 345 barrels @ 2.50, on six months' credit.
May 26th, 9 barrels @ 2.00, on six months' credit.

What is the equated time for the payment of the whole?

Ans. November 2nd.

13. Purchased goods of J. R. Worthington & Co., at different times, and on various terms of credit, as by the following statement:

March 1st, 1863, a bill of $675.25, on 3 months' credit.
July 4th, 1863, a bill of 376.18, on 4 months' credit.
September 25th, 1863, a bill of 821.75, on 2 months' credit.
October 1st, 1863, a bill of 961.25, on 8 months' credit.
January 1st, 1864, a bill of 144.50, on 3 months' credit.
February 10th, 1864, a bill of 811.30, on 6 months' credit.
March 12th, 1864, a bill of 567.70, on 5 months' credit.
April 15th, 1864, a bill of 369.80, on 4 months' credit.

What is the equated time for the payment of the whole?

Ans. March 16th, 1864.

## AVERAGING ACCOUNTS.

When one merchant trades with another, exchanging merchandise, or giving and receiving cash, the memorandum of the transactions is called an *Account Current.* If the goods be purchased at different dates, or for different terms of credit, and some are not due while others are overdue, the fixing on a time when all may be settled, so that no interest shall be chargeable to either party, is called *Averaging the Account.*

Since interest is the standard to which is referred the benefit of receiving money before it is due, so that in the meantime it can be used in trade, and also the damage of not getting it when due, it is fair and proper that interest should be charged on all sums overdue, and deducted from all not due. In illustration, let us suppose that A sells goods to B, March 2, on 4 months' credit, and again an equal amount on March 20, on 6 months' credit; the first will be due on July 2, and the second on September 20. Should B tender payment of the whole on June 2, he would be entitled to claim interest for one month on the first purchase, and for three months and eighteen days on the second. But if payment be delayed till August 2, A would be entitled to one month's interest on the first purchase, and B to the interest on the second for one month and eighteen days, so that there would be in favour of B, on the whole, a balance of interest for eighteen days. Again, supposing the settlement is not made till September 20, when all is due, no interest can be either charged or claimed on the second purchase, the term of credit having just then expired; but as the first debt is two months and eighteen days overdue, A is entitled to interest on it for that period. If neither is paid till after September 20, A has a right to claim interest on each for the period it has been overdue. But this regulates only one side of the account. In order to settle the other, let us suppose that B has, in the meantime, sold goods to A, it is obvious that B's claims on A must be settled on the very same principle, and that therefore the final result must be simply the finding of the balance. It is more usual, however, in accounts current, to fix on a time such that the interest due by A shall exactly balance that due by B. To illustrate this, let us suppose a case corresponding to a ledger account:

## R. EVANS.

| 1865. | | Dr. |
|---|---|---|
| July 21, | To Merchandise on 2 months' credit... | $200 |
| July 25, | To Cash.................................... | 150 |
| Aug. 24, | To Merchandise on 4 months' credit... | 100 |
| Sept. 21, | To Merchandise on 3 months' credit... | 250 |
| | | $700 |

| 1865. | | Cr. |
|---|---|---|
| August 1, | By Cash.................................. | $100 |
| August 20, | By Merchandise at 22 days.......... | 110 |
| Sept'r 30, | By Cash.................................. | 180 |
| Balance.......................................... | | 310 |
| | | $700 |

To find in this case at what time the account may be settled so that interest shall be chargeable to neither party. Equating the time, as in equation of payments, we have the following operation:

**Dr.**

1865

July 25.......150× 0

Sept. 21.......200× 58=11600

Decr. 24.......250×152=38000

Decr. 21.......100×149=14900

700 64500

64500÷700=92 days.

Due 92 days from July 25, viz., on October 25.

**Cr.**

1865.

August 1. ......100× 0

Sept. 12........110×22= 2420

Sept. 30........180×71=12780

390 15200

15200—390=39 days.

Due 39 days from August 1, viz., on September 9.

Time from September 9, to October 25=46 days.

Excess of debit above credit 700—390=310.

390×46=17940, and 17940÷310=58 days, nearly.

Counting 58 days forward, from October 25, will bring us to December 22, the time required for a settlement, with interest chargeable to neither party. Here the time is counted forward from the average date of the larger side which becomes due last, but had it become due first, we should have counted backward.

The first transaction on the debit side being two months' credit from July 21, is not to be taken into consideration till September 21. The second transaction, being a cash one, and therefore considered as so much due, will therefore mark the date from which all others shall be reckoned; and, since there is no interval of time, we write it without a multiplier. The next transaction has a term of credit extending to 152 days, and therefore we write 250×152= 11600.

The term of the next extends from September 21 to December 21, a period of 149 days, and we write 100×149=14900. The sum of the debits is $700, and the sum of the results obtained by multiplying each item by the number of days it has to run from July 25 is $64500. Then 64500÷700=92, the equated time in days for the debit side. Now, as already explained, the interest for $700 for 92 days will be the same as the interest of $64500 for 1 day. Hence, the debits are due 92 days from July 25, viz., on October 25.

In like manner, on the credit side, the first transaction being a cash one, we start from its date, August 1, and, as there is no interval, we have no multiplier. The second being merchandise, on 22 days' credit, we write 110×22=2420. The third is cash paid 71 days after August 1, and we write 180×71=12780.

Had the account been settled on September 9, the debits would have been paid 46 days before coming due, and the credit side would have gained and the debit side lost the interest for that time.

Again, we must consider how long it would take the balance, $310, to produce the same interest that $390 would produce in 46 days. It is obvious that whatever interest $390 gives in 46 days will require 46 times $390 for $1 to produce the same interest, that is, 390×46=17940 days, and it will require 17940÷310=58 days, for $310 to produce the same interest. If the settlement is made on October 25, the latest date, then the credit has been due 46 days, and therefore bearing interest; and in order that the debit side may be increased by an equal amount, the time must be extended beyond October 25, that is, it must be counted *forward*. For the same reason, if the greater side had become due first, then the balance must be considered as due at a *previous* date, and therefore we must count *backward*.

An account may be averaged from any date, but either the first or the last will be found the most convenient. The first due is generally used.

On the principles now explained may be founded the following

### RULE.

*Find the equated time when each side becomes due.*

*Multiply the amount of the smaller side by the number of days between the two average dates, and divide the product by the balance of the account.*

*The quotient thus obtained will be the time that the balance becomes due, counted from the average date of the larger side,* FORWARD *when the amount of that side becomes due* LAST, *but* BACKWARD *when it becomes due* FIRST.

The *cash value* of a balance depends on the time of settlement. If the settlement be made *before* the balance is due, the interest for the unexpired time is to be *deducted;* but if the settlement is not made till *after* the balance is due, interest is to be *added* for the time it is overdue.

### EXERCISES.

In J. H. Marsden's Ledger, we find the following accounts, which, on being equated, stand as follows; at what time should the respective balances commence to draw interest:

1. *Dr.* J. S. Peckham. *Cr.*
May 16th, 1865.........$724.45. | July 29th, 1865.........$486.80.
Ans. December 15th, 1864.

2. *Dr.* Nelson Bostford. *Cr.*
November 19th, 1865 ......$635. | December 12th, 1865......$950.
Ans. January 27th, 1866.

3. *Dr.* James Crow & Co. *Cr.*
February 24th, 1866....$512.25. | June 10th, 1865.........$309,70.
Ans. March 27th, 1867.

4. *Dr.* J. H. Burritt & Co. *Cr.*
March 17th, 1866..........$145. | January 15th, 1866.....$695.60.
Ans. December 30th, 1865.

5. *Dr.* M. McDonald. *Cr.*
August 27th, 1865..........$341. | November 7th, 1865.......$247.

6. *Dr.* James I. Musgrove. *Cr.*
July 20th, 1866.............$711. | April 14th, 1866..........$1260.
Ans. December 9th, 1865.

7. *Dr.* Thos. A. Bryce & Co. *Cr.*
June 24th, 1864...........$1418. | September 7th, 1865......$2346.

8. *Dr.* E. R. Carpenter. *Cr.*
December 2nd, 1865...$1040.80. | August 13th, 1865....$1112.40.

9. Required the time when the balance of the following account becomes subject to interest, allowing the merchandise to have been on 8 months' credit?

*Dr.* A. B. Smith & Co. *Cr.*

| 1864. | | | 1865. | | |
|---|---|---|---|---|---|
| May 1, | To Mdse............ | $300.00 | Jan. 1, | By Cash.... | $500.00 |
| July 7, | “ “ ........... | 759.96 | Feb. 18, | “ Mdse. .. | 481.75 |
| Sep. 11, | “ “ ............ | 417.20 | Mar. 19, | “ Cash.... | 750.25 |
| Nov. 25, | “ “ ............ | 287.70 | April 1, | “ Draft... | 210.00 |
| Dec. 20, | “ “ ............ | 571.10 | May 25, | “ Cash.... | 100.00 |

Ans. August 5, 1865.

10. When will the balance of the following account fall due, the merchandise items being on 6 months' credit?

*Dr.* J. K. White. *Cr.*

| | | | | | |
|---|---|---|---|---|---|
| 1865. | | | 1865. | | |
| May 1, | To Mdse............ | $312.40 | June 14, | By Cash.... | $200.00 |
| May 23, | " " ............ | 85.70 | July 30, | " Mdse.... | 185.90 |
| June 12, | " Cash paid dft.. | 105.00 | Aug. 10, | " Cash.... | 100.00 |
| July 29, | " Mdse............ | 243.80 | Aug. 21, | " Mdse.... | 58.00 |
| Aug. 4, | " " ............ | 92.10 | Sept. 28, | " " .... | 45.10 |
| Sept. 18, | " Cash............ | 50.00 | | | |

Ans. January 12, 1866.

11. When does the balance of the following account become subject to interest?

*Dr.* W. H. Musgrove. *Cr.*

| | | | | | |
|---|---|---|---|---|---|
| 1864. | | | 1864. | | |
| Aug. 10, | To Mdse 4 mos. | $285.30 | Oct. 13, | By Cash....... | $400.00 |
| Aug. 17, | " " 60 days | 192.60 | Oct. 26, | " " ....... | 150.00 |
| Sept. 21, | " " 30 " | 256.80 | Dec. 15, | " Mdse 2 mos | 345.80 |
| Oct. 13, | " Cash p'd dft. | 190.00 | Dec. 30, | " " 4 " | 230.40 |
| Nov. 25, | " Mdse 6 mos. | 432.20 | 1865. | | |
| Nov. 30, | " " 90 days | 215.25 | Jan. 4, | " Cash....... | 340.30 |
| Dec. 18, | " " 2 mos. | 68.90 | Jan. 21, | " " ....... | 180.00 |
| 1865. | | | | | |
| Jan. 31, | " Cash.......... | 100.00 | | | |

12. In the following account, when did the balance become due, the merchandise articles being on 6 months' credit?

*Dr.* R. J. Bryce in account with D. Hicks & Co. *Cr.*

| | | | | | |
|---|---|---|---|---|---|
| 1864. | | | 1864. | | |
| Jan. 4, | To Mdse............ | $ 96.57 | Jan. 30, | By Cash... | $240.00 |
| Jan. 18, | " " ........... | 57.67 | April 3, | " " ... | 48.88 |
| Feb. 4, | " Cash paid draft. | 80.00 | May 22, | " " ... | 50.00 |
| Feb. 4, | " Mdse............ | 38.96 | | | |
| Feb. 9, | " Cash paid draft. | 50.26 | | | |
| Mar. 3, | " Mdse............ | 154.46 | | | |
| Mar. 24, | " " ........... | 42.30 | | | |
| April 9, | " " ........... | 23.60 | | | |
| May 15, | " " ........... | 28.46 | | | |
| May 21, | " " ........... | 177.19 | | | |

Ans. December 22nd, 1864.

13. When, in equity, should the balance of the following account be payable?

*Dr.* J. McDonald & Co. *Cr.*

| | | | | | |
|---|---|---|---|---|---|
| 1865. | | | 1864. | | |
| Jan. 3, | To Cash.... | $200 | Sept. 20, | By Mdse, 6 mos... | $583.17 |
| Jan. 31, | “ “ .... | 300 | Oct. 27, | “ “ 4 “ .. | 321.00 |
| Feb. 8, | “ “ .... | 75 | Dec. 5, | “ “ 6 “ .. | 137.00 |
| Feb. 21, | “ “ .... | 100 | 1865. | | |
| Mar. 10, | “ “ .... | 350 | Jan. 18, | “ “ 60 days.. | 98.75 |
| Mar. 24, | “ “ .... | 25 | Feb. 26, | “ “ 6 mos... | 53.98 |
| Apr. 12, | “ “ .... | 40 | Apr. 15, | “ “ 4 “ .. | 634.00 |
| June 1, | “ “ .... | 80 | June 12, | “ “ 2 “ .. | 97.23 |
| June 20, | “ “ .... | 125 | Sept. 21, | “ “ 6 “ .. | 84.00 |
| July 4, | “ “ .... | 268 | Dec. 29, | “ “ 6 “ .. | 132.14 |
| Sept. 27, | “ “ .... | 250 | | | |
| Dec. 9, | “ “ .... | 100 | | | |

**Ans. October 10, 1866.**

* * *

**To find the true cash balance of an account current when each item draws interest.**

### EXAMPLE.

What is the balance of the following account on January 19th, 1866, a credit of three months being allowed on the merchandise, money being worth 6 per cent.?

*Dr.* Musgrove & Wright. *Cr.*

| | | | | | |
|---|---|---|---|---|---|
| 1865. | | | 1865. | | |
| Mar. 12, | To Merchandise... | $340.00 | Apr. 20, | By Mdse ... | $200.00 |
| Apr. 21, | “ “ ... | 150.00 | May 4, | “ Cash ... | 110.00 |
| May 6, | “ Cash paid draft | 165.00 | June 15, | “ “ ... | 230.00 |
| May 27, | “ Mdse........... | 215.00 | Aug. 10, | “ Mdse ... | 180.00 |
| July 16, | “ Cash........... | 100.00 | Sept. 23, | “ Cash ... | 50.00 |
| Sept. 10, | “ Mdse........... | 310.00 | Nov. 12, | “ “ ... | 50.00 |
| Oct. 19, | “ “ ........... | 120.00 | Dec. 15, | “ “ ... | 100.00 |

### SOLUTION.

| Debits. | | | Credits. | | |
|---|---|---|---|---|---|
| Due. | | | Due. | | |
| June 12, | $340×221= | 75140 | July 20, | $200×183= | 36600 |
| July 21, | 150×182= | 27300 | May 4, | 110×260= | 28600 |
| May 6, | 165×258= | 42570 | June 15, | 230×218= | 50140 |
| Aug. 27, | 215×145= | 31175 | Nov. 10, | 180× 70= | 12600 |
| July 16, | 100×187= | 18700 | Sept. 23, | 50×118= | 5900 |
| Dec. 10, | 310× 40= | 12400 | Nov. 12, | 50× 68= | 3400 |
| Jan. 19, | 120× 0= | 0 | Dec. 15, | 100× 35= | 3500 |
| | $1400 | 6)207285 | | $920 | 6)140740 |
| | | $34.547 | | | $23.456 |

The different items on the debit and credit sides of the account being on interest from the date on which it becomes due until the time of settlement, the total interest of all the debit items will be the same as the interest of $207285 for one day, or the interest of $1 for 207285 days, which is $34.547. So also, the total interest of all the credit items will be the same as the interest of $140740 for one day, or the interest of $1 for 140740 days, which is $23.456. Now, since each side of the account is to be increased by its interest, the cash balance will be represented by the number denoting the difference between the two sides of the account, after the interest is added; thus, $1400+$34.547=$1434.547, amount of debit side, and $920 +$23.456=$943.456, amount of credit side, then $1434.547—$943.456=$49.109, cash balance.

### SECOND METHOD.

| Debits. | | | | | Credits. | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | Days. | Int. | | | | Days. | Int. |
| Int. on | $340 | for | 221= | $12.523 | Int. on | $200 | for | 183= | $6.100 |
| " | 150 | " | 182= | 4.550 | " | 110 | " | 260= | 4.766 |
| " | 165 | " | 258= | 7.095 | " | 230 | " | 218= | 8.356 |
| " | 215 | " | 145= | 5.195 | " | 180 | " | 70= | 2.100 |
| " | 100 | " | 187= | 3.116 | " | 50 | " | 118= | .983 |
| " | 310 | " | 40= | 2.066 | " | 50 | " | 68= | .566 |
| " | 120 | " | 0 | | " | 100 | " | 35= | .583 |
| | $1400 | | | $34.545 | | $920 | | | $23.454 |

Now, $34.545 debit interest—$23.454 credit interest=$11.09, the balance of interest, and $1400, amount of debit items+$11.09 =$1411.09, and $1411.09—$920 amount of credit items=$491.09 the cash balance, which is the same as obtained by the first solution. Hence from the foregoing we deduce the following

### RULE.

*Multiply each item of debit and credit by the number of days intervening between its becoming due and the time of settlement. Then consider the sums of the products of the debit and credit items as so many dollars, and find the interest on each for one day, which will be the interest, respectively, of the debit and credit items.*

*Place the balance of interest on its own side of the account, and the difference then between the two sides will be the true balance ; or,*

*Find the interest on each item from the date on which it becomes due to the time of settlement. The difference of the sums of interests, on the debit and credit sides of the account will represent the balance of interest, which is placed on its own side of the account, and the difference then between the two sides will be the true balance.*

NOTE.—If any item should not come due until after the time of settlement, the side upon which it is, should be diminished, or the opposite side increased by the interest of such item from the time of settlement until due.

### EXERCISES.

1. What will be the cash balance of the following account if settled on January 1, 1865, allowing interest at 8 per cent. on each item after it is due ?

*Dr.* R. EVANS in account with JOHN JONES. *Cr.*

| | | | | | |
|---|---|---|---|---|---|
| 1864. | | | 1864. | | |
| June 11, | To Mdse, 4 mos. | $315.00 | Apr. 15, | By Mdse, 3 mos. | $350.00 |
| June 29, | " " 6 " | 180.00 | May 10, | " " 4 " | 120.00 |
| July 18, | " Cash p'd dft. | 200.00 | June 12, | " " 6 " | 240.00 |
| Aug. 25, | " Cash......... | 75.00 | June 30, | " Cash......... | 100.00 |
| Aug. 31, | " Mdse,2 mos. | 50.00 | July 15, | " " ......... | 90.00 |
| Sept. 3, | " " 1 " | 100.00 | July 27, | " " ......... | 80.00 |
| Sept. 20, | " Cash......... | 80.00 | Aug. 6, | " Mdse,as cash | 100.00 |
| Oct. 14, | " " ......... | 150.00 | Aug. 20, | " Cash......... | 175.00 |
| Oct. 19. | " Mdse,as cash | 300.00 | Aug. 30, | " Mdse, 3 mos. | 75.00 |

Ans. $110.86.

2. A. B. Smith is in account and interest with J. K. Amos & Co., as follows :—Debtor, January 1, 1865, to merchandise, on 6 months,

$156.10; February 3, to cash paid draft, $100; March 20, to merchandise, on 4 months, $316.90; March 30, to merchandise, on 4 months, $162; May 15, to cash paid draft, $100; August 20, to merchandise, on 6 months, $213. Creditor, February 1, by cash, $120; March 20, by merchandise, on 4 months, $420.16; May 1, by merchandise, on 6 months, $300: July 1, by merchandise, on 4 months, $50; September 10, by merchandise, on 4 months, $99.84. Required, the true balance, if settled on December 1, 1865, interest being at 6 per cent.? Ans. $61.36.

3. Required the true balance, March 25, 1865, on the following account, each item drawing 7 per cent. interest from its date. A. B. Lyman in account and interest with John Russell & Co.:— Debtor, July 4, 1864, to merchandise, $200; September 8, to merchandise, $300; September 25, to merchandise, $250; October 1, to merchandise, $600; November 20, to merchandise, $400; December 12, to merchandise, $500; January 15, 1865, to merchandise, $100; March 11, to merchandise, $120. Creditor, July 20, 1864, by cash, $300; August 15, by cash, $350; September 1, by cash, $400; November 1, by cash, $320; December 6, by merchandise, $600; December 20, by cash, $100; February 1, 1865, by cash, $200; February 28, by merchandise, $150. Ans. $50.64.

## ALLIGATION.

*Alligation* is the method of making calculations regarding the compounding of articles of different kinds or different values. It is a Latin word, which means *binding to*, or binding together.

It is usual to distinguish alligation as being of two kinds, *medial* and *alternate.*

## ALLIGATION MEDIAL.

*Alligation medial* relates to the average value of articles compounded, when the actual quantities and rates are given.

### EXAMPLE.

A miller mixes three kinds of grain: 10 bushels, at 40 cents a bushel; 15 bushels, at 50 cents a bushel; and 25 bushels, at 70 cents a bushel; it is required to find the value of the mixture.

10 bushels, at 40 cents a bushel, will be worth 400 cents.,
15 bushels, at 50 cents a bushel, will be worth 750 cents.,
25 bushels, at 70 cents a bushel, will be worth 1750 cents.,

giving a total of 50 bushels and 2900 cents, and hence the mixture is $2900 \div 50 = 58$ cents, the price of the mixture per bushel. Hence the

### RULE.

*Find the value of each of the articles, and divide the sum of their values by the number denoting the sum of the articles, and the quotient will be the price of the mixture.*

### EXERCISES.

1. A farmer mixes 20 bushels of wheat, worth $2.00 per bushel, with 40 bushels of oats, worth 50 cents per bushel; what is the price of one bushel of the mixture? Ans. $1.

2. A grocer mixes 10 pounds of tea, at 40 cents per pound; 20 pounds, at 45 cents per pound, and 30 pounds, at 50 cents per pound; what is a pound of this mixture worth? Ans. $46\frac{2}{3}$ cents.

3. A liquor merchant mixed together 40 gallons of wine, worth 80 cents a gallon; 25 gallons of brandy, worth 70 cents a gallon; and 15 gallons of wine, worth $1.50 a gallon; what was a gallon of this mixture worth? Ans. 90 cents.

4. A farmer mixed together 30 bushels of wheat, worth $1 per bushel; 72 bushels of rye, worth 60 cents per bushel; and 60 bushels of barley, worth 40 cents per bushel; what was the value of $2\frac{1}{2}$ bushels of the mixture? Ans. $1.50.

5. A goldsmith mixes together 4 pounds of gold, of 18 carats fine; 2 pounds, of 20 carats fine; 5 pounds, of 16 carats fine; and 3 pounds, of 22 carats fine; how many carats fine is one pound of the mixture? Ans. $18\frac{3}{7}$.

## ALLIGATION ALTERNATE.

*Alligation alternate* is the method of finding how much of several ingredients, the quantity or value of which is known, must be combined to make a compound of a given value.

### CASE I.

Given, the value of several ingredients, to make a compound of a given value.

### EXAMPLE

How much sugar that is worth 6 cents, 10 cents, and 13 cents per pound, must be mixed together, so that the mixture may be worth 12 cents per pound?

### SOLUTION.

| | | | |
|---|---|---|---|
| 12 cents. | 1 lb., at 6 cents, is a gain of 6 cents. | Gain. | |
| | 1 lb., at 10 cents, is a gain of 2 cents. | 8 | Loss. |
| | 1 lb., at 13 cents, is a loss of 1 cent. | | 1 |
| | 7 lbs. more, at 13 cents, is a loss of...... | | 7 |
| | | Gain 8 | Loss 8 |

It is evident, in forming a mixture of sugar worth 6, 10 and 13 cents per pound so as to be worth 12 cents, that the gains obtained in putting in sugar of *less* value than the average price must exactly balance the losses sustained in putting in sugar of *greater* value than the average price. Hence in our example, sugar that is worth 6 cents per pound when put in the mixture will sell for 12, thereby giving a gain of 6 cents on every pound of this sugar put in the mixture. So also sugar that is worth 10 cents per pound, when in the mixture will bring 12, so that a gain of 2 cents is obtained on every pound of this sugar used in the compound. Again, sugar that is worth 13 cents per pound, on being put into the mixture will sell for only 12 cents, consequently a loss of 1 cent is sustained on every pound of this sugar used in forming the mixture. In this manner we find that in taking *one* pound of each of the different qualities of sugar there is a gain of 8 cents, and a loss of only 1 cent. Now, our losses must equal our gains, and therefore we have yet to lose 7 cents, and as there is only one quality of sugar in the mixture by which we can lose, it is plain that we must take as much more sugar at 13 cents as will make up the loss, and that will require 7 pounds. Therefore, to form a mixture of sugar worth 6, 10 and 13 cents per pound, so as to be worth 12 cents per pound, we will require 1 pound at 6 cents, 1 pound at 10 cents, and 1 pound at the 13 cents+7 pounds of the same, which must be taken to make the loss equal to the gain.

By making a mixture of any number of times these answers, it will be observed, that the compound will be correctly formed. Hence we can readily perceive that any number of answers may be obtained to all exercises of this kind. From what has been said we deduce the following

### RULE.

*Find how much is gained or lost by taking one of each kind of the proposed ingredients. Then take one or more of the ingredients, or such parts of them as will make the gains and losses equal.*

### EXERCISES.

1. A grocer wishes to mix together tea worth 80 cents, $1.20, $1.80 and $2.40 per pound, so as to make a mixture worth $1.60 per pound; how many pounds of each sort must he take?

Ans. 1 lb. at 80 cents; 1 lb. at $1.20; 2 lbs. at $1.80, and 1 lb. at $2.40.

2. How much corn, at 42 cents, 60 cents, 67 cents, and 78 cents per bushel, must be mixed together that the compound may be worth 64 cents per bushel?

Ans. 1 bush. at 42 cts.; 1 bush. at 60 cts.; 4 bush. at 67 cts.; and 1 bush. at 78 cts.

3. It is required to mix wine, worth 60 cents, 80 cents, and $1.20 per gallon, with water, that the mixture may be worth 75 cts. per gallon; how much of each sort must be taken?

Ans. 1 gal. of water; 1 gal. of wine at 60 cts.; 9 gal. at 80 cts.; and 1 gal at $1.20.

4. In what proportion must grain, valued at 50 cents, 56 cents, 62 cents, and 75 cents per bushel, be mixed together, that the compound may be 62 cents per bushel?

Give, at least, three answers, and prove the work to be correct.

5. A produce dealer mixed together corn, worth 75 cents per bushel; oats, worth 40 cents per bushel; rye, worth 65 cents per bushel, and wheat, worth $1 per bushel, so that the mixture was worth 80 cents per bushel; what quantity of each did he take?

Give four answers, and prove the work to be correctly done in each case.

### CASE II.

When one or more of the ingredients are limited in quantity, to find the other ingredients.

### EXAMPLE.

How much barley, at 40 cents; oats, at 30 cents, and corn, at 60 cents per bushel, must be mixed with 20 bushels of rye. at 85 cents per bushel, so that the mixture may be worth 60 cents per bushel?

### SOLUTION.

| Bush. | Cents. | | Gain. | Loss. |
|---|---|---|---|---|
| 1 | at 40, | gives.................. | .20 | ... |
| 1 | at 30, | gives.................. | .30 | ... |
| 1 | at 60, | gives.......... ...... | .00 | .00 |
| 20 | at 85, | gives.................. | ... | 5.00 |
| | | | .50 | 5.00 |
| 9 | at 40, | gives.................. | 1.80 | ... |
| 9 | at 30, | gives.................. | 2.70 | ... |
| | | | $5.00 | $5.00 |

By taking 1 bushel of barley, at 40 cents, 1 bushel of oats at 30 cents, and 1 bushel of corn at 60, in connection with 20 bushels of rye at 85 cents per bushel, we observe that our gains amount to 50 cents and our losses to $5.00. Now, to make the gains equal the losses, we have to take 9 bushels more at 40 cents, and 9 bushels more at 30 cents. This gives us for the answer 1 bushel+9=10 bushels of barley, 1 bushel+9=10 bushels of oats, and 1 bushel of corn. From this we deduce the

### RULE.

*Find how much is gained or lost, by taking one of each of the proposed ingredients, in connection with the ingredient which is limited, and if the gain and loss be not equal, take such of the proposed ingredients, or such parts of them, as will make the gain and loss equal.*

### EXERCISES.

6. How much gold, of 16 and 18 carats fine, must be mixed with 90 ounces, of 22 carats fine, that the compound may be 20 carats fine?

Ans. 41 ounces of 16 carats fine, and 8 of 18 carats fine.

7. A grocer mixes teas worth $1.20, $1, and 60 cents per pound, with 20 pounds, at 40 cents per pound; how much of each sort must he take to make the composition worth 80 cents per pound?

Ans. 20 at $1.20; 10 lbs. at $1; 10 lbs. at 60 cents.

8. How much barley, at 50 cents per bushel, and at 60 cents per bushel. must be mixed with ten bushels of pease, worth 80 cents

per bushel, and 6 bushels of rye, worth 85 cents per bushel, to make a mixture worth 75 cents per bushel?

Ans. 3 bushels, at 50 cents; 2⅓ bushels, at 60 cents.

9. How many pounds of sugar, at 8, 14, and 13 cents per pound, must be mixed with 3 pounds, worth 9¼ cents per pound; 4 pounds, worth 10½ cents per pound; and 6 pounds, worth 13½ cents per pound, so that the mixture may be worth 12½ cents per pound?

Ans. 1 lb., at 8 cts.; 9 lbs., at 14 cts.; and 5½ lbs., at 13 cts

### CASE III.

To find the quantity of each ingredient, when the sum of the ingredients and the average price are given.

### EXAMPLE.

A grocer has sugar worth 8, 10, 12 and 14 cents per pound, and he wishes to make a mixture of 240 pounds, worth 11 cents per pound; how much of each sort must he take?

### SOLUTION.

| | Gain. | Loss. |
|---|---|---|
| 1 lb., at 8 cents, gives...................... | 3 | . |
| 1 lb., at 10 cents, gives...... .............. | 1 | . |
| 1 lb., at 12 cents, gives..................... | . | 1 |
| 1 lb., at 14 cents, gives..................... | . | 3 |
| 4 lbs. | 4 | 4 |

240 lbs.÷4=60 lbs. of each sort.

By taking 60 lbs. of each sort we have the required quantity, and it will be observed that the gains will exactly balance the losses, consequently the work is correct. Hence the

### RULE.

*Find the least quantity of each ingredient by* CASE I., *Then divide the given amount by the sum of the ingredients already found, and multiply the quotient by the quantities found for the proportional quantities.*

10. What quantity of three different kinds of raisins, worth 15 cents, 18 cents, and 25 cents per pound, must be mixed together to fill a box containing 680 lbs., and to be worth 20 cents per pound?

Ans. 200 lbs., at 15 cents; 200 lbs., at 18 cents; and 280 lbs., at 25 cents.

11. How much sugar, at 6 cents, 8 cents, 10 cents, and 12 cents per pound, must be mixed together, so as to form a compound of 200 pounds, worth 9 cents per pound? Ans. 50 lbs. of each.

12. How much water must be mixed with wine, worth 80 cents per gallon, so as to fill a vessel of 90 gallons, which may be offered at 50 cents per gallon? Ans. 56$\frac{2}{8}$ gals. wine, and 33$\frac{6}{8}$ gals. water.

13. A wine merchant has wines worth $1, $1.25, $1.50, $1.75, and $2 per gallon, and he wishes to form a compound to fill a 150 gallon cask that will sell at $1.40 per gallon; how many gallons of each sort must he take? Ans. 54 of $1, and 24 of each of the others.

14. A grocer has sugars worth 8 cents, 10 cents, 12 cents, and 20 cents per pound; with these he wishes to fill a hogshead that would contain 200 pounds; how much of each kind must he take, so that the mixture may be worth 15 cents per pound?

Ans. 33$\frac{1}{3}$ lbs. of 8, 10, and 12 cents, and 100 lbs. of 20 cents.

15. A grocer requires to mix 240 pounds of different kinds of raisins, worth 8 cents, 12 cents, 18 cents, and 24 cents per lb., so that the mixture shall be worth 10 cents per pound; how much must be taken of each kind?

Ans. 192 lbs. of 8 cents, and 16 lbs. of each of the other kinds.

## MONEY; ITS NATURE AND VALUE.

Money is the medium through which the incomes of the different members of the community are distributed to them, and the measure by which they estimate their possessions.

The precious metals have, among almost all nations, been the standard of value from the earliest time. Except in the very rudest state of society, men have felt the necessity of having some article, of more or less intrinsic value, that can at any time be exchanged for different commodities. No other substances were so suitable for this purpose as gold and silver. They are easily divisible, portable, and among the least imperishable of all substances. The work of dividing the precious metals, and marking or coining them, is generally undertaken by the Government of each State.

Money is a commodity, and its value is determined, like that of other commodities, by demand and supply, and cost of production. When there is a large supply of money it becomes cheap; in other words, more of it is required to purchase other articles. If all the money in circulation were doubled, prices would be doubled. The usefulness of money depends a great deal upon the rapidity of its circulation. A ten-dollar bill that changes hands ten times in a month, purchases, during that time, a hundred dollars' worth of goods. A small amount of money, kept in rapid circulation, does the same work as a far larger sum used more gradually. Therefore, whatever may be the quantity of money in a country, only that part of it will effect prices which goes into circulation, and is actually exchanged for goods.

Money hoarded, or kept in reserve by individuals, does not act upon prices. An increase in the circulating medium, conformable in duration and extent to a temporary activity in business, does not raise prices, it merely prevents the fall that would otherwise ensue from its temporary scarcity.

## PAPER CURRENCY.

PAPER CURRENCY may be of two kinds—convertible and inconvertible. When it is issued to represent gold, and can at any time be exchanged for gold, it is called convertible. When it is issued by the sovereign power in a State, and is made to pass for money, by merely calling it money, and from the fact that it is received in payment of taxes, and made a legal tender, it is known as an inconvertible currency. Nothing more is needful to make a person accept anything as money, than the persuasion that it will be taken from him on the same terms by others. That alone would ensure its currency, but would not regulate its value. This evidently cannot depend, as in the case of gold and silver, upon the cost of production, for that is very trifling. It depends, then, upon the supply or the quantity in circulation. While the issue of inconvertible currency is limited to something under the amount of bullion in circulation, it will on the whole maintain a par value. But as soon as gold and silver are driven out of circulation by the flood of inconvertible currency, prices begin to rise, and get higher with every additional issue. Among other commodities the price of gold and silver articles will rise, and the coinage will rise in value as mere bullion. The paper currency will then become proportionably depreciated, as compared with the metallic currency of other countries. It would be quite impossible for these results to follow the issue of convertible paper for which gold could at any time be obtained.

All variations in the value of the circulating medium are mischievous; they disturb existing contracts and expectations, and the liability to such disturbing influences renders every pecuniary engagement of long date entirely precarious.

A convertible paper currency is, in many respects, beneficial. It is a more convenient medium of circulation. It is clearly a gain to the issuers, who, until the notes are returned for payment, obtain the use of them as if they were a real capital, and that, without any loss to the community.

## THE CURRENCY OF CANADA.

In Canada there are two kinds of currency; the one is called the old or Halifax currency, reckoned in pounds, shillings, pence and fractions of a penny; the other is reckoned by dollars and cents as already explained under the head of Decimal Coinage. The equivalent in gold of the pound currency is 101.321 grains Troy weight of the standard of fineness prescribed by law for the gold coins of the united kingdom of Great Britain and Ireland. The only gold coins now in circulation in Britain are the sovereign, value one pound, or twenty shillings sterling; and the half sovereign, ten shillings. The dollar is one-fourth of the pound currency, and the pound sterling is equal to $4.86⅔. In the year 1786, the congress of the United States adopted the decimal currency, the dollar being the unit, and the system was introduced into Canada in 1858. By the term *legal tender* is meant the proffer of payment of an account in the currency of any country as established by law. Copper is a legal tender in Canada to the amount of one shilling or twenty cents, and silver to the amount of ten dollars. The British sovereign of lawful weight passes current, and is a legal tender to any amount paid in that coin. There is a silver currency proper to Canada, though United States' coins are most in circulation. The gold eagle of the United States, coined before July 1, 1834, is a legal tender for $10.66⅔ of the coin current in this province. The same coin issued after that is a legal tender for $10.

# EXCHANGE.

It often becomes necessary to send money from one town or country to another for various purposes, generally in payment for goods. The usual mode of making and receiving payments between distant places is by bills of exchange. A merchant in Liverpool, whom we shall call A. B., has received a consignment of flour from C. D., of Montreal; and another man, E. F., in Liverpool, has shipped a quantity of cloth, in value equal to the flour, to G. H. in Montreal. There arises, in this transaction, an indebtness to Montreal for the flour, as well as an indebtedness from Montreal for the cloth. It is evidently unnecessary that A. B., in Liverpool, should send money to C. D. in Montreal, and that G. H., in Montreal, should send an equal sum to E. F. in Liverpool. The one debt may be applied in payment of the other, and by this plan the expense and risk attending the double transmission of the money may be saved. C. D. draws on A. B. for the amount which he owes to him; and G. H. having an equal amount *to pay* in Liverpool, buys this bill from C. D., and sends it to E. F., who, at the maturity of the bill, presents it to A. B. for payment. In this way the debt due from Montreal to Liverpool, and the debt due from Liverpool to Montreal are both paid without any coin passing from one place to the other.

An arrangement of this kind can always be made when the debts due between the different places are equal in amount. But if there is a greater sum due from one place than from the other, the debts cannot be simply written off against one another. Indeed, when a person desires to make a remittance to a foreign country, he does not make a personal search for some one who has money to receive from that country, and ask him for a bill of exchange. There are exchange brokers and bankers whose business this is. They buy bills from those who have money to receive, and sell bills to those who have money to pay. A person going to a broker to buy a bill may very likely receive one that has been bought the same day from a merchant. If the broker has not on hand any exchange that he has bought, he will often give a bill on his own foreign correspondent; and to place his correspondent in funds to meet it, he will remit to him all the exchange which he has bought and not re-sold.

When brokers find that they are asked for more bills than are offered to them, they do not absolutely refuse to give them. To enable their correspondents to meet the bills at maturity, as they have no exchange to send, they have to remit funds in gold and silver. There are the expenses of freight and insurance upon the specie, besides the occupation of a certain amount of capital involved in this; and an increased price, or premium, is charged upon the exchange to cover all.

The reverse of this happens when brokers find that more bills are offered to them than they can sell or find use for. Exchange on the foreign country then falls to a discount, and can be purchased at a lower rate by those who require to make payments.

There are other influences that disturb the exchange between different countries. Expectations of receiving large payments from a foreign country will have one effect, and the fear of having to make large payments will have the opposite effect.

## AMERICAN EXCHANGE.

EXCHANGE between United States and Canada is a matter of every day importance on account of the proximity of the two countries, and the incessant intercourse between them both of a social and commercial character. Much inconvenience has been felt latterly in Canada on account of the depreciation of American currency. The immediate cause of this was the late war. The exigencies of the Northern States compelled them to issue, to enormous amounts, an inconvertible paper currency known by the name of "Greenbacks." As the value of these depended mainly on the stability of the government, and the success of the war, public confidence wavered, and in consequence the value of this issue sunk materially. Much damage was accordingly suffered by Canada, as all her commercial intercourse with the States was cramped by this depreciation of the currency, as well as by the fluctuation of the money market generally, and the doubtful issue of the struggle.

From these causes the value of gold rose gradually until it reached the enormous premium of nearly two hundred per cent., or quotation of nearly three hundred. But though gold was quoted at nearly two hundred per cent. premium, it must not be understood that

American money, or greenbacks, was at that rate per cent. discount. For example, when gold is quoted at 150, or 50 per cent. premium, greenbacks are not at a discount of 50 per cent., as many might be led to suppose, but only at 33$\frac{1}{3}$. The error will be more apparent from the consideration that when gold is quoted at 100 per cent. premium, the discount would not be 100, for if it were, money would be worth absolutely nothing.

### CASE I.

To find the value of $1, American currency, when gold is at a premium.

### EXAMPLE.

When gold is quoted at 140, or 40 per cent. premium, what is the value of $1, American currency?

### SOLUTION.

Since gold is at a premium of 40 per cent., it requires 140 cents of American funds to equal in value $1, or 100 cents in gold. Hence the value of $1, American money, will be represented by the number of times 140 is contained in 100, which is .71$\frac{3}{7}$, or 71$\frac{3}{7}$ cents. Hence to find the value of $1 of any depreciated currency reckoned in dollars and cents, we deduce the following

### RULE.

*Divide* 100 *cents by* 100 *plus the rate of premium on gold, and the quotient will be the value of* $1.

*Subtract this from* $1, *and the remainder will be the rate of discount on the given currency.*

### CASE II.

To find the value of any given sum of American currency when gold is at a premium.

### EXAMPLE.

What is the value of $280, American money, when gold is quoted at 140, or 40 per cent. premium?

### SOLUTION.

We find by Case I. the value of $1 to be 71$\frac{3}{7}$ cents. Now it is evident that if 71$\frac{3}{7}$ cents be the value of $1, the value of $280 will be 280 times 71$\frac{3}{7}$ cents, which is $200, or $\$280 \div 1.40 = 28000 \div 140 = \$200$. Hence we have the following

### RULE.

*Multiply the value of* $1 *by the number denoting the given amount of American money, and the product will be the gold value; or,*

*Divide the given sum of American money by* 100 (*the number of cents in* $1,) *plus the premium, and the quotient will be the value in gold.*

### CASE III.

To find the premium on gold when American money is quoted at a certain rate per cent. discount.

### EXAMPLE.

When the discount on American money is 40 per cent., what is the premium on gold ?

### SOLUTION.

If American money is at a discount of 40 per cent., the discount on $1 would be 40 cents, and consequently the value of $1 would be equal to $1.00—40 cents, equal to 60 cents. Now, if 60 cents in gold be worth $1 in American currency, $1 or 100 cents in gold would be worth 100 times 1⁄60 of $1, which is $1.66⅔, from which if we subtract $1, the remainder will be the premium. Therefore, if American currency be at a discount of 40 per cent., the premium on gold would be 66⅔ per cent. Hence we deduce the following

### RULE.

*Divide* 100 *cents by the number denoting the gold value of* $1, *American currency, and the quotient will be the value, in American currency, of* $1 *in gold, from which subtract* $1, *and the remainder will be the premium.*

### CASE IV.

To find the value in American currency of any given amount of gold.

### EXAMPLE.

What is the value of $200 of gold, in American currency, gold being quoted at 150 ?

### SOLUTION.

When gold is quoted at 150, it requires 150 cents, in American currency, to equal in value $1 in gold. Now, if $1 in gold be worth $1.50 in American currency, $200 will be worth 200 times $1.50, which is $300. Hence the

### RULE.

*Multiply the value of $1 by the number denoting the amount of gold to be changed, and the product will be the value in American currency; or,*

*To the given sum add the premium on itself at the given rate, and the result will be the value in American currency.*

### EXERCISES.

1. If American currency is at a discount of 50 per cent., what is the value of $450? Ans. $225.

2. The quotation of gold is 140, what is the discount on American currency? Ans. 28$\frac{4}{7}$ per cent.

3. A person exchanged $750, American money, at a discount of 35 per cent. for gold; how much did he receive? Ans. $487.50.

4. Purchased a draft on New York for $1500, at a discount of 31$\frac{1}{2}$ per cent.; what did it cost me? Ans. $1027.50.

5. American currency is quoted at 33$\frac{1}{3}$ per cent discount; what is the premium on gold? Ans. 50 per cent.

6. Purchased a suit of clothes in Toronto for $35, but on paying for the same in American funds, the tailor charged me 32 per cent. discount; how much had I to pay him? Ans. $51.47.

7. What would be the difference between the quotations of gold, if greenbacks were selling at 40 and 60 per cent. discount.? Ans. 83$\frac{1}{3}$ per cent.

8. P. Y. Smith borrowed from C. R. King, $27 in gold, and wished to repay him in American currency, at a discount of 38 per cent.; how much did it require? Ans. $43.55.

9. J. E. Peckham bought of Sidney Leonard a horse and cutter for $315.50, American currency, but only having $200 of this sum, he paid the balance in gold, at a premium of 65 per cent.; how much did it require? Ans. $70.

10. An American drover purchased of a farmer a yoke of oxen valued at $135 in gold, but paid him $112 in American currency, at a discount of 27$\frac{1}{2}$ per cent.; how much gold did it require to pay the balance? Ans. $53.80.

11. W. H. Hounsfield & Co., purchased in New York City, merchandise amounting in value to $4798.40, on 3 months' credit, premium on gold being 79$\frac{3}{8}$ per cent. At the expiration of the three months they purchased a draft on Adams, Kimball and Moore of

New York, for the amount due, at a discount of 57¾ per cent.; what was the gain by exchange? Ans. $647.75.

12. A makes an exchange of a horse for a carriage with B; the horse being valued at $127.50 in gold, and the carriage at $210, American currency. Gold being at a premium of 65 per cent; what was the difference, and by whom payable?

Ans. B pays A 23 cents.

13. A merchant takes $63 in American silver to a broker, and wishes to obtain for the same greenbacks which are selling at a discount of 30 per cent. The broker takes the silver at 3½ per cent. discount; what amount of American currency does the merchant receive? Ans. $86.85.

14. I purchased of B. W. Smith a farm in the township of Essa containing 100 acres, at $15 per acre. Sold 50 acres to an American speculator, at $23 per acre, American money; exchanged ⅕ of the remainder with Isaiah Wright, Esq., for a town lot in Brantford, allowing him for the difference of barter $400, for which he agreed to take its equivalent in American money, at a discount of 30 per cent.; the balance I sold to J. R. Forster, at a profit of 20 per cent., receiving in payment his note at 30 days, which I immediately sold to an American for $900 greenbacks. The amount of American currency I then had on hand I exchanged with my broker for Canada money, which was quoted at a premium of 50 per cent.; required the amount of profit, and the rate per cent. of gain.

15. A merchant left Toronto for New York City to purchase his stock of spring goods, taking with him to defray expenses $95 in gold. After purchasing his ticket to the Suspension Bridge for $2.40, he expended the balance in greenbacks, which were at a discount of 41½ per cent. When in New York he drew from this amount $23.85 to "square" an old account then past due. On arriving home he found that he still had in greenbacks $16.40, which he disposed of at a discount of 43¾ per cent., receiving in payment American silver at a discount of 3¼ per cent., which he passed off at 2½ per cent. discount for gold. What were his expenses in gold; the actual amount in greenbacks paid for expenses, and the amount of silver received?

Ans. Total expenses in gold, $71.76; expenses in greenbacks, $118.04; silver received, $9.53.

# STERLING EXCHANGE.

IN Britain money is reckoned by pounds, shillings and pence, and fractions of a penny, and is called *sterling money*, the gold sovereign, consisting of 22 parts gold and 2 alloy, being the standard, and the shilling, one-twentieth part of this, a silver coin of 37 parts silver and 3 copper, and the penny, one-twelfth part of this, a copper coin, the ingredients and size of which have frequently been altered. This is called *sterling money*, and the estimating of that currency in terms of the currency of another country, which in the case of Canada is done by dollars and cents, is called *sterling exchange*. If a house in Canada has transactions with one in Britain, it is necessary that each should be able to estimate what given sum in the currency of the one country is worth in that of the other.

The comparative value of the pound sterling, or gold sovereign and Canadian money, was formerly $\$4.44\frac{4}{9}$, but as this was less than the intrinsic value, and besides as the commercial value, though fluctuating, was always considerably higher, it was settled by Act of Provincial Parliament that the standard value of the pound sterling, estimated in dollars and cents, should be raised to $4.86⅔. The increase in the standard value was therefore equal to 9½ per cent. of its nominal value.

Sterling exchange is usually quoted in the form : 108, 109, 109½, 110, &c., which indicates that it is at 8, 9, 9½, or 10 per cent. premium. The REAL PAR VALUE of the British sovereign in Canadian money is $4.86⅔, the exact equivalent of the pound sterling. The COMMERCIAL VALUE is the amount in dollars and cents required to purchase one pound sterling at any given rate of the money market. If exchange is quoted at 109½, this means that it will require £109½ at par value to purchase £100 at commercial value. The rate of exchange between Canada and Britain is usually reckoned from the old par value. Though the commercial value fluctuates, yet 9½ per cent. above par may be taken as its average, so that if we add the premium of $\$4.44\frac{4}{9}$ to itself at 9½ per cent., we shall have $4.86⅔, which is nearly the average of the commercial value, and is generally taken as such.

### EXAMPLE.

A merchant in Canada wishes to remit £648.17.6 to Britain. The old par value is $\$4.44\frac{4}{9}=\$\frac{40}{9}=\frac{1}{9}$ of \$40 by reducing to an improper fraction. Then if the quotation is 108, or 8 per cent. above the nominal value, we find the premium on \$40 at 8 per cent., which is \$3.20, which added to \$40 will give \$43.20, and \$43.20÷9=\$4.80 to be remitted for every pound sterling, and therefore £648.17.6 multiplied by 4.80 or 4.8 will be the value in Canadian money. 17s. 6d=.875, and the operation is as follows

$$\begin{array}{r}
648.875\phantom{0} \\
4.8\phantom{0} \\
\hline
5191000\phantom{0} \\
2595500\phantom{00} \\
\hline
\$3114.60
\end{array}$$

### RULE.

*To \$40 add the premium on itself at the quoted rate, multiply the sum by the number representing the amount of sterling money, and divide the result by 9, the quotient will be the equivalent of the sterling money in dollars and cents.*

NOTE.—If there be shillings, pence, &c., in the sterling money, they are to be reduced to the decimal of £1.

To find the value of Canadian money in sterling money at any given rate above par.

Let it be required to find the value of \$465 in sterling money, at 8 per cent. above its nominal value. Here we have exactly the converse of the last problem, and therefore, having found the value of £1 sterling, we divide the given sum instead of multiplying; thus, the premium on \$40, at 8 per cent., is \$3.20, which added to \$40 makes \$43.20, and 43.20÷9=4.80, and \$465÷4.80=£96.17.6.

### RULE.

*Divide the given sum by the number denoting the value of one pound sterling at the given rate above par, and if there be a decimal remaining reduce it to shillings and pence.*

### EXERCISES.

1. When sterling exchange is quoted at 108, what is the value of £1 ? Ans. \$4.80.

2. If £1 sterling be worth \$4.84$\frac{4}{9}$, what is the premium of exchange between London and Canada? Ans. 9 per cent.

3. At 10 per cent. above its nominal value, what is the worth of £50 sterling, in Canadian currency ? Ans. \$244.44.

4. When sterling exchange is quoted at 9$\frac{1}{4}$ per cent. premium, what is the value of \$1000 ? Ans. £205 18s. 11$\frac{3}{4}$d.

5. At 12 per cent. above its nominal value, what will a bill for £1800 cost in dollars and cents ? Ans. \$8960.

6. A merchant sold a bill of exchange on London for £7000, at an advance of 11 per cent.; what did he receive for it more than its real value ? Ans. \$466.66$\frac{2}{3}$.

7. Bought a bill on London for £1266 15s., at 9$\frac{1}{2}$ per cent. prepremium; what shall I have to pay for it ? Ans. \$6164.85.

8. A merchant sells a bill on London for £4000, at 8 per cent. above its nominal value, instead of importing specie at an expense of 2 per cent.; what does he save ? Ans. \$122.66$\frac{2}{3}$.

9. A merchant in Kingston paid \$7300 for a draft of £1500 on Liverpool; at what per cent. of premium was it purchased ?

Ans. 9$\frac{1}{2}$.

10. A broker sold a bill of exchange for £2000, on commission, at 10 per cent. above its nominal value, receiving a commission of $\frac{1}{10}$ per cent. on the real value, and 5 per cent. on what he obtained for the bill above its real value; what was his commission ?

Ans. \$11.95$\frac{2}{9}$.

11. I owe A. N. McDonald & Co., of Liverpool, \$7218, net proceeds of sales of merchandise effected for them, which I am to remit them in a bill of exchange on London for such amount as will close the transaction, less $\frac{1}{4}$ per cent. on the face of the bill for my commission for investing. Bills on London are at 8 per cent. premium. Required the amount of the bill, in sterling, to be remitted.

Ans. £1500 5s. 6d.

## TABLE OF FOREIGN MONEYS.

| Cities and Countries. | Denominations of Money. | Value. |
|---|---|---|
| London, Liverpool, &c. | 12 pence=1 shilling; 20 shillings =1 pound....................= | $\$4.86\frac{2}{3}$ |
| Paris, Havre, &c........ | 100 centimes=1 franc............= | $.18\frac{3}{5}$ |
| Amsterdam, Hague, &c. | 100 cents=1 guilder or florin...= | .40 |
| Bremen ................. | 5 swares=1 grote; 72 grotes=1 *rix dollar*.......................= | $.78\frac{3}{4}$ |
| Hamburg, Lubec, &c... | 12 pfennings=1 schilling; 16s.= 1 *mark banco*..................= | .35 |
| Berlin, Dantzic.......... | 12 pfennings=1 groschen; 30 gro. =1 thaler.......... ..........= | .69 |
| Belgium.................. | 100 centimes=1 franc..........= | $.18\frac{3}{5}$ |
| St. Petersburg.......... | 100 kopecks=1 ruble............= | .75 |
| Stockholm .............. | 12 rundstycks=16 skillings; 48s. =1 *rix dollar specie* ..........= | 1.06 |
| Copenhagen............. | 16 skillings=1 mark; 6 m.=1 *rix dollar*.........................= | 1.05 |
| Vienna, Trieste, &c.... | 60 kreutzers=1 florin...........= | $.48\frac{1}{2}$ |
| Naples .................. | 10 grani=1 carlino; 10 car.=1 *ducat*..........................= | .80 |
| Venice, Milan, &c....... | 100 centesimi=1 *lira*...........= | .16 |
| Florence, Leghorn, &c. | 100 centesimi=1 *lira*...........= | .16 |
| Genoa, Turin, &c....... | 100 centesimi=1 *lira*..........= | $.18\frac{3}{5}$ |
| Sicily...................... | 20 grani=1 taro; 30 tari=1 oz.= | 2.40 |
| Portugal................. | 1000 reas=1 millrea............= | 1.12 |
| Spain.................... | 34 maravedis=1 *real vellon*= | .05 |
| | 68 maravedis=1 *real plate*..= | .10 |
| Constantinople ......... | 100 aspers=1 *piaster*.... ........= | .05 |
| British India............ | 12 pice=1 anna; 16 annas=1 *rupee*...........................= | $.44\frac{1}{2}$ |
| Canton.................... | 100 candarines=1 mace; 10 m.= 1 *tael*...........................= | 1.48 |
| Mexico .................. | 8 rials=1 *dollar*................= | 1.00 |
| Monte Video............ | 100 centesimas=1 rial; 8 rials=1 *dollar*................. .......= | $.83\frac{3}{10}$ |
| Brazil..................... | 1000 reas=1 *milrea*............= | $.82\frac{4}{5}$ |
| Cuba...................... | 8 reals plate or 20 reals vellon=1 *dollar*........................= | 1.00 |
| Turkey.................... | 100 aspers=1 *piaster*... ........= | .05 |
| United States........... | 10 mills=1 cent; 10 cents=1 dime; 10 dimes=1 *dollar*....= | variable. |
| New Brunswick.........<br>Nova Scotia............<br>Newfoundland......... | 4 farthings=1 penny; 12 pence =1 shilling; 20 shillings=1 pound.[*]..................= | 4.00 |

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[*] The Government of New Brunswick now issues postage stamps in the decimal currency, but so for as we have been able to ascertain, the currency of

# ARBITRATION OF EXCHANGE.

*Arbitration of Exchange* is the method of finding the rate of exchange between two countries through the intervention of one or more other countries. The object of this is to ascertain what is the most advantageous channel through which to remit money to a foreign country.

Three things have here to be considered. *First*, what is the most secure channel; *secondly*, what is the least expensive, and *thirdly*, the comparative value of the currencies of the different countries. Regarding the two first considerations no general rule can be given, as there must necessarily be a continual fluctuation arising from political and other causes. We are therefore compelled to confine our calculation to the third, viz., the comparative value of the coin current of different countries.

For this purpose we shall investigate a rule, and append tables.

Let us suppose an English merchant in London wishes to remit money to Paris, and finds that owing to certain international relations, he can best do it through Hamburg and Amsterdam, and that the exchange of London on Hamburg is 13½ marcs per pound sterling; that of Hamburg on Amsterdam, 40 marcs for 36¼ florins, and that of Amsterdam on Paris, 56¾ florins for 120 francs, and thus the question is to find the rate of exchange between London and Paris.

### SOLUTION:

We write down the equivalents in ranks, the equivalent of the first term being placed to the right of it, and the other pairs below them in a similar order. Hence the first term of any pair will be of the same kind as the second term of the preceding pair. As the answer is to be the equivalent of the first term, the first term in the last rank corresponds to the third term of an analogy, and is therefore a multiplier, it must be placed below the second rank. The

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these three Provinces is, as usual, in pounds, shillings and pence. It is to be hoped that when the Confederation of the British Provinces takes place, the decimal currency will be speedily adopted in the Lower Provinces, and that the efforts now being made in Britain to adapt the same currency will prove successful.

terms being thus arranged, we divide the product of the second rank by that of the first, and the quotient will be the equivalent, as exhibited below:

$$\begin{array}{rcl} \pounds 1 \text{ sterling} & = & 13\frac{1}{2} \text{ marcs.} \\ 40 \text{ marcs} & = & 36\frac{1}{4} \text{ florins.} \\ 56\frac{3}{4} \text{ florins} & = & 120 \text{ francs.} \\ & & \pounds 1 \text{ stg.} \end{array}$$

As it is most convenient to express the fractions decimally, we have

$$\frac{13\cdot5 \times 36\cdot25 \times 120 \times 1}{1 \times 40 \times 56\cdot75} = 25.87 \text{ francs.}$$

The foregoing explanations may be condensed into the form of a

### RULE.

*Write down the first term, and its equivalent to the right of it, and the other pairs in the same order, the odd term being placed under the second rank, and then divide the product of the second rank by the product of the first, the quotient will be the required equivalent.*

NOTE.—The true principle on which this operation is founded is, that each pair consists of the antecedent and consequent which are to each other in the ratio of equality IN POINT OF INTRINSIC VALUE, though not in regard to THE NUMBERS BY WHICH THEY ARE EXPRESSED, and therefore the required term and its equivalent must have the same relation to each other, that is, they will be an antecedent and a consequent in the ratio of equality as regards their *value*, but not as regards the *numbers* by which they are expressed.

### EXERCISES.

1. If the exchange of London on Paris is 28 francs per pound sterling, and that of Canada on Paris 18 cents per franc; what is the rate of exchange of Canada on London, through Paris?

Ans. $5.04 per £ sterling.

2. If exchange between Montreal and London is at 8 per cent. premium, and between London and Paris $25\frac{1}{4}$ francs per pound sterling; what sum in Montreal is equal to 7000 francs in Paris?

Ans.

3. When exchange between Quebec and Hamburg is at 34 cents per mark banco, and between Hamburg and St. Petersburg is 2 marks, 8 schillings per ruble; how much must be paid in St. Petersburg for a draft on Quebec for $650?

Ans. 764 rubles, $70\frac{10}{17}$ kopecks.

4. If a merchant buys a bill in London, drawn on Paris, at the rate of 25.87 francs per pound sterling, and if this bill be sold in Amsterdam at 120 francs for 56¾ florins, and the proceeds be invested in a bill on Hamburg, at the rate of 36¼ florins for 40 marcs; what is the rate of exchange between London and Hamburg, or what is £1 sterling worth in Hamburg? Ans. 13.449+marcs.

5. A merchant of St. Louis wishes to pay a debt of $5000 in New York; the direct exchange is 1½ per cent. in favour of New York, but on New Orleans it is ½ per cent. discount, and between New Orleans and New York at a ¼ per cent. premium; how much would be saved by the circular exchange compared with the direct?

Ans. $87.56.

6. A merchant in Toronto wishes to remit to J. B. Gladstone & Co., of London, £3600 sterling. Exchange on London, in Toronto, is at a premium of 10 per cent. Exchange on London can be obtained at Halifax, Nova Scotia, for 9 per cent. premium. If Toronto Bills on Halifax are at a discount of ¼ per cent., and the merchant remits a draft to Halifax, and pays his agent ½ per cent. for investing it in bills on London; what will he gain over the direct exchange? Ans. $123.80.

7. A merchant in London remits to Amsterdam £1000, at the rate of 18 pence per guilder, directing his correspondent at Amsterdam to remit the same to Paris at 2 francs, 10 centimes per guilder, less ½ per cent. for his commission; but the exchange between Amsterdam and Paris happened to be, at the time the order was received, at 2 francs, 20 centimes per guilder. The merchant at London, not apprised of this, drew upon Paris at 25 francs per pound sterling. Did he gain or lose, and how much per cent.?

Ans. $16\frac{56}{75}$ per cent. gain.

### MIXED EXERCISES IN EXCHANGE.

1. When gold is quoted at 150 per cent. premium; what is the reason American money is not at a discount of 50 per cent.?

2. I wish to invest $3760.80 in a sterling bill of exchange; for how many pounds must the bill be drawn, exchange being at a premium of 8 per cent.? Ans. £783 10s.

3. What sum in Canada money must I pay for a bill on London of £76 14s. 1d., exchange being 9½ per cent. premium, and the broker's commission for negociating the bill being ½ per cent.?

Ans. $375.

4. A merchant shipped 2560 barrels of flour to his agent in Liverpool, who sold it at £1 8s. 6d. per barrel, and charged 2 per cent. commission; what was the net amount of the flour in Canada money, allowing exchange to be at a premium of 8 per cent.? Ans. $17160.19.

5. What is the cost of a 30 days' bill on Montreal, at ½ per cent. premium, the face of the bill being $1500? Ans. $1507.50.

6. What must be the face of a 60 days' draft on Halifax, Nova Scotia, to yield $1641.75, when sold at a discount of ½ per cent.? Ans. $1650.

7. What is the cost of a 30 days' bill on Quebec, at ⅜ per cent. premium, and interest off at 6 per cent.; the face of the bill being $9256.40?* Ans. $9240.20.

8. A merchant paid $14400.12 for a bill on Havre for 79000 francs; how much was exchange below par? Ans. 2 per cent.

9. I have in possession the net proceeds of a sale of cotton amounting to $3765, which my correspondent desires me to remit to him in New Orleans; exchange on New Orleans is at a discount of 2½ per cent., and I invest the whole in a draft at that rate, which I remit to him; what is the face of the draft? Ans. $3861.54.

10. The proceeds of a sale of goods, consigned to me from Bremen, is $2764.67, on which I am to charge a commission of 10 per cent., and remit the balance to my consignor in such a way as shall be most advantageous to him. Exchange on Paris can be had at 92 cents per 5 francs, and in Paris exchange on Bremen is 17 francs to 4 thalers. Exchange on Liverpool can be had at 9 per cent. premium, and in Liverpool exchange on Bremen is 6 thalers to the pound sterling. Direct exchange is 80¼ cents per thaler. Which course will be the best, allowing ½ per cent. brokerage to correspondents both in Liverpool and Paris? Ans. By way of Paris.

11. A, of Hamilton, sent articles to the World's Fair in London, which were afterwards sold by B, of London, on A's account, net proceeds £1266 15s. sterling. B was instructed to invest this amount in bills on Toronto, and remit to A, which was accordingly done. B charged ¼ per cent. brokerage on the face of the bills for investing, and purchased the bills at 7 per cent discount. Required

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* When there is interest to be computed, it must be reckoned on the face of the bill or draft. When other than the value or cost of the bill is to be found, proceed as in percentage.

the amount of the bill A must receive in Canada money to close the transaction. Ans. $6037.53 nearly.

12. A merchant in Toronto having to remit £434 15s. to Liverpool, wishes to know which is the most profitable, to buy a set of exchange on Liverpool at 10½ per cent. premium, or send it by way of France; exchange on the latter place being 19¾ cents per franc, and exchange on Liverpool can be bought in France at the rate of 24½ francs per pound sterling, and he has to pay his correspondent in France ¾ of 1 per cent. for purchasing the bill on Liverpool.

Ans. By way of France, $15.69.

13: John McDonald & Co. wish to remit to a creditor in London £1241 15s. 9d. Exchange on London can be bought in Toronto at 109¾, but exchange on London can be purchased in New York for gold at 108½. In New York it takes $1.85 greenbacks to equal $1 in gold. The broker in New York charges ¾ per cent. on the greenback value for investing. If exchange on New York is at 47 per cent. discount, at which place would it be the most advantageous to purchase, and how much gain, and if the remittance be made by the way of New York, what would be the face of the draft?

Ans. New York by $141.72; face of draft, $11161.21.

In the above exercise, suppose that instead of purchasing a draft on New York, they remit specie at an expense of ⅝ per cent., the New York broker's commission being ¾ per cent. on the gold value of the bill; what would be his gain or loss? Ans. Loss $13.64.

14. Hughes, Bros. & Co., purchase of E. Chaffey & Co., a sterling bill at 60 days on Gladstone & Hart, of London, for £3956 10s. They remit this bill to James Aldler, in London, where it is accepted by Gladstone & Hart, and falls due on the 20th November, at which time it is protested, causing an expense of £2 19s. Gladstone & Hart having failed, E. Chaffey & Co.'s agent in London pays James Aldler on the 20th November, £2000 on account. How much must E. Chaffey & Co. pay to Hughes, Brothers & Co., on the 24th December, to cover the amount still due in London, allowing interest at the rate of 10 per cent. from November 20th, to the maturity of a 60 days' bill at date of 24th December, and ¼ of 1 per cent. commission for their trouble in negociating a new bill? Ans. $9815.91.

# INVOLUTION.

*Involution* is the process of finding a given power of a given number.

We have noted already, under the head of multiplication, that the product of any number of equal factors is called the second, third, fourth, &c., power of the number, according as the factor is taken two, three, four, &c., times. Thus: $9=3\times3$ is the second power of 3; $27=3\times3\times3$ is the third power of three; $81=3\times3\times3\times3$ is the fourth power of 3. These are often written thus: $3^2$, $3^3$, $3^4$, &c. The small figures, 2, 3, 4, indicate the number of factors, and therefore each is called the *index* or *exponent* of the power. Hence to find any required power of a given quantity, we have the

### RULE.

*Multiply the quantity continually by itself until it has been used as a factor as often as there are units in the index.*

Since the first multiplication exhausts two factors, the number of operations will be *one* less than the number of factors.

Involution, then, is nothing more than multiplication, and for any power above the second, it is a case of continual multiplication. For the sake of uniformity the original quantity is called the *first power*, and also the root in relation to higher powers. Again, if we multiply $3\times3$ by $3\times3\times3$, we have five factors, or $3\times3\times3\times3\times3$, but this being an inconvenient form, it is written briefly $3^5$, the 5 indicating the number of times that 3 is to be repeated as a factor. Hence, since $3\times3$ is written $3^2$, and $3\times3\times3$ is written $3^3$, it follows that $3^2\times3^3=3^5$, and therefore we may multiply quantities so expressed by *adding their indices*, and so also we may divide such quantities by *subtracting the index of the divisor from that of the dividend.* For example $3^3\div3^2=3$ or $3^1$. If we divide $3^1$ by $3^1$ by subtracting the index of the divisor from that of the dividend, we obtain $3^0$, but 3 or $3^1$ divided by 3 or $3^1$ is equal to 1, and therefore any quantity with an index *zero* is equal to unity.

When high powers are to be found, the operation may be shortened in the following manner:—Let it be required to find the sixteenth power of 2. We first find the second power of 2, which is 4, then $4\times4=16$, which is the fourth power, and $16\times16=256$, the eighth power, and $256\times256=65536$, the sixteenth power. If we wished to find the nineteenth power, we should only have to multiply the last result by 8, which is the third power of 2, for $2^{16}\times2^3=2^{19}$.

### EXERCISES.

1. Find the second power of 697. Ans. 485809.
2. What is the third power of 854 ? Ans. 622835864.
3. What is the second power of 4.367 ? Ans. 19.070689.
4. Find the fourth power of 75. Ans. 31640625.
5. What is the sixth power of 1.12? Ans. 1.9738+.
6. What is the second power $.\dot{7}$, correct to six places ? Ans. .060893+.
7. What is the fifth power of 4 ? Ans. 1024.
8. Find the third power of $.\dot{3}$ to three places? Ans. .036963.
9. What is the third power of $\frac{7}{9}$ ? Ans. $\frac{343}{729}$.
10. What is the fifteenth power of 1.04 ?* Ans. 1.800943.
11. Raise 1.05 to the thirty-first power. Ans. 4.538039.
12. What is the eighth power of $\frac{3}{5}$ ? Ans. $\frac{6561}{390625}$.
13. What is the second power of $4\frac{7}{8}$ ? Ans. $23\frac{49}{64}$.
14. Expand the expression $6^5$. Ans. .7776.
15. What is the second power of $5\frac{1}{2}$ ? Ans. $\frac{121}{4}=30\frac{1}{4}$.
16. What part of $8^3$ is $2^6$ ? Ans. $\frac{1}{8}$.
17. What is the difference between $5^6$ and $4^6$? Ans. 11529.
18. Expand $3^5\times2^4$. Ans. 3888.
19. Express, with a single index, $47^3\times47^5\times47^6$ ? Ans. $47^{14}$.
20. How many acres are in a square lot, each side of which is 135 rods ? Ans. 113 acres, 3 roods, 25 rods.
21. What is the sixth power of .1 ? Ans. .000001.
22. What is the fourth power of .03 ? Ans. .00000081.
23. What is the fifth power of 1.05 ? Ans. .1.2762815625.
24. What is the third power of .001 ? Ans. .000000001.
25. What is the second power of $.004\dot{4}$ ? Ans. .00001836.

The second power of any number ending with the digit 5 may be readily found by taking all the figures except the 5, and multi-

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[*] This exercise will be most readily worked by finding the sixteenth power, and dividing by 1.04. So in the next exercise, find the thirty-second power, and divide by 1.05. A still more easy mode of working such questions will be found under the head of logarithms.

plying that by itself, increased by a unit, and annexing 25 to the result.

Thus, to find the second power of 15, cut off the 5, and 1 remains, and this increased by 1 gives 2, and $2 \times 1 = 2$, and 25 annexed will give 225, the second power of 15. So also,

| | | | | | |
|---|---|---|---|---|---|
| 2,5 | 3,5 | 6,5 | 10,5 | 21,5 | 57,5 |
| 3 | 4 | 7 | 11 | 22 | 58 |
| 625 | 1225 | 4225 | 11025 | 46225 | 330625 |

### EXERCISES ON THIS METHOD.

26. What is the second power of 135? Ans. 18225.
27. What is the second power of 205? Ans. 42025.
28. What is the second power of 335? Ans. 112225.
29. What is the second power of 455? Ans. 207025.
30. What is the second power of 585? Ans. 342225.
31. What is the second power of 795? Ans. 632025.

NOTE.—The square root of any quantity ending in 9, must end in either 3 or 7.

No second power can end in 8, 7, 3 or 2.

The second root of any quantity ending in 6, must end in 4 or 6.

The second root of any quantity ending in 5, must end also in 5.

The second root of any quantity ending in 4, must end either in 8 or 2.

The second root of any quantity ending in 1, must end either in 1 or 9.

The second root of any quantity ending in 0, must also end in 0.

## EVOLUTION.

The root of any quantity is a number such that when repeated, as a factor, the specified number of times, will produce that quantity. Thus, 3 repeated twice as a factor gives 9, and therefore 3 is called the *second root* of 9, while 3 taken *three times* as a factor will give 27, and therefore 3 is called the *third root* of 27, and so also it is called the *fourth root* of 81.

There are two ways of indicating this. First, by the mark $\sqrt{}$ which is merely a modified form of the letter r, the initial letter of the English word root, and the Latin word *radix* (root). When no mark is attached, the simple quantity or *first root* is indicated. When the *second root* is meant, the mark $\sqrt{}$ alone is placed before the quantity, but if the third, fourth, &c., roots are to be indicated, the figures 3, 4, &c., are written in the angular space. Thus: $3=\sqrt{9}=\sqrt[3]{27}=\sqrt[4]{81}=\sqrt[5]{243}$, &c., &c. The other method is to write the index as a fraction. Thus, $9^{\frac{1}{2}}$ means the second root of the first power of 9, *i. e.* 3. So also, $27^{\frac{1}{3}}$ is the third root of the first power of 27. In the same manner $64^{\frac{2}{3}}$ means the third root of the second power of 64, or the second power of the third root of 64. Now the third root of 64 is 4, and the second power of 4 is 16, or the second power of 64 is 4096, and the third root of 4096 is 16, so that both views give the same result.

*Evolution* is the process of finding any required root of a given quantity.

## SECOND OR SQUARE ROOT.

Extracting the square or second root of any number, is the finding of a number which, when multiplied by itself, will produce that number.

To find the second root, or square root of any quantity.

By inspecting the table of second powers, it will be found that the second power of any whole number less than 10, consists of either *one* or *two* digits; the second power of any number greater than 9, and less than 100, will in like manner be found to consist of *three* or *four* digits; and, universally, the second power of any number will consist of either *twice* the number of digits, or *one less than twice* the number of digits that the root itself consists of. Hence, if we begin at the units' figure, and mark off the given number in periods of two figures each, we shall find that the number of digits contained in the root will be the same as the number of periods. If the number of digits is even, each period will consist of two figures, but if the number of digits be odd, the last period to the left will consist of only one figure.

Let it now be required to find the second root of 144. We know by the rule of involution that 144 is the second power of 12. Now 12 may be resolved into *one ten* and *two units*, or 10+2, and 10+2 multiplied by itself, as in the margin, gives 100+40+4, and since 100 is the second power of 10, and 4 the second power of 2, and 40 is twice the product of 10 and 2, we conclude that the second power of any number thus resolved is equal to the sum of the second powers of the parts, *plus* twice the product of the parts. Hence to find the second root of 144, let us resolve it into the three parts 100+40+4, and we find that the second root of the first part is 10, and since 40 is twice the product of the parts, 40 divided by twice 10 or 20 will give the other part 2, and 10+2=12, the second root of 144. We should find the same result by resolving 12 into 11+1, or 9+3, or 8+4, or 7+5, or 6+6, but the most convenient mode is to resolve into the tens and the units. In the same manner, if it be required to find the second root of 1369, we have by resolution 900+420+49, of which 900 is the second power of 30, and 30×2=60, and 420÷60=7, the second part of the root, and 30+7=37, the whole root.

```
10+2
10+2
------
100+20
    20+4
----------
100+40+4
```

Again, let it be required to find the second root of 15129. This may be resolved as below:

10000 is the second power of 100.
400 is the second power of 20.
9 is the second power of 3.
4000 is twice the product of 20 and 100.
600 is twice the product of 100 and 3.
120 is twice the product of 20 and 3.

15129 is the sum of all, and hence 1 is the root of the hundreds, 2 the root of the tens, and 3 the root of the units.

Generalizing these investigations, we find that the second power of a number consisting of *units alone* is the product of that number by itself; that the second power of a number consisting of *tens and units* is the second power of the *tens*, *plus* the second power of the units, *plus* twice the product of the tens and units; that the second power of a number, consisting of hundreds, tens and units, is the sum of the squares of the hundreds, the tens, and the units, *plus* twice the product of each pair. Now since the complement of the full second power, to the sum of the second powers of the parts, is twice the product of the parts, it follows that, when the first figure of the root has been found, it must be doubled before used as a divisor to find the second term, and for the same reason each figure, when found, must be doubled to give correctly the next divisor. Hence the

### RULE.

*Beginning at the units' figure, mark off the whole line in periods of two figures each; find the greatest power contained in the left hand period, and subtract it from that period; to the remainder annex the next period; for a new dividend, place the figure thus obtained as a quotient, and its double as a divisor, and find how often that quantity is contained in the second partial dividend, omitting the last figure; annex the figure thus found to both divisor and quotient, multiply and subtract as in common division, and to the remainder annex the next period; double the last obtained figure of the divisor, and proceed as before till all the periods are exhausted,—if there be a remainder, annex to it two ciphers, and the figure thence obtained will be a decimal, as will every figure thereafter obtained.*

### EXAMPLES.

1. To find the second root of 797449.

First, commencing with the units' figure, we divide the line into periods, viz., 49, 74 and 79,—we then note that the greatest square contained in 79 is 64,—this we subtract from 79, and find 15 remaining, to which we annex the next period 74, and place 8, the second root of 64, in the quotient, and its double 16 as a divisor, and try how often 16 is contained in 157, which we find to be 9 times; placing the 9 in both divisor and quotient, we multiply and subtract as in common division, and find a remainder of 53, to which we annex the last period 49, and proceeding as before, we find 3, the last figure of the root, without remainder, and now we have the complete root 893.

| | | |
|---|---|---|
| 8 | 797449 | 893 |
| | 64 | |
| 169 | 1574 | |
| 1783 | 1521 | |
| | 5349 | |
| | 5349 | |

2. This operation may be illustrated as follows:

To find the second root of 273529.

| | | |
|---|---|---|
| 500 | 273529 | 500+20+3=523 |
| 500×2=1000+20, or | 250000 | |
| 1020 | 23529 | |
| 1000+2×20+3=1043 | 20400 | |
| | 3129 | |
| | 3129 | |

3. To find the second root of 153687.

Here we obtain, by the same process as in the last example, the whole number 392, with a remainder of 23, which can produce only a fraction.

| | | |
|---|---|---|
| 3 | 153687 | 392.029+ |
| | 9 | |
| 69 | 636 | |
| 782 | 621 | |
| | 1587 | |
| 78402 | 1564 | |
| 784049 | 230000 | |
| | 156804 | |
| | 7319600 | |
| | 7056441 | |
| | 263159 | |

We now annex two ciphers, placing the decimal point after the root already found, but as the divisor is not contained in this new dividend, we place a cipher in both quotient and divisor, and annex two ciphers more to the dividend, and by continuing this process we find the decimal part of the root, and the whole root is 392.029+.

## EXERCISES.

1. What is the second root of 279841? Ans. 529.
2. What is the second root of 74684164? Ans. 8642.
3. What is the second root of 459684? Ans. 678.
4. What is the second root of 785? Ans. 28.01785+.
5. What is the second root of 1728? Ans. 41.569219+.
6. What is the second root of 666? Ans. 25.8069+.
7. What is the second root of 123456789? Ans. 11111.11106+.
8. What is the second root of 5 to three places? Ans. 2.236.
9. What is the side of a square whose area is 19044 square feet? Ans. 138 feet.
10. What is the length of each side of a square field containing 893025 square rods? Ans. 945 linear rods.

The second root of a fraction is found by extracting the roots of its terms, for $\frac{16}{25}=\frac{4}{5}\times\frac{4}{5}$ and therefore $\sqrt{\frac{16}{25}}=\sqrt{\frac{4}{5}\times\frac{4}{5}}=\frac{4}{5}$. So also, $\sqrt{\frac{49}{81}}=\frac{7}{9}$. Again, since $\sqrt{\frac{81}{100}}=\frac{9}{10}=.09$, and $.3\times.3=.09$, the second root of .09 is .3. This follows from the rules laid down for the multiplication of decimals.

To find the second root of a decimal or of a whole number and a decimal:

*Point off periods of two figures each from the decimal point towards the right and left, adding a cipher, or a repetend, if the number of figures be odd.*

From what has been said, it is plain that every period, except the first on the left, must consist of two digits, and every decimal presupposes something going before, for .5 indicates the half of some unit under consideration, and .5 is equivalent to .50, *and not to* .05, from which it is obvious that the second root of .5 is not the root of .05, but of .50, and therefore the second root of .5 is not .2+, as the beginner would naturally suppose, but .7+, for .2+ is the approximate root of .05.

### ADDITIONAL EXERCISES.

11. What is the second root of .7 to five places of decimals? Ans. .83666.
12. Find the second root of .07 to six places. Ans. 264575.
13. What is the second root of .05? Ans. .2236+.
14. What is the second root of $.\dot{7}$? Ans. .8819+.
15. Find the second root of $.\dot{5}$. Ans. .74535+.
16. What is the second root of .1? Ans. .3162277+.
17. What is the second root of $.\dot{1}$? Ans. $.\dot{3}$.
18. What is the second root of 1.375? Ans. 1.1726, &c.*
19. What is the second root of .375? Ans. 61237, &c.*
20. What is the second root of 6.4? Ans. 2.52982+.
21. Find to four decimal places $\sqrt{3\frac{3}{20}}$. Ans. 1.7748.
22. Find $\sqrt{2}$ to four decimal places. Ans. 1.4142.
23. Find the value of $\sqrt{3271.4207}$. Ans. 57.196+.
24. Find the second root of .005 to five places. Ans. 07071.
25. Find the square root of 4.372594. Ans. 2.09107+.
26. What is the second root of .01? Ans. .1.
27. What is the second root of .001? Ans. 03162+.
28. What is the square root of .0001? Ans. .01.
29. What is the second root of .000001? Ans. .001.
30. What is the second root of 19.0968?

---

* The young student would naturally expect that the decimal figures of $\sqrt{1.375}$ and $\sqrt{.375}$ would be the same, but it is not so. If it were so, $\sqrt{1}+\sqrt{.375}$ would be equal to $\sqrt{1.375}$. That such is not the case, may be shown by a very simple example: $\sqrt{16}+\sqrt{9}=4+3=7$, but $\sqrt{16+9}=\sqrt{25}=5$. Let it be carefully observed, therefore, that *the sum of the second roots is not the same as the second root of the sum.*

OPERATION

| | | |
|---|---|---|
| 4 | $\dot{1}9.0\dot{9}6\dot{8}$ | 4.37 trial. |
| 83 | 16 | 4.36 true. |
| | 309 | |
| Trial 867 | 249 | |
| | 6068 | |
| Too great by 1 | 6069 | |
| True 866 | 6068 | |
| | 5196 | |
| | 872 | |

Here we find the remainder, 872, is greater than the divisor, 866, which seems inconsistent with ordinary rules; but it must be observed that we are not seeking an exact root, but only the closest possible approximation to it. If the given quantity had been 19.0969, we should have found an exact root 4.37. The remainder 872 being greater than the divisor, shows that the last figure of the root is too small by $\frac{99}{100}$, whereas 7 would be too great by $\frac{1}{100}$, and that 866 is not a correct divisor but an approximate one, and that the true root lies between 4.36 and 4.37.

When the root of any quantity can be found exactly, it is called a *perfect power* or *rational quantity*, but if the root cannot be found exactly, the quantity is called *irrational* or *surd.*

A number may be rational in regard to one root, and irrational in regard to another. Thus, 64 is rational as regards $\sqrt{64}=8$, $\sqrt[3]{64}=4$ and $\sqrt[6]{64}=2$, but it is irrational regarding any other root expressed by a whole number. But 64, with the fractional index $\frac{2}{3}$, *i. e.*, $64^{\frac{2}{3}}$, is rational, because it has an even root as already shown. We may call $64^{\frac{2}{3}}$ either the second power of the third root of 64, or the third root of the second power. In the former view, the third root of 64 is 4, and the second power of 4 is 16, and according to the second view, $64^2$ is 4096, and the third root of 4096 is 16, the same as before. $\sqrt[4]{81}=3$ is rational, and $\sqrt{81}=9$ is rational, but 81 is not rational regarding any other root; while $\sqrt{25}$ is rational only regarding the second root, and $\sqrt[3]{8}=2$ only regarding the third root.

The second root of an even square may be readily found by resolving the number into its prime factors, and taking each of these factors once,—the product will be the root. Thus, 441 is $3\times3\times7\times7$ and each factor taken once is $3\times7=21$, the second root. Here let it be observed, that if we used each factor *twice* we should obtain the *second power*, but if we use each factor half the number of times that it occurs, we shall have the second root of that power. 64 is $2\times2\times2\times2\times2\times2=2^6$, *i. e.*, 2 repeated six times as a factor gives the number 64, and therefore half the number of these factors will give the second root of 64, or $2\times2\times2=8$, and $2\times2\times2$ multiplied by $2\times2\times2=8\times8=64$.

As this cannot be considered more than a trial method, though often expeditious, we would observe that the smallest possible divisors should be used in every case, and that if the number cannot be thus resolved into factors, it has no even root, and must be carried out into a line of decimals, or those decimals may be reduced to common fractions.

## THIRD ROOT OR CUBE ROOT.

As extracting the second root of any quantity is the finding of what two equal factors will produce that quantity, so extracting the third root is the finding of what *three* equal factors will produce the quantity.

By inspecting the table of third powers, it will be seen that no third power has more than three digits for each digit of the first power, nor fewer than two less than three times the number of digits. Hence, if the given quantity be marked off in periods of three digits each, there will be one digit in the first power for each period in the third power. The left hand period may contain only one digit.

From the mode of finding the third power from the first, we can deduce, by the converse process, a rule for finding the first power from the third. We know by the rule of involution that the third power of 25 is 15625. If we resolve 25 into

$20+5$, and perform the multiplication in that form,
we have $20+5$

$$\begin{array}{l}
\quad 20+5\\
\quad 20+5\\
\hline
400+100\\
\qquad\;\; 100+25\\
\hline
400+200+25=(20+5)^2\\
\quad 20+5\\
\hline
8000+4000+500\\
\qquad\;\; 2000+1000+125\\
\hline
8000+6000+1500+125=(20+5)^3=15625
\end{array}$$

Now, 8000 is the third power of 20, and 125 is the third power of 5; also, 6000 is three times the product of 5, and the second power of 20, and 1500 is three times the product of 20, and the second power of 5. Let $a$ represent 20 and $b$ represent 5, then

$$\begin{array}{rcr}
a^3=20^3 & = & 8000\\
3\,a^2\,b=3\times20^2\times5 & = & 6000\\
3\,a\,b^2=3\times20\times5^2 & = & 1500\\
b^3=5^3 & = & 125\\
\hline
 & & 15625
\end{array}$$

By using these symbols we obtain the simplest possible method of extracting the third root of any quantity, as exhibited by the subjoined scheme:

| | |
|---|---|
| Given quantity.......... .............. | 15625 |
| $a^3=20^3=20\times20\times20$ ..........= | 8000 |
| Remainder ............................ | 7625 |
| $3\,a^2\,b=3\times20^2\times5$ .............. = | 6000 |
| Remainder ............................ | 1625 |
| $3\,a\,b^2=3\times20\times5^2$ ...............= | 1500 |
| Remainder ............................ | 125 |
| $b^3=5^3=5\times5\times5$ ................= | 125 |

From this and similar examples we see that a number denoted by more than one digit may be resolved into tens and units. Thus, 25 is 2 tens and 5 units, 123 is 12 tens and 3 units, and so of all numbers.

To find the third root of 1860867:

As this number consists of three periods, the root will consist of three digits, and the first period from the left will give hundreds, the second tens, and the third units, and so also in case of remainder, each period to the right will give one decimal place, the first being tenths, the second hundredths, &c., &c.

We may denote the digits by $a$, $b$ and $c$.

| | | |
|---|---|---|
| $a=100$ | $\dot{1}860\dot{8}6\dot{7}(100+20+3=123$ | |
| $a^3=100^3=$ | 1000000 | |
| $b=3a^2\ b\div3\ a^2=\frac{860867}{30000}=20+$, | 860867 | remainder. |
| and $30000\times20=$ | 600000 | |
| | 260867 | remainder. |
| $3\ a\ b^2=3\times100\times400=$ | 120000 | |
| | 140867 | remainder. |
| $b^3=20^3=$ | 8000 | |
| Now $(a+b)=120$ ∴ $3\ (a+b)^2=$ | 132867 | remainder. |
| 43200, which is contained 3 times+ | | |
| in 132867, ∴ $c=3$, and $3\ (a+b)^2c^2$ | | |
| $=3\times120^2\times3=$ | 129600 | |
| | 3267 | remainder. |
| And $3\ (a+b)\ c^2=3\times120\times9=$ | 3240 | |
| | 27 | |
| And lastly, $c^3=3^3=$ | 27 | |
| | ...... | no remainder. |

## RULE.

*Mark off the given number in periods of three figures each.*

*Find the highest third power contained in the left hand period, and subtract it from that period. Divide the remainder and next period by three times the second power of the root thus found, and the quotient will be the second term of the root.*

*From the first remainder subtract three times the product of the second term, and the square of the first,* PLUS *three times the product of the first term, and the square of the second,* PLUS *the third power of the second.*

*Divide the remainder by three times the square of the sum of the first and second terms, and the quotient will be the third term.*

*From the last remainder subtract three times the product of the term last found, and the square of the* SUM *of the preceding terms,* PLUS *the product of the square of the last found term by the* SUM *of the preceding ones,* PLUS *the third power of the last found term, and so on.*

## EXERCISES.

1. What is the third root of 46656? Ans. 36.
2. What is the third root of 250047? Ans. 63.
3. What is the third root of 2000576? Ans. 126.
4. What is the third root of 5545233? Ans. 177.
5. What is the third root of 10077696? Ans. 216.
6. What is the third root of 46268279? Ans. 359.
7. What is the third root of 85766121? Ans. 441.
8. What is the third root of 125751501? Ans. 501.
9. What is the third root of 153990656? Ans. 536.
10. What is the third root of 250047000? Ans. 630.
11. What is each side of a square box, the solid content of which is 59319? Ans. 39 inches.
12. What is the third root of 926859375? Ans. 975.
13. Find the third root of 44.6. Ans. 3.456+.
14. What is the third root of 9? Ans. 2.08008+.
15. What is the length of each side of a cubic vessel whose solid content is 2936.493568 cubic feet? Ans. 1432 feet.
16. Find the third root of 5. Ans. 1.7099.
17. A store has its length, breadth and height all equal; it can hold 185193 cubic feet of goods; what is each dimension? Ans. 57 feet.
18. How many linear inches must each dimension of a cubic vessel be which can hold 997002999 cubic inches of water? Ans. 999 inches.
19. What is the third root of 1? Ans. 1.
20. What is the third root of 144? Ans. 5.241483.

The third root of a fraction is found by extracting the third root of the terms. The result may be expressed either as a common fraction, or as a decimal, or the given fraction may be reduced to a decimal, and the root extracted under that form.

### EXERCISES.

1. What is the third root of $\frac{27}{64}$? Ans. $\frac{3}{4}=.75$.

Otherwise:

$\frac{27}{64}=.421875$. To find the third root of this we have

$.42\dot{1}87\dot{5}(.70+.05=.75$

$70^3=$ 343000

78875 remainder.

$$\left.\begin{array}{l}3\times70^2\times5=73500\\3\times70\times5^2=\ \ 5250\\5^3=\ \ \ 125\end{array}\right\}=78875$$

...... no remainder.

The third root of a mixed quantity will be most readily found by reducing the fractional part to the decimal form, and applying the general rule.

It has been already explained that the second root of an even power may be obtained by dividing the given number by the smallest possible divisors in succession, and taking half the number of those divisors as factors. The same principle will apply to any root. If the given quantity is not an even power, it may yet be found approximately. If we take the number 46656, we notice that as the last figure is an even number, it is divisible by 2, and by pursuing the same principle of operation we find six *twos* as factors, and afterwards six *threes;* and, as in the case of the second root, we take *each* factor *half* the number of times it occurs, so in the case of the third root, we take *each factor one-third* the number of times it occurs.

The same principle on which the extraction of the second and third depends may be applied to any root, the line of figures being divided into periods, consisting of as many figures as there are units in the index; for the fourth root, periods of four figures each; for the fifth, five, &c., &c. We may remark, however, that these modes are now superseded by the grand discovery of Logarithmic Computation. (See Logarithms.

# PROGRESSION.

A *series* is a succession of quantities increasing or decreasing by a *Common Difference*, or a *Common Ratio*.

*Progression by a Common Difference* forms a series by the addition or subtraction of the same quantity. Thus 3, 7, 11, 15, 19, 23 forms a series increasing by the constant quantity 4, and 28, 21, 14, 7, forms a series decreasing by the constant quantity 7. Such a progression is also called an equidifferent series.*

*Progression by a Common Ratio* forms a series increasing or decreasing by multiplying or dividing by the same quantity. Thus, 3, 9, 27, 81, 243, is a series increasing by a constant multiplier 3, and 64, 32, 16, 8, 4, 2, is a series decreasing by a constant divisor 2.

The quantities forming such a progression are also called *Continual Proportionals*,* because the ratio of 3 to 9 is the same as the ratio of 9 to 27, &c., &c. From this it is plain that in a progression by ratio, each term is a mean proportional between the two adjacent ones, and also between any two that are equally distant from it.

The first and last terms are called the *Extremes*, and all between them the *Means*.

## PROGRESSION BY A COMMON DIFFERENCE.

In a series increasing or decreasing by a common difference, the sum of the extremes is always equal to the sum of any two that are equally distant from them. Thus, in the first example $3+23=7+19=11+15=26$, and in the second $28+7=21+14=35$.

If the number of terms be odd, the sum of the extremes is equal to twice the middle term. Thus in the series 3, 7, 11, 15, 19, $3+19=2\times11=22$, and hence the middle term is half the sum of the extremes.

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* The names *Arithmetical Progression* and *Geometrical Progression* are often applied to quantities so related, but these terms are altogether inappropriate, as they would indicate that the one kind belonged solely to arithmetic, and the other solely to geometry, whereas, in reality, each belongs to both these branches of science.

In treating of progressions by difference or equidifferent series, there are five things to be considered, viz., the first term, the last term, the common difference, the number of terms, and the sum of the series. These are so related to each other that when any three of them are known we can find the other two.

Given the first term of a series, and the common difference, to find any other term.

Suppose it is required to find the seventh term of the series 2, 5, 8, &c. Here, as the first term is given, no addition is required to find it, and therefore six additions of the common difference will complete the series on to seven terms. In other words, the common difference is to be added to the first term as often as there are units in the number of terms diminished by 1. This gives 7—1=6, and 6×3=18, which added to the first term 2 gives 20 for the seventh term. If we had taken the series on the descending scale, 20, 17, 14, &c., we should have had to subtract the 18 from the first term 20 to find the seventh term 2. The term thus found is usually designated *the last term*, not because the series terminates there, for it does not, but simply because it is the last term *considered* in each question proposed. From these illustrations we derive the

### RULE (1.)

*Subtract* 1 *from the number of terms, and multiply the remainder by the common difference; then if the series be an increasing one, add the result to the first term, and if the series be a decreasing one, subtract it.*

### EXAMPLES.

To find the fifty-fourth term of the increasing series, the first term of which is 33¾, and the common difference 1¼. Here 54—1=53, and 53×1¼=66¼, and 66¼+33¾=100, the fifty-fourth term.

Given 64 the first term of a decreasing series, and 7 the common difference, to find the eighth term. Here 8—1=7, and 7×7=49, and 64—49=15, the eighth term.

### EXERCISES.

1. Find the eleventh term of the decreasing series, the first term of which is 248¾, and the common difference 3¼. Ans. 216¼.

2. The hundredth term of a decreasing series is 392⅖, and the common difference is 3⅗, what is the last term? Ans. 36.

3. What is the one-thousandth term of the series of the odd figures? Ans. 1999.

4. What is the five-hundredth term of the series of even digits? Ans. 1000.

5. What is the sixteenth term of the decreasing series, 100, 96, 92, &c.? Ans. 40.

---

To find the sum of any equidifferent series, when the number of terms, and either the middle term or the extremes, or two terms equidistant from them, are given.

We have seen already that in any such series the sum of the extremes is equal to the sum of any two terms that are equidistant from them, and when the number of terms is odd, to twice the middle term. Hence the middle term, or half the sum of any two terms equi-distant from the extremes, will be equal to half the sum of those extremes. Thus, in the series $2+7+12+17+22+27+32$, we have $\frac{2+32}{2}=\frac{7+27}{2}=17$, the middle term. It is plain, therefore, that if we take the middle term and half the sum of each equi-distant pair, the series will be equivalent to $17+17+17+17+17+17+17$, or 7 times 17, which will give 119, the same as would be found by adding together the original quantities. The same result would be arrived at when the number of terms is even, by taking half the sum of the extremes, or of any two terms that are equi-distant from them. From these explanations we deduce the

### RULE (2.)

*Multiply the middle term, or half the sum of the extremes, or of any two terms that are equidistant from them, by the number of terms.*

NOTE.—If the sum of the two terms be an odd number, it is generally more convenient to multiply by the number of terms before dividing by 2.

### EXAMPLES.

Given 23, the middle term of a series of 11 numbers, to find the sum. Here we have onl yto multiply 23 by 11, and we find at once the sum of the series to be 253.

Given 7 and 73, the extremes of an increasing series of 12 numbers, to find the sum. The sum of the extremes is 80, the half of which is 40, and $40\times12=480$, the sum required.

Two equidistant terms of a series, 35 and 70, are given in a series of 20 numbers, to find the sum of the series. In this case, we have $35+70=105$, and $105\times20=2100$, and $2100\div2=1050$, the sum required.

### EXERCISES.

1. Find the sum of the series, consisting of 200 terms, the first term being 1 and the last 200. Ans. 20100.
2. What is the sum of the series whose first term is 2, and twenty-first 62? Ans. 672.
3. What is the sum of 14 terms of the series, the first term of which is $\frac{1}{2}$ and the last 7? Ans. $52\frac{1}{2}$.
4. Find the sum to 10 terms of the decreasing series, the first term of which is 60 and the ninth 12. Ans. 360.
5. A canvasser was only able to earn $6 during the first month he was in the business, but at the end of two years was able to earn $98 a month; how much did he earn during the two years, supposing the increase to have been at a constant monthly rate? Ans. $1248.
6. If a man begins on the first of January by saving a cent on the first, two on the second, three on the third, four on the fourth, &c., &c., how much will he have saved at the end of the year, not counting the Sabbaths? Ans. $490.41.
7. How many strokes does a clock strike in 13 weeks? Ans. 14196.
8. If $8\frac{3}{4}$ is the fourth part of the middle term of a series of 99 numbers, what is the sum? Ans. 3465.
9. In a series of 17 numbers, 53 and 33 are equidistant from the extremes; what is the sum of the series? Ans. 731.
10. In a series of 13 numbers, 33 is the middle term; what is the sum? Ans. 429.

---

To find the number of terms when the extremes and common difference are given:

As in the rule (1), we found the *difference of the extremes* by multiplying by *one less* than the number of terms, and added the first term to the result, so now we reverse the operation and find the

### RULE (3.)

*Divide the difference of the extremes by the common difference and add* 1 *to the result.*

### EXAMPLE.

Given the extremes 7 and 109, and the common difference, 3, to find the number of terms.

In this case we have 109—7=102, and 102÷3=34, and 34+1=35, the number of terms.

### EXERCISES.

1. What is the number of terms when the extremes are 35 and 333, and the common difference 2? Ans. 150.

2. Two equidistant terms are 31 and 329, and the common difference 2; what is the number of terms? Ans. 150.

3. The first term of a series is 7, and the last 142, and the common difference $\frac{1}{4}$; what is the number of terms? Ans. 541.

4. The first and last terms of a series are $2\frac{1}{2}$ and $35\frac{1}{2}$, and the common difference $\frac{1}{3}$; what is the number of terms? Ans. 100.

5. The first term of a series is $\frac{1}{2}$ and last $12\frac{1}{2}$, and the common difference $\frac{1}{2}$; what is the number of terms? Ans. 25.

Given one extreme, the sum of the series and the number of terms, to find the other extreme.

This case may be solved by reversing Rule (2), for in it the data are the same, except that there the second extreme was given to find the sum, and now the sum is given, to find the second extreme. Therefore, as in that rule we *multiplied* the sum of the extremes by the number of terms and *halved* the product, so now we must *double* the sum of the series and *divide* by the number of terms to find the sum of the extremes, and from this subtract the given extreme, and the remainder will be the required extreme. This will illustrate the

### RULE (4.)

*Divide twice the sum of the series by the number of terms, and from the quotient subtract the given extreme, and the remainder will be the required extreme.*

### EXAMPLE.

Given 5050, the sum of a series, 1 the first term, and 100 the number of terms, to find the other extreme.

Twice the sum is 10100, which, divided by 100, gives 101, and 101—1=100, the number of terms.

### EXERCISES.

1. Given 50, the greater extreme of a decreasing series, 442, the sum, and 17 the number of terms, to find the other extreme. Ans. 2.

2. If 121268 be the sum of a series, 8 the less extreme, and 142 the number of terms; what is the greater extreme? Ans. 1700.

3. The sum of a series of 7 terms is 105, the greater extreme is 21, and the number of terms 7; what is the less extreme? Ans. 9.

4. The sum of a series is 576, the number of terms 24, and the greater extreme is 47; what is the less extreme? Ans. 1.

5. The sum of a series is $30204\frac{1}{2}$, the greater extreme 312, and the number of terms 193; what is the less extreme? Ans. 1.

Given the extremes and number of terms, to find the common difference.

As explained in the introduction to Rule (1), the number of common differences must be *one less* than the number of terms. It is obvious also, that the sum of these differences constitutes the difference between the extremes, and that therefore the sum of the differences is the same as 1 less than the number of terms. Therefore the difference of the extremes, divided by *the sum of the differences*, will give *one* difference, *i. e.*, the common difference. This gives us the

### RULE (5.)

*Subtract* 1 *from the number of terms, and divide the difference of the extremes by the remainder.*

### EXAMPLE.

If the extremes of an increasing series be 1 and 47, and the number of terms 24, we can find the common difference thus:— $47-1=46$, and $46\div23=2$, the common difference.

### EXERCISES.

1. If the extremes are 2 and 36, and the number of terms 18; what the common difference? Ans. 2.

2. What is the common difference if the extremes are 58 and 3, and the number of terms 12? Ans. 5.

3. In a decreasing series given 1000 the less extreme, and 1793 the greater, and 367 the number of terms, to find the common difference. Ans. $2\frac{1}{6}$.

4. If 6 and 60 are the extremes in a series of 10 numbers, what is the common difference? Ans. 6.

5. What is the common difference in a decreasing series of 42 terms, the extremes of which are 9 and 50? Ans. 1.

There are fifteen other cases, but they may all be deduced from the five here given.

We subjoin the Algebraic form as it is more satisfactory and complete, and also more easy to persons acquainted with the symbols of that science.

Let $a$ be the first term, $d$ the common difference, $n$ the number of terms, $s$ the sum of the series; the series will be represented by $a+(a+d)+(a+2d)+(a+3d)+$&c., to $\left\{a+(n-1)d.\right\}$ By inspecting this series it will be seen that the co-efficient of $d$ is always 1 less than the number of terms, for in the second term where $d$ first appears, its co-efficient is 1, in the third it is 2, and therefore since $n$ represents the number of terms, the co-efficient of $d$ in the last term is $n-1$, and that term therefore is $a+(n-1)d$. If the series were a decreasing one, that is, one formed by a succession of subtractions, the last term would be $a-(n-1)d$.

To find the sum of an equidifferent series.

We have here $s=a+(a+d)+(a+2d)+(a+3d)+$ &c......... $+\left\{a+(n-1)d.\right\}$ Since $a+(n-1)d$ is the last term, the last but one will be $a+(n-2)d$, and the last but two will be $a+(n-3)d$, &c., &c. But the sum of any number of quantities is the same in whatever order they may be written. Let us therefore write this series both as above, and also in reversed order:

$s=a+(a+d)+(a+2d)+(a+3d)+(a+4d)+$&c............. . $+a+(n-3)d+a+(n-2)d+a+(n-1)d.$

$s=a+(n-1)d+a+(n-2)d+a+(n-3)d+$&c.......... ...... $(a+4d)+(a+3d)+(a+2d)+(a+d)+a.$

Adding the two members of the second to those of the first, we obtain $2s=\left\{2a+(n-1)d\right\}+\left\{2a+(n-1)d\right\}+\left\{2a+(n-1)d\right\}+\left\{2a+(n-1)d\right\}+$&c., to $n$ terms.

In the last expression all the terms are the same, but there are $n$ terms, and therefore the whole will be

$$2s=n\left\{2a+(n-1)d\right\} \text{ and therefore}$$

$$s=\frac{n}{2}\left\{2a+(n-1)d\right\}\ldots\ldots(1.)$$

As we have used no single symbol to represent the last term, we must now show how it may be obtained from the other data. We have seen that the last term is $a+(n-1)d$, which we may denote by $l$, which will give us the formula

$$l=a+(n-1)d.$$

This formula, in the case of a decreasing series, will become

$$l=a-(n-1)d, \text{ and generally}$$
$$l=a\pm(n-1)d. \ (2.)$$

This formula is the same as Rule (1.)

We may modify (1) by (2) by substituting $l$ for $a+(n-1)d$. Thus:

$$s=\frac{n}{2}(a+l). \ (3.)$$

This is a convenient form when the last term is given. Using $l$ for the last term, we have five quantities to consider, viz., $a$, $l$, $d$, $n$, $s$, and, as already stated, any three of these being given. the other two can be found from (1) and (2.)

To find $d$ when $a$, $l$, $n$ are given :

By (2.)

$$l=a+(n-1)d$$
$$\therefore l-a=(n-1)d$$
$$\therefore \quad d=\frac{l-a}{n-1}. \ (4.)$$

This finds the common difference, when the extremes and number of terms are given, and corresponds to Rule (5.)

If $a$, $n$, $s$ are given, we have

By (1.)

$$s=\frac{n}{2}\left\{2a+(n-1)d\right\}$$
$$\therefore 2s=2an+n\ (n-1)d$$
$$\therefore dn\ (n-1)=2\ (s-an)$$
$$\therefore \quad d=\frac{2\ (s-an)}{n\ (n-1)}.$$

If $n$ is to be found from $a$, $d$, $s$, we have

by (1.) $$s=\frac{n}{2}\left\{2a+(n-1)d\right\}$$

$$\therefore 2s=2an=dn^2-dn$$
$$\therefore dn^2+n(2a-d)=2s$$

And by solving this quadratic equation, we find

$$n=\frac{d-2a\pm\sqrt{\left\{8ds+(2a-d)^2\right\}}}{2d}$$

EXAMPLES.

Given $a=6$, $d=4$, $n=20$, to find $s$.

First by (2) $l=a+(n-1)d$
$=6+(20-1)4$
$=82$

and hence by (3) $s=\frac{20}{2}(6+82)$
$=880.$

Given $a=3$, $l=300$, $n=33$, to find $d$.

By (4) $$d=\frac{l-a}{n-1}$$
$$=\frac{297}{32}=9\tfrac{9}{32}.^*$$

MIXED EXERCISES.

1. Given 70, the less extreme, 10 the common difference, and 44 the number of terms, to find the sum. Ans. 12540.

2. What is the less extreme when the greater is 579, the common difference 9, and the sum of the series 18915? Ans. 3.

3. What is the series when $s=143$, $d=2$, $n=11$?
Ans. 3, 5, 7, 9, 11, 13, 15, &c.

4. Given 4 and 49, the extremes, and 6 the number of terms, to find the series. Ans. 4, 13, 22, 31, 40, &c.

5. If 120 stones are laid in a straight line, on level ground, at a regular distance of a yard and a quarter, how far must a person travel to pick them all up one by one and carry them singly and place them in a heap at the distance of 6 yards from the first, and in the same line with the stones? Ans. 10 m. 7 fur., 27 rds., $1\frac{1}{2}$ yds.

6. Insert three means between the extremes 117 and 477.
Ans. 207, 297, and 387.

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* The other variations are left as exercises for the student.

7. A courier agreed to ride 100 miles on condition of being paid 1 cent for the first mile, 5 for the second, 9 for the third, and so on; how much did he get per mile on an average, how much for the last mile, and how much altogether?

Ans. $1.99 per mile, $3.97 for the last, and $199 for all.

8. A man performed a journey in 11 days on horseback—the first day he rode 45 miles, but, his horse getting lame, he was forced to slacken the pace at a certain rate per day, so that on the last day he made only five miles; what was the length of the journey, and at what rate did he slacken his speed?

Ans. The journey was 275 miles, and the slackening of speed 4 m. per day.

9. Find the series of which 72 is the sum, 17 the first term, and number of terms 6. Ans. 17, 15, 13, 11, 9, 7.

10. The Venetian clocks strike the hours for the whole day; how many strokes will one of these strike in a year. Ans. 109500.

11. An Eastern monarch being threatened with invasion, offered his commander-in-chief a reward equivalent to a mill for the first soldier he would enlist within a month, two for the second, three for the third, and so on; the officer enlisted 999,999 men; what was his reward equal to in our money. Ans. $499,999,500.

12. One hundred sailors were drawn up in line at a distance from each other of 2 yards, including the breadth of the body—the paymaster, seated a distance of two yards from the first, sent a lieutenant to hand to the first a sum of prize money, then back again to the second, and so on to each singly; how far had the lieutenant to walk?

Ans. 11 miles, 3 fur., 32 rods, 4 yds.

## PROGRESSIONS BY RATIO.

There are in progression by ratio, as in progression by difference, the same five quantities to be considered, except that in place of a common difference we have a common ratio; that is, instead of increase or decrease by addition and subtraction, we have increase or decrease by multiplication or division. If any three of these are known the other two can be found.

We have noticed already that if any quantity, 2, be multiplied by itself, the product, 4, is called the square, or second power of that quantity; if this be again multiplied by 2, the product, 8, is called the cube, or third power of that quantity; if this again be multiplied by 2, the product is called the fourth power of that quantity, and so on to the fifth, sixth, &c., powers. To show the short mode of indicating this, let us take $3\times3\times3\times3\times3=243$. For brevity this is written $3^5$, which means that there are 5 factors, all 3, to be continually multiplied together, and 5 is called the *index*, because it *indicates* the number of equal factors.

Given the first term and the common ratio to find the last proposed term.

Let it be required to find the sixth term of the increasing series, of which the first term is 3 and the ratio 4.

This may obviously be found by successive multiplications of the first term, 3, by the ratio, 4,—thus:—

$$\begin{array}{r} 3=\text{1st term.}\\ 3\times4=\ \ 12=\text{2nd term.}\\ 12\times4=\ \ 48=\text{3rd term.}\\ 48\times4=\ 192=\text{4th term.}\\ 192\times4=\ 768=\text{5th term.}\\ 768\times4=3072=\text{6th term.} \end{array}$$

The series, therefore, is 3, 12, 48, 192, 768, 3072. From this, it is plain, that as to find the last of 6 terms, only 5 multiplications of the first are required, in all cases the number of multiplications will be one less than the number of terms. But to multiply five times by 4 is the same as to multiply by 1024, the fifth power of 4, for $4\times4\times4\times4\times4=1024$, and $1024\times3=3072$.*

This gives us the general

### RULE (1.)

*Multiply the first term by that power of the given ratio which is a unit less than the number of terms.*

If the series be a decreasing one, *divide instead of multiplying.*

### EXAMPLES.

Given in a series of 12 numbers, the first term 4 and the ratio 2, to find the last term.

Since 11 is one less than the number of terms, we find the 11th power of 4, which is 2048, and this, multiplied by the first term, 4, gives 8192 for the twelfth term.

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* For the abbreviated mode see Involution.

Given the ninth term of a decreasing series, 39366, and the ratio 3, to find the first term.

As there are 9 terms, we take the 8th power of the ratio, 3, which we find to be 6561, and the first term 39366÷6561=6, the first term.

### EXERCISES.

1. What is the ninth term of the increasing series of which 5 is the first term and 4 the ratio? Ans. 327680.

2. What is the twelfth term of the increasing series, the first term of which is 1 and the ratio 3? Ans. 177147.

3. In a decreasing series the first term is 78732, the ratio 3, and the number of terms, 10; what is the last term? Ans. 4.

4. What is the 20th term of an increasing series, the first of which is 1.06, and also the ratio 1.06? Ans. 3.207135.

5. In a decreasing series the first term is 126.2477, the ratio 1.06; what is the last of 5 terms? Ans. 100.

Given the extremes and ratio, to find the sum of the series.

It is not easy to give a direct *proof* of this rule without the aid of Algebra, but the following illustration may be found satisfactory, and, in some sort, be accounted a proof.

Let it be required to find the sum of a series of continual proportions, of which the first term is 5, the ratio 3, and the number of terms 4.

Since 3 is the common ratio, we can easily find the terms of the series by a succession of multiplications. These are—

$$\begin{array}{r} 5+15+45+135, \text{ and the sum is } 200 \\ 15+45+135+405\phantom{, \text{ and the sum is } 200} \\ \hline 400\phantom{, \text{ and the sum is } 200} \end{array}$$

Let us now multiply each term by the ratio, 3, and, for convenience and clearness, place each term of the second line below that one of the first to which it is equal. Let us now subtract the upper from the lower line, and we find that there is no remainder, except the difference of the two extreme quantities, viz., 400. Now, it will be seen that this remainder is exactly double of the sum of the series, 200, and consequently 400 divided by 2, will give the sum 200. Also, 405 is the product of the last term by the ratio, and 400 is the difference between that product and the first term, and the divisor, 2, is a unit less than the ratio, 3. Hence the

### RULE (2.)

*Multiply the last term by the ratio, from this product subtract the first term, and divide the remainder by the ratio, diminished by unity.*

### EXAMPLE.

Given the first term of an increasing series, equal 4, the ratio 3, and the number of terms 6, to find the sum of the series.

By the former rule we find the last term to be 972. This, multiplied by the ratio, gives 2916, and the first extreme, 4, subtracted from this, leaves 2912, and this divided by 2, which is less than the ratio, gives 1456, the sum of the series.

### EXERCISES.

1. What is the sum of the series, of which the less extreme is 4, the ratio 3, and the number of terms 10? Ans. 118096.

2. What is the sum of the series, of which 1 is the less extreme, 2 the ratio, and 14 the number of terms? Ans. 16383.

3. What is the sum of the series, of which the greater extreme is 18.42015, the less 1, and the ratio 1.06? Ans. 308.755983.

4. A cattle dealer offered a farmer 10 sheep, at the rate of a mill for the first, a cent for the second, a dime for the third, a dollar for the fourth, &c., &c.; in what amount was he "taken in," supposing that each sheep was worth $11.111? Ans. $1111100.00.

5. What is the sum of six terms of the series, of which the greater extreme is $\frac{1}{2}$ and the ratio $\frac{3}{5}$? Ans. $\frac{3724}{3125}$, or $1\frac{599}{3125}$.

To find the ratio when the extremes and number of terms are given:

Let it be required to find the ratio when the extremes are 3 and 192, and the number of terms 7. This is effected by simply reversing the first rule, and therefore we divide 192 by 3 and find 64, and take the 6th root of 64, which is 2, the ratio. Hence the

### RULE (3.)

*Divide the greater extreme by the less, and find that root of the quotient, the index of which is one less than the number of terms.*

### EXAMPLE.

If the greater extreme is 1024, and the less 2, and the number of terms 10, we divide 1024 by 2, and find 512, and then by extracting the ninth root of 512, we find the ratio, 2.

### EXERCISES.

1. If the first yearly dividend of a joint stock company be $1, and the dividends increase yearly, so as to form a series of continual proportionals, what will all amount to in 12 years, the last dividend being $2048, and what will be the ratio of the increase?
Ans. ratio, 2; sum, $4095.

2. What is the ratio, in the series of which the less extreme is 3 and the greater 98034, and the number of terms 16. Ans. .196605.

3. What is the ratio of a series, the extremes of which are 4 and 324, and the number of terms 5? Ans. 3.

4. What is the ratio of a series, the number of terms being 7 and the extremes 3 and 12288? Ans. 4.

5. In a series of 23 terms the extremes are 2 and 8388608; what is the ratio? Ans. 2.

To insert any number of means between two given extremes:

*Find the ratio by Rule* (3), *and multiply the first extreme by this ratio, and the second will be obtained, and divide the last by the ratio, and the last but one will be obtained; continue this operation until the required term or terms be procured.*

NOTE.—A mean proportional is found by *taking the square root of the product of the extremes.*

### EXAMPLE.

Let it be required to insert between the extremes 5 and 1280 *three* terms, so that the numbers constituting the series shall be continual proportionals.

The number of terms here is 5, and hence, by Rule (3), we find the ratio to be 4, and 5 multiplied by this will give the second term, 20, and that again multiplied by 4 will give 80, the third, and that again multiplied by 4 will give the fourth term, 320, so that the full series is found to be 5, 20, 80, 320, 1280. The same result would be found by dividing the greater extreme by 4, and so on downwards, thus: 1280, 320, 80, 20, 5.

### EXERCISES.

1. Between 5 and 405 insert three terms, which shall make the whole a series of continual proportionals. Ans. 5, 15, 45, 135, 405.

2. Insert between $\frac{1}{9}$ and 27 four terms to form a series, and give the ratio. Ratio, 3; series, $\frac{1}{9}$, $\frac{1}{3}$, 1, 3, 9, 27.

3. What three numbers inserted between 7 and 4375 will form a series of continual proportionals? Ans. 35, 175, 875.

4. What is the mean proportional between 23 and 8464?
Ans. 441.2164+.

5. Find a mean proportional between $\frac{16}{27}$ and $\frac{4}{3}$. Ans. $\frac{8}{9}$.

### ALGEBRAIC FORM.

Let $a$ represent the first term, $l$ the last, $r$ the ratio, $n$ the number of terms, and $s$ the sum.

Then $s=a+ar+ar^2+ar^3+ar^4+\&c. \ldots\ldots ar^{n-2}+ar^{n-1}$.

Multiplying the whole equation by $r$, we obtain

$$rs=ar+ar^2+ar^3+ar^4+ar^5+\&c. \ldots\ldots ar^{n-1}+ar^n.$$

But $s=a+ar+ar^2+ar^3+ar^4+ar^5+\&c. \ldots\ldots ar^{n-1}$.

Subtracting, we obtain

$$rs-s=s(r-1)=ar^n-a, \text{ and therefore}$$

$$s=\frac{ar^n-a}{r-1} \ldots\ldots (1.)$$

But we found the last term of the series to be $ar^{n-1}$, calling this $l$, we have from (1.) $s=\frac{rl-a}{r-1} \ldots\ldots (2.)$

If $r$ is a fraction, $r^n$ and $ar^n$ decrease as $n$ increases, as already shown under the head of fractions, so that if $n$ become indefinitely great, $ar^n$ will become unassignably small, compared with any finite quantity, and may be reckoned as nothing. In this case (1) will become $s=\frac{-a}{r-1}=\frac{a}{1-r} \ldots\ldots (3.)$

By this formula we can find the sum of any infinite series so closely as to differ from the actual sum by an amount less than any assignable quantity. This is called the *limit*, an expression more strictly correct than *the sum*.

From the formula $s=\frac{rl-a}{r-1}$, any three of the quantities $a$, $r$, $l$, $s$ being given, the fourth can be found.

Let it be required to find the sum of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+$ &c., to infinity.

Here $a=1$ and $r=\frac{1}{2}$ $\therefore$ $s=1-\frac{1}{2}=\frac{1}{\frac{1}{2}}=1\times2=2$. Therefere, 2 is the number to which the sum of the series continually approaches, by the increase of the number of its terms, and is the limit from which it may be made to differ by a quantity less than any assignable quantity, and is also the limit beyond which it can never pass.

By adding the first two terms, we find $1+\frac{1}{2}=\frac{3}{2}=2-\frac{1}{2}=1\frac{1}{2}$.

By adding the first three terms, we find $\frac{3}{2}+\frac{1}{4}=\frac{7}{4}=2-\frac{1}{4}=1\frac{3}{4}$.

By adding the first four terms, we find $\frac{7}{4}+\frac{1}{8}=\frac{15}{8}=2-\frac{1}{8}=1\frac{7}{8}$.

By adding the first five terms, we find $\frac{15}{8}+\frac{1}{16}=\frac{31}{16}=2-\frac{1}{16}=1\frac{15}{16}$.

By adding the first six terms, we find $\frac{31}{16}+\frac{1}{32}=\frac{63}{32}=2-\frac{1}{32}=1\frac{31}{32}$.

It will be observed here that the difference from 2 is continually decreasing. The next term would differ from 2 by $\frac{1}{64}$, and the next by $\frac{1}{128}$, &c., &c. Thus, when the series is carried to infinity, 2 may be taken as the sum, because it differs from the actual sum by a quantity less than any assignable quantity.

### EXAMPLES.

To find the sum of the first twelve terms of the series $1+3+9+27+$&c.:

Here $a=1$, $r=3$,

And $s=\frac{rl-a}{r-1}=\frac{3\cdot3^{11}-1}{3-1}=\frac{3\times177147-1}{2}=265720$.

To find the sum of the series $1, -3, 9, -27$, &c., to twelve terms,

$s=\frac{-3\times-3^{11}-1}{-3-1}=\frac{-3\times-177147-1}{-4}=-132860$.

In the case of infinite series, if $a$ is sought, $s$ and $r$ being given, we have from (3) $a=s\ (1-r)$, and if $r$ is sought, $a$ and $s$ being given, we have $r=\frac{s-a}{s}$ or $1-\frac{a}{s}$.

### EXERCISES.

1. Find the sum of the series 2, 6, 18, 54, &c., to 8 terms.
Ans. 6560.

2. Find the sum of the infinite series $\frac{1}{3}-\frac{1}{6}+\frac{1}{12}-\frac{1}{24}$. Observe here $r=-\frac{1}{2}$.
Ans. $\frac{2}{9}$.

3. What is the sum of the series $1, \frac{1}{3}, \frac{1}{9}$, &c., to infinity?
Ans. $\frac{3}{2}$.

4. Find the sum of the infinite series $1-\frac{2}{3}+\frac{4}{9}-\frac{8}{27}+$&c.
Ans. 3.

5. What is the sum of nine terms of the series 5, 20, 80, &c.?
Ans. 436905.

6. Find the sum of $\sqrt{\frac{1}{2}}+\frac{1}{2}+\sqrt{\frac{1}{8}}+$&c., to infinity.
Ans. $\sqrt{\frac{1}{2}}-1$.

7. What is the limit to which the sum of the infinite series $\frac{3}{4}, \frac{1}{2}, \frac{1}{3}, \frac{2}{9}$, &c., continually approaches?
Ans. $\frac{9}{4}$.

8. What is the sum of ten terms of the series 4, 12, 36, &c.? Ans. 118096.

9. Insert three terms between 39 and 3159, so that the whole shall be a series of continual proportionals. Ans. 117, .351 and 1053.

10. Insert four terms between $\frac{1}{9}$ and 27, so that the whole shall form a series of continual proportionals. Ans. $\frac{1}{3}$, 1, 3, 9.

11. The sum of a series of continual proportionals is $10\frac{1}{8}$, the first term $3\frac{3}{8}$; what is the ratio? Ans. $\frac{2}{3}$.

12. The limit of an infinite series is 70, the ratio $\frac{3}{7}$; what is the first term? Ans. 40.

## ANNUITIES.

The word *Annuity* originally denoted a sum paid *annually*, and though such payments are often made half-yearly, quarterly, &c., still the term is applied, and quite properly, because the calculations are made for the year, at what time soever the disbursements may be made.

By the term *annuities certain* is indicated such as have a fixed time for their commencement and termination.

By the term *annuities contingent* is meant annuities, the commencement or termination of which depends on some contingent event, most commonly the death of some individual or individuals.

By the term annuity *in reversion* or *deferred*, is meant that the person entitled to it cannot enter on the enjoyment of it till after the lapse of some specified time, or the occurrence of some event, generally the death of some person or persons.

An annuity *in perpetuity* is one that "lasts for ever," and therefore is a species of hereditary property.

An annuity *forborne* is one the payments of which have not been made when due, but have been allowed to accumulate.

By the *amount* of an annuity is meant the sum that the principal and compound interest will amount to in a given time.

The *present worth* of an annuity is the sum to which it would amount, at compound interest, at the end of a given time, if forborne for that time.

Tables have been constructed showing the present and final values per unit for different periods, by which the value of any annuity may be found according to the following

### RULES.

To find either the amount or the present value of an annuity,—

*Multiply the value of the unit, as found in the tables, by the number denoting the annuity.*

If the annuity be in perpetuity,—

*Divide the annuity by the number denoting the interest of the unit for one year.*

If the annuity be in reversion,—

*Find the value of the unit up to the date of commencement, and also to the date of termination, and multiply their difference by the number denoting the annuity.*

To find the annuity, the time, rate and present worth being given.

*Divide the present worth by the worth of the unit.*

Tables are appended varying from 20 to 50 years.

### EXAMPLES.

To find what an annuity of $400 will amount to in 30 years, at 3½ per cent.

We find by the tables the amount of $1, for 30 years, to be $51.622677, which multiplied by 400 gives $20649.07 nearly.

To find the present worth of an annuity of $100 for 45 years, at 3 per cent.

By the table we find $24.518713, and this multiplied by 100 gives $2451.88.

To find the present worth of a property on lease for ever, yielding $600, at 3½ per cent.

The rate per unit for one year is .035, and 600 divided by this gives $17142.86.

To find the present worth of an annuity on a lease in reversion, to commence at the end of three years and to last for 5, at 3½ per cent.

By the table we find the rate per unit for 3 years to be $2.801637, and for 8 years, the time the lease expires, $6.873956; the difference is $4.072319, which, multiplied by 300, gives $1221.70.

Given $207.90, the present worth of an annuity continued for 4 years, at 3 per cent., to find the annuity.

By the tables, the value for $1 is $3.717098, and $207.90, divided by this, gives $55.93.

# TABLE,

### SHOWING THE AMOUNT OF AN ANNUITY OF ONE DOLLAR PER ANNUM, IMPROVED AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

| Years. | 3 per cent. | 3½ per cent. | 4 per cent. | 5 per cent. | 6 per cent. | 7 per cent. |
|---|---|---|---|---|---|---|
| 1 | 1.000 000 | 1.000 000 | 1.000 000 | 1.000 000 | 1.000 000 | 1.000 000 |
| 2 | 2.030 000 | 2.035 000 | 2.040 000 | 2.050 000 | 2.060 000 | 2.070 000 |
| 3 | 3.090 900 | 3.106 225 | 3.121 600 | 3.152 500 | 3.183 600 | 3.214 900 |
| 4 | 4.183 627 | 4.214 943 | 4.246 464 | 4.310 125 | 4.374 616 | 4.439 943 |
| 5 | 5.309 136 | 5.362 466 | 5.416 323 | 5.525 631 | 5.637 093 | 5.750 739 |
| 6 | 6.468 410 | 6.550 152 | 6.632 975 | 6.801 913 | 6.975 319 | 7.153 291 |
| 7 | 7.662 462 | 7.779 408 | 7.898 294 | 8.142 008 | 8.393 838 | 8.654 021 |
| 8 | 8.892 336 | 9.051 687 | 9.214 226 | 9.549 109 | 9.897 468 | 10.259 803 |
| 9 | 10.159 106 | 10.368 496 | 10.582 795 | 11.026 564 | 11.491 316 | 11.977 989 |
| 10 | 11.463 879 | 11.731 393 | 12.006 107 | 12.577 893 | 13.180 795 | 13.816 448 |
| 11 | 12.807 796 | 13.141 992 | 13.486 351 | 14.206 787 | 14.971 643 | 15.783 599 |
| 12 | 14.192 030 | 14.601 962 | 15.025 805 | 15.917 127 | 16.869 941 | 17.888 451 |
| 13 | 15.617 790 | 16.113 030 | 16.626 838 | 17.712 983 | 18.882 138 | 20.140 643 |
| 14 | 17.086 324 | 17.676 986 | 18.291 911 | 19.598 632 | 21,015 066 | 22.550 488 |
| 15 | 18.598 914 | 19.295 681 | 20.023 588 | 21.578 564 | 23.275 970 | 25.129 022 |
| 16 | 20.156 881 | 20.971 030 | 21.824 531 | 23.657 492 | 25.670 528 | 27.888 054 |
| 17 | 21.761 588 | 22.705 016 | 23.697 512 | 25.840 366 | 28.212 880 | 30.840 217 |
| 18 | 23.414 435 | 24.499 691 | 25.645 413 | 28.132 385 | 30.905 653 | 33.999 033 |
| 19 | 25.116 868 | 26.357 180 | 27.671 229 | 30.539 004 | 33.759 992 | 37.378 965 |
| 20 | 26.870 374 | 28.279 682 | 29.778 079 | 33.065 954 | 36.785 591 | 40.995 492 |
| 21 | 28.676 486 | 30.269 471 | 31.969 202 | 35.719 252 | 39.992 727 | 44.865 177 |
| 22 | 30.536 780 | 32.328 902 | 34.247 970 | 38.505 214 | 43.392 290 | 49.005 739 |
| 23 | 32.452 884 | 34.460 414 | 36.617 889 | 41.430 475 | 46.995 828 | 53.436 141 |
| 24 | 34.426 470 | 36,666 528 | 39,082 604 | 44.501 999 | 50.815 577 | 58.176 671 |
| 25 | 36.459 264 | 38.949 857 | 41.645 908 | 47.727 099 | 54.864 512 | 63.249 030 |
| 26 | 38.553 042 | 41.313 102 | 44.311 745 | 51.113 454 | 59.156 383 | 68.676 470 |
| 27 | 40.709 634 | 42.759 060 | 47.084 214 | 54.669 126 | 63.705 766 | 74.483 823 |
| 28 | 42.930 923 | 46.290 627 | 49.967 583 | 58.402 583 | 68.528 112 | 80.697 691 |
| 29 | 45.218 850 | 48.910 799 | 52.966 286 | 62.322 712 | 73.639 798 | 87.346 529 |
| 30 | 47.575 416 | 51.622 677 | 56.084 938 | 66.438 848 | 79.058 186 | 94.460 786 |
| 31 | 50.002 678 | 54.429 471 | 59.328 335 | 70.760 790 | 84.801 677 | 102.073 041 |
| 32 | 52.502 759 | 57.334 502 | 62.701 469 | 75.298 829 | 90.889 778 | 110.218 154 |
| 33 | 55.077 841 | 60.341 210 | 66.209 527 | 80.063 771 | 97.343 165 | 118.933 425 |
| 34 | 57.730 177 | 63.453 152 | 69.857 909 | 85.066 959 | 104.183 755 | 128.258 765 |
| 35 | 60.462 082 | 66.674 013 | 73.652 225 | 90.320 307 | 111.434 780 | 138.236 878 |
| 36 | 63.271 944 | 70.007 603 | 77.598 314 | 95.836 323 | 119.120 867 | 148.913 460 |
| 37 | 66.174 223 | 73.457 869 | 81.702 246 | 101.628 139 | 127.268 119 | 160.337 400 |
| 38 | 69.159 449 | 77.028 895 | 85.970 336 | 107.709 546 | 135.904 206 | 172.561 020 |
| 39 | 72.234 233 | 80.724 906 | 90.409 150 | 114.095 023 | 145.058 458 | 185.640 292 |
| 40 | 75.401 260 | 84.550 278 | 95.025 516 | 120.799 774 | 154.761 966 | 199.635 112 |
| 41 | 78.663 298 | 88.509 537 | 99.826 536 | 127.839 763 | 165.047 684 | 214.609 570 |
| 42 | 82.023 196 | 92.607 371 | 104.819 598 | 135.231 751 | 175.950 645 | 230.632 240 |
| 43 | 85.483 892 | 96.848 629 | 110.012 382 | 142.993 339 | 187.507 577 | 247.776 496 |
| 44 | 89.048 409 | 101.238 331 | 115.412 877 | 151.143 006 | 199.758 032 | 266.120 851 |
| 45 | 92.719 861 | 105.781 673 | 121.029 392 | 159.700 156 | 212.743 514 | 285.749 311 |
| 46 | 96.501 457 | 110.484 031 | 126.870 568 | 168.685 164 | 226.508 125 | 306.751 763 |
| 47 | 100.396 501 | 115.350 973 | 132.945 390 | 178.119 422 | 241.098 612 | 329.224 386 |
| 48 | 104.408 396 | 120.388 297 | 139.263 206 | 188.025 393 | 256.564 529 | 353.270 093 |
| 49 | 108.540 648 | 125.601 846 | 145.833 734 | 198.426 663 | 272.958 401 | 378.999 000 |
| 50 | 112.796 867 | 130.999 910 | 152.667 084 | 209.347 976 | 290.335 905 | 406.528 929 |

# TABLE,

SHOWING THE PRESENT WORTH OF AN ANNUITY OF ONE DOLLAR PER ANNUM, TO CONTINUE FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

| Years. | 3 per cent. | 3½ per cent | 4 per cent. | 5 per cent. | 6 per cent. | 7 per cent. | Years. |
|---|---|---|---|---|---|---|---|
| 1 | 0.970 874 | 0.966 184 | 0.961 538 | 0.952 381 | 0.943 396 | 0.934 579 | 1 |
| 2 | 1.913 470 | 1.899 694 | 1.886 095 | 1.859 410 | 1.833 393 | 1.808 017 | 2 |
| 3 | 2.828 611 | 2.801 637 | 2.775 091 | 2.723 248 | 2.673 012 | 2.624 314 | 3 |
| 4 | 3.717 098 | 3.673 079 | 3.629 895 | 3.545 951 | 3.465 106 | 3.387 209 | 4 |
| 5 | 4.579 707 | 4.515 052 | 4.451 822 | 4.329 477 | 4.212 364 | 4.100 195 | 5 |
| 6 | 5.417 191 | 5.328 553 | 5.242 137 | 5.075 692 | 4.917 324 | 4.766 537 | 6 |
| 7 | 6.230 283 | 6.114 544 | 6.002 055 | 5.786 373 | 5.582 381 | 5.389 286 | 7 |
| 8 | 7.019 692 | 6.873 956 | 6.732 745 | 6.463 213 | 6.209 744 | 5.971 295 | 8 |
| 9 | 7.786 109 | 7.607 687 | 7.435 332 | 7.107 822 | 6.801 692 | 6.515 228 | 9 |
| 10 | 8.530 203 | 8.316 605 | 8.110 896 | 7.721 735 | 7.360 087 | 7.023 577 | 10 |
| 11 | 9.252 624 | 9.001 551 | 8.760 477 | 8.306 414 | 7.886 875 | 7.498 669 | 11 |
| 12 | 9.954 004 | 9.663 334 | 9.385 074 | 8.863 252 | 8.383 844 | 7.942 671 | 12 |
| 13 | 10.634 955 | 10.302 738 | 9.985 648 | 9.393 573 | 8.852 683 | 8.357 635 | 13 |
| 14 | 11.296 073 | 10.920 520 | 10.563 123 | 9.898 641 | 9.294 984 | 8.745 452 | 14 |
| 15 | 11.937 935 | 11.517 411 | 11.118 387 | 10.379 658 | 9.712 249 | 9.107 898 | 15 |
| 16 | 12.561 102 | 12.094 117 | 11.652 296 | 10.837 770 | 10.105 895 | 9.446 632 | 16 |
| 17 | 13.166 118 | 12.651 321 | 12.165 669 | 11.274 066 | 10.477 260 | 9.763 206 | 17 |
| 18 | 13.753 513 | 13.189 682 | 12.659 297 | 11.689 587 | 10.827 603 | 10.059 070 | 18 |
| 19 | 14.323 799 | 13.709 837 | 13.133 939 | 12.085 321 | 11.158 116 | 10.335 578 | 19 |
| 20 | 14.877 475 | 14.212 403 | 13.590 326 | 12.462 210 | 11.469 421 | 10.593 997 | 20 |
| 21 | 15.415 024 | 14.697 974 | 14.029 160 | 12.821 153 | 11.764 077 | 10.835 527 | 21 |
| 22 | 15.936 917 | 15.167 125 | 14.451 115 | 13.163 003 | 12.041 582 | 11.061 241 | 22 |
| 23 | 16.443 608 | 15.620 410 | 14.856 842 | 13.488 574 | 12.303 379 | 11.272 187 | 23 |
| 24 | 16.935 542 | 16.058 368 | 15.246 963 | 13.798 642 | 12.550 358 | 11.469 334 | 24 |
| 25 | 17.413 148 | 16.481 515 | 15.622 080 | 14.093 945 | 12.783 356 | 11.653 583 | 25 |
| 26 | 17.876 842 | 16.890 352 | 15.982 769 | 14.275 185 | 13 003 166 | 11.825 779 | 26 |
| 27 | 18.327 031 | 17.285 365 | 16.329 586 | 14.643 034 | 13.210 534 | 11.986 709 | 27 |
| 28 | 18.764 108 | 17.667 019 | 16.663 063 | 14.898 127 | 13.406 164 | 12.137 111 | 28 |
| 29 | 19.188 455 | 18.035 767 | 16.983 715 | 15.141 074 | 13.590 721 | 12.277 674 | 29 |
| 30 | 19.600 441 | 18.392 045 | 17.292 033 | 15.372 451 | 13.764 831 | 12.409 041 | 30 |
| 31 | 20.000 428 | 18.736 276 | 17.588 494 | 15.592 811 | 13.929 086 | 12.531 814 | 31 |
| 32 | 20.338 766 | 19.068 865 | 17.873 552 | 15.802 677 | 14.084 043 | 12.646 555 | 32 |
| 33 | 20.765 792 | 19.390 208 | 18.147 646 | 16.002 549 | 14.230 230 | 12.753 790 | 33 |
| 34 | 21.131 837 | 19.700 684 | 18.411 198 | 16.192 204 | 14.368 141 | 12.854 009 | 34 |
| 35 | 21.487 220 | 20.000 661 | 18.664 613 | 16.374 194 | 14.498 246 | 12.947 672 | 35 |
| 36 | 21.832 252 | 20.290 494 | 18.908 282 | 16.546 852 | 14.620 987 | 13.035 208 | 36 |
| 37 | 22.167 235 | 20.570 525 | 19.142 579 | 16.711 287 | 14.736 780 | 13.117 017 | 37 |
| 38 | 22.492 462 | 20.841 087 | 19.367 864 | 16.867 893 | 14.846 019 | 13.193 473 | 38 |
| 39 | 22.808 215 | 21.102 500 | 19.584 485 | 17.017 041 | 14.949 075 | 13.264 928 | 39 |
| 40 | 23.114 772 | 21.355 072 | 19.792 774 | 17.159 086 | 15.046 297 | 13.331 709 | 40 |
| 41 | 23.412 400 | 21.599 104 | 19.993 052 | 17.294 368 | 15.138 016 | 13.394 120 | 41 |
| 42 | 23.701 359 | 21.834 883 | 20.185 627 | 17.423 208 | 15.224 543 | 13.452 449 | 42 |
| 43 | 23.981 902 | 22.062 689 | 20.370 795 | 17.545 912 | 15.306 173 | 13.506 962 | 43 |
| 44 | 24.254 274 | 22.282 791 | 20.548 841 | 17.662 773 | 15.383 182 | 13.557 908 | 44 |
| 45 | 24.518 713 | 22.495 450 | 20.720 040 | 17.774 070 | 15.455 832 | 13.605 522 | 45 |
| 46 | 24.775 449 | 22.700 918 | 20.884 654 | 17.880 067 | 15.524 370 | 13.650 020 | 46 |
| 47 | 25.024 708 | 22.899 438 | 21.042 936 | 17.981 016 | 15.589 028 | 13.691 608 | 47 |
| 48 | 25.266 707 | 23.091 244 | 21.195 131 | 18.077 158 | 15.650 027 | 13.730 474 | 48 |
| 49 | 25.501 657 | 23.276 564 | 21.341 472 | 18.168 722 | 15.707 572 | 13.766 799 | 49 |
| 50 | 25.729 764 | 23.455 618 | 21.482 185 | 18.255 925 | 15.761 861 | 13.800 746 | 50 |

## PARTNERSHIP SETTLEMENTS.

The circumstances under which partnerships are formed, the conditions on which they are made, and the causes that lead to their dissolution, are so varied that it is impossible to do more than give general directions deduced from the cases of most common occurrence. In forming a partnership, the great requisite is to have the terms of agreement expressed in the most clear and yet concise language possible, setting forth the sum invested by each, the duration of partnership, the share of gains or losses that fall to each, the sum that each may draw from time to time for private purposes, and any other circumstances arising out of the peculiarities of each case. The ease and satisfaction of making an equitable settlement, in case of dissolution, depends mainly on the clearness of the original agreement, and hence the necessity for its being distinct and explicit. Even when no dissolution is contemplated, settlements should be frequently made, in order that the parties may know how they stand to each other, and how the business is succeeding. This is of great importance in preserving unanimity and securing vigour and regularity in all the transactions of a mercantile house.

A dissolution may take place from various causes. If the partnership is formed for a term of years, the expiration of those years necessarily involves either a dissolution or a new agreement. The death of one of the partners may or may not cause dissolution, for the deceased partner may have, by his will, left his share in the business to his son, or some other relative or friend. In no case, however, can an equitable settlement be made, except by the mutual consent of the parties, or else in exact accordance with the terms of agreement. It is also necessary that when a dissolution takes place public notice should be given thereof, in order that all parties having dealings with the firm may be apprized of the change, and have their accounts arranged. For the same reason, it is necessary that some one of the partners, or some trustworthy accountant appointed by them, should be authorized to collect all debts due to the firm, and pay all accounts owing by it.

Partnerships are sometimes formed for a specific speculation, and therefore, of course, cease with the completion of the transaction, and a settlement must necessarily be then made. No matter for what time the partnership has been made, any partner is at liberty, at any time, to withdraw, on showing sufficient cause and giving proper notice. This is a just provision, for the circumstances of any partner may so change, from various causes, as to make it undesirable for him to remain in the business. If one partner is deputed to settle the accounts of the house, it would be reckoned fraudulent for any other partner to collect any moneys due, except that on receipt of them he hands them directly over to the person so deputed.

The resources and liabilities, with the net investment on commencing business, being given, to find the net gain or loss.

1. W. Smith and R. Evans are partners in business, and invested when commencing $1000 each. On dissolving the partnership, the assets and liabilities are as follows:—Merchandise valued at $1295; cash, $344; notes against sundry individuals, $790; W. H. Monroe owes on account $86.40; E. R. Carpenter owes $132.85, and C. F. Musgrove owes $67.50. They owe on sundry notes, as per bill book, $212.40; E. G. Conklin, on account, $29.45, and H. C. Wright, on account, $41.30. What has been the net gain?

SOLUTION.

| *Assets.* | | *Liabilities.* | |
|---|---|---|---|
| Merchandise on hand | $1295.00 | Bills Payable | $212.40 |
| Cash on hand | 344.00 | Amt. due E. G. Conklin. | 29.45 |
| Bills Receivable | 790.00 | Amt. due H. C. Wright. | 41.30 |
| Amt. due from W. H. Monroe | 86.40 | W. Smith's investment | 1000.00 |
| Amt. due from E. R. Carpenter | 132.85 | R. Evan's investment | 1000.00 |
| Amt. due from C. F. Musgrove | 67.50 | | $2283.15 |
| Total amount Assets | $2715.75 | | |
| " " Liabilities, | 2283.15 | | |
| Net gain | $432.60 | | |

RULE.

*Find the sum of the assets and liabilities; from the assets subtract the liabilities, (including the net amount invested) and the difference will be the net gain; or, if the liabilities be the larger, subtract the assets from the liabilities, and the difference will be the net loss.*

2. Harvey Miller and James Carey are partners in a dry goods business; Harvey Miller investing $1400, and James Carey $1250. When closing the books, they have on hand--cash, $1125.30; merchandise, as per inventory book, $1855.75; amount deposited in Bank of Toronto, $1200; amount invested in oil lands, $963; a site of land for building purposes, valued at $1600; Adam Dudgeon owes them, on account, $104.92; William Fleming owes $246.80; a note against Alfred Mills for $69.43, and a due bill for $30, drawn by James Laing. They owe W. S. Hope & Co., on account, $849.21; R. J. King & Co., $608.12, and on notes, $1326.14. What has been the net gain or loss? Ans. $1759.73 gain.

3. James Henning and Adam Manning have formed a co-partnership for the purpose of conducting a general dry goods and grocery business, each to share gains or losses equally.

At the end of one year they close the books, having $1280 worth of merchandise on hand; cash, $714.27; Royal Bank stock, $500; deposited in Royal Bank, $320.60; store and fixtures valued at $3100; amount due on notes and book accounts, $3471.49. The firm owes on notes $3400, and on open accounts $747.10.

James Henning invested $1200, and Adam Manning, $1000; what is each partner's interest in the business at closing?

Ans. James Henning's interest, $2719.63; Adam Manning's interest, $2519.63.

NOTE.—Where the interest of each partner at closing is required, the gain or loss is first found, as in former examples, then the share of gain or loss is added to or subtracted from each partner's investment, and the sum, or difference, is the interest of each partner. If a partner has withdrawn anything from the business, the amount thus withdrawn must be deducted from the sum of his investment, *plus* his share of the gain, or *minus* his share of the loss, and the remainder will be his net capital or interest.

4. F. A. Clarke, W. H. Marsden, and J. M. Musgrove, are conducting business in partnership; F. A. Clarke is to be $\frac{1}{2}$ gain or loss, W. H. Marsden and J. M. Musgrove, each $\frac{1}{4}$.

On dissolving the partnership, they have cash on hand $712.90; merchandise, as per Inventory Book, $4360; bills receivable, as per Bill Book, $1450.75; amount deposited in Bank of Montreal, $3475; merchandise shipped to Montreal, to be sold on own account and risk, valued at $995; debts due from individuals on book account, $2644.67. They owe on notes $3760, and to Manning & Munson, $1312.60.

F. A. Clarke invested $5750, and has drawn out $875; W. H. Marsden invested $2500, and has drawn out $500; J. M. Musgrove invested $3000, and has drawn out $750. What has been the net gain or loss, and what is each partner's interest in the business?

Ans. Net loss, $559.28; F. A. Clarke's interest, $4595.36; W. H. Marsden's interest, $1860.18; J. M. Musgrove's interest, $2110.18.

NOTE.—In this and succeeding examples, net interest is to be allowed on investment, or charged on amounts withdrawn, unless so specified.

5. A, B, and C are partners. The gains and losses are to be shared as follows: A, $\frac{5}{12}$; B, $\frac{3}{12}$; and C, $\frac{4}{12}$. A invested $3000, and has withdrawn $2,500, with the consent of B and C, upon which no interest is to be charged; B invested $2700, and has withdrawn $1150; C invested $2500, and has withdrawn $420. After doing business 14 months, C retires. Their assets consist of bills receivable, $2937.20; merchandise, $1970; cash, $1243.80; 50 shares of the Canada Permanent Building and Savings' Society Stock, the par value of which is $50 per share; cash deposited in the Ontario Bank, $1850; store and furniture, $3130; amount due from W. Smith, $360.80; G. S. Brown, $246.40; and E. R. Carpenter, $97.12. Their liabilities are as follows: amount due Samuel Harris, $1675; unpaid on store and furniture, $935; and notes unredeemed, $3388.76. C, in retiring, agrees to allow the firm 10 per cent. advance on the Savings' Bank stock. What is the amount due C, and what is A's, and what is B's interest in the business?

Ans. Due C, $815.52; A's interest, $2356.90; B's interest, $2664.14.

6. E, F, G and H are partners in business, each to share $\frac{1}{4}$ of profits and losses. The business is carried on for one year, when E and F purchase from G and H their interest in the business, allowing each $100 for his good will. Upon examination, their resources are found to be as follows: cash deposited in Quebec Bank, $3645; cash on hand, $1422; bills receivable, $1685; bonds and mortgages, $2746, upon which there is interest due $106; Royal Canadian Bank stock, $1000; Quebec Bank stock, $500; store and fixtures, $3500; house and lot, $1800; span of horses, carriages, harness, &c., $495; outstanding book debts due the firm, $4780. Their liabilities are: notes payable, $2345, upon which there is interest due $57; due on book debts, $1560. E invested $5000; F, $4500;

G, $4000; and H, $3000. E has drawn from the business $1200, upon which he owes interest $32; F has drawn $1000—owes interest $24.50; G has drawn $950—owes interest $12; and H has drawn nothing. In the settlement a discount of 10 per cent., for bad debts, is allowed, on the book debts due the firm and on the bills receivable. G takes the Royal Bank stock, allowing on the same a premium of 5 per cent.; and H takes the Quebec Bank stock, at a premium of 8 per cent; E and F take the assets and assumes the liabilities, as above stated. What has been the net gain or loss, the balances due G and H, and what are E and F each worth after the settlement?

Ans. Due G, $3057.75; due H, $3529.75; E's net capital, $4637.75; F's net capital, $4345.25.

7. H. C. Wright, W. S. Samuels, and E. P. Hall, are doing business together—H. C. W. to have ½ gain or loss; W. S. S. and E. P. H. each ¼. After doing business one year, W. S. S. and E. P. H. retire from the firm. On closing the books and taking stock, the following is found to be the result: merchandise on hand, $3216.50; cash deposited in Quebec Bank, $1627.35; cash in till, $134.16; bills receivable, $940.60; G. Brown owes, on account, $112.40; Thos. A. Bryce owes $94.12; W. McKee owes $143.95; J. Anderson owes $54.20; R. H. Hill owes $43.60; and S. Graham owes $260.13. They owe on notes not redeemed $1864; H. T. Collins, on account, $124.45; and W. F. Curtis, $79.40. H. C. Wright invested $3200, and has drawn from the business $350. W. S. Samuels invested $2455, and has drawn $140; E. P. Hall invested $2100, and has drawn $2000. A discount of 10 per cent. is to be allowed on the bills receivable and book accounts due the firm, for bad debts. H. C. Wright takes the assets and assumes the liabilities, as above stated. What has been the net gain or loss, and what does H. C. Wright pay W. S. Samuels and E. P. Hall on retiring?

Ans. Net loss, $970.74; H. C. W. pays W. S. S., $2072.31½; and E. P. Hall pays H. C. W. $142.68½.

8. T. P. Wolfe, J. P. Towler and E. R. Carpenter have been doing business in partnership, sharing the gains and losses equally. After dissolution and settlement of all their liabilities they make a division of the remaining effects without regard to the proper proportion each should take. The following is the result according to their ledger:—T. P. Wolfe invested $3495, and has drawn $2941;

J. P. Towler invested $2900, and has drawn $2200; E. R. Carpenter invested $3150, and has drawn $3000. How will the partners settle with each other?

Ans. E. R. Carpenter pays T. P. Wolfe $86, and J. P. Towler $232.

9. I, J, K, L and M have entered into co-partnership, agreeing to share the gains and losses in the following proportion:—I, $\frac{4}{15}$; J, $\frac{3}{15}$; K, $\frac{2}{15}$; L, $\frac{5}{15}$; and M, $\frac{1}{15}$. When dissolving the partnership the resources consisted of cash, $4700; merchandise, $9855; notes on hand, $7680; debentures of the city of Toronto, valued at $6780, on which there is interest due, $123; horses, waggons, &c., $1280; Montreal bank stock, $5000; Ontario bank stock, $5000; mortgages and bonds, $3600; interest due on mortgages, $345.80; store and fixtures, $8000; amount due from W. P. Campbell & Co., $2418; due from R. B. Smith, $712.60; due from J. W. Jones, $1000. The liabilities are:—Mortgage on store and fixtures, $5000; interest due on the same, $212.25; due the estate of R. M. Evans, $14675; notes and acceptances, $11940, on which interest is due, $85; sundry other book debts, $7500; I invested $7800, interest on his investment to date of dissolution, $702; J invested $6400, interest on investment, $576; K invested $6100, interest on investment, $549; L invested $5800, interest on investment, $522; M invested $5000, interest on investment, $450. I has withdrawn from the firm at different times, $2425, upon which the interest calculated to time of dissolution is $183.40; J has drawn $2960, interest, $267.85; K has drawn $1850, interest $87.30; L has drawn $3000, interest, $460; M has drawn $895, interest, $63.45. What is the net gain or loss of each partner, and what is the net capital of each partner?

Ans. I's net loss, $1233.29; I's net capital, $4660.31. J's net loss, $924.97; J's net capital, $2823.18. K's net loss, $616.65; K's net capital, $4095.05. L's net loss, $1541.62; L's net capital, $1320.38. M's net loss, $308.32; M's net capital, $4183.23.

10. A, B, C and D are partners. At the time of dissolution, and after the liabilities are all cancelled, they make a division of the effects, and upon examination of their ledger it shows the following result:—A has drawn from the business $3465, and invested on commencement of business, $4240; B has drawn $4595, and invested $3800; C has drawn $5000, and invested $3200; D has drawn $2200, and invested $2800. The profit or loss was to be divided in

proportion to the original investment. What has been each partner's gain or loss, and how do the partners settle with each other?

Ans. A's net gain, $368.43; B's net gain, $330.20; C's net gain, $278.06; D's net gain, $243.31. B has to pay in $464.80; C has to pay in $1521.94. A receives $1143.43; D receives $843.31.

11. Three mechanics, A. W. Smith, James Walker and P. Ranton, are equal partners in their business, with the understanding that each is to be charged $1.25 per day for lost time. At the close of their business, in the settlement it was found that A. W. Smith had lost 14 days, James Walker 21 days, and P. Ranton 30 days. How shall the partners properly adjust the matter between them?

Ans. P. Ranton pays A. W. Smith, $9.58$\frac{1}{3}$, and James Walker, 83$\frac{1}{3}$ cents.

12. There are 5 mechanics on a certain piece of work in the following proportions:—A is $\frac{3}{20}$; B, $\frac{2}{20}$; C, $\frac{4}{20}$; D, $\frac{6}{20}$, and E, $\frac{5}{20}$. A is to pay $1.25 per day for all lost time; B, $1; C, $1.50; D, $1.75, and E, $1.62$\frac{1}{2}$. At settlement it is found that A has lost 24; B, 19; C, 34; D, 12; and E, 45 days. They receive in payment for their joint work, $2500. What is each partner's share of this amount according to the above regulations?

Ans. A's share, $374.12; B's, $250.41; C's, $487.83; D's, $787.24; E's, $600.40.

13. A. B. Smith and T. C. Musgrove commenced business in partnership January 1st, 1864. A. B. Smith invested, on commencement, $9000; May 1st, $2400; June 1st, he drew out $1800; September 1st, $2000, and October 1st, he invested $800 more. T. C. Musgrove invested on commencing, $3000; March 1st, he drew out $1600; May 1st, $1200; June 1st, he invested $1500 more, and October 1st, $8000 more. At the time of settlement, on the 31st December, 1864, their merchandise account was—*Dr.* $32000; *Cr.* $27000; balance of merchandise on hand, as per inventory, $10500; cash on hand, $4900; bills receivable, $12400; R. Draper owes on account, $2450. They owe on their notes, $1890, and G. Roe on account, $840. Their profit and loss account is, *Dr.* $866; *Cr.* $1520. Expense account is, *Dr.* $2420. Commission account is, *Cr.* $2760. Interest account is *Dr.* $480; *Cr.* $950. The gain or loss is to be divided in proportion to each partner's capital, and in proportion to the time it was invested. Required each partner's share of the gain or loss, the net balance

due each, and a ledger specification exhibiting the closing of all the accounts, and the balance sheet.

Ans. A. B. S.'s net gain, $6671.73; his net balance, $15071.73.
T. C. M.'s net gain, $2748.27; his net balance, $12448.27.

## PROPERTIES OF NUMBERS.

The term *Integer*, or *Whole Number*, is used in contradistinction to the term *Fraction*. All numbers expressed by the natural series 1, 2, 3...10...20...100, &c., are called integers, so that 3 and 4 are integers, but $\frac{3}{4}$ is a fraction.

All numbers in the natural series 1, 2 3, &c., that can be resolved into factors, are called *Composite*, while those that cannot be so resolved are called *Prime*. Since 4=2×2, it is called composite, and so 6, 8, 9, 10, &c., but 1, 2, 3, 5, 7, 11, &c., are called prime because they cannot be resolved into factors. Thus, 11 can only be resolved into 11×1, or 1×11, and these are not factors in the strict meaning of the word.

A *Prime Factor* is a prime number, which is a factor of a composite number. The factors of 10 are 2 and 5, both prime numbers.

A composite number may have composite factors, as 36, which has 4 and 9 as factors, and both of these are composite.

When any number will divide two or more others, it is called a *Common Factor*. Thus, 3 is called a common factor of 6, 9, 12, 15, &c.

Numbers that have no common factor, as 4, 5, 9, are said to be *prime to each other*.

To resolve a composite number into its prime factors, *divide it by the least possible factor that it contains, and repeat the process till a prime number is obtained.*

### EXAMPLES.

2)96
2)48
2)24
2)12
2) 6
3

so that the prime factors of 96 are 2×2×2×2×2×3.

Also, because $5\times7\times11=385$, we see that 5, 7 and 11 are the prime factors of 385.

### EXERCISES.

1. What are the prime factors of 2310? Ans. 2, 3, 5, 7, 11.
2. What are the prime factors of 1764? Ans. 2, 2, 3, 3, 7, 7.
3. What are the prime factors of 180642? Ans. 2, 3, 7, 11, 17, 23.
4. What are the prime factors of 95? Ans. 5, 19.
5. What are the prime factors of 51? Ans. 3, 17.
6. What are the prime factors of 99? Ans. 3, 3, 11.
7. What are the prime factors of 651? Ans. 3, 7, 31.
8. What are the prime factors of 362880? Ans. 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 5, 7.
9. What factors are common to 84, 105, and 147? Ans. 3, 7.
10. What are the prime factors of 308? Ans. 4, 7, 11.

Whether a number is prime or composite can only be found by trial.

The only even prime number is 2; for 4, 6, 8, 10, &c., are all multiples of 2.

The only prime number ending in the digit 5 is 5 units, and all other numbers ending in either 5 or 0 are multiples of 5.

### ADDITIONAL EXERCISES.

11. Is 101 prime or composite? Ans. Prime.
12. Is 198 prime or composite? Ans. It has the factors 2, 3, 3, 11.
13. Is 171 prime or composite? Ans. It has the factors 3, 3, 19.
14. Is 473 prime or composite? Ans. Prime.
15. Is 477 prime or composite? Ans. Composite.
16. Is 549353259 prime or composite? Ans. Composite.
17. Is 674041 prime or composite? Ans. Composite.
18. Is 199 prime or composite? Ans. Prime.
19. What are the prime factors of 210? Ans. 5, 6, 7.
20. What are the prime factors of 51051? Ans. 3, 7, 11, 13, 17.

NOTE.—We have thought it sufficient under this head to give only the leading and most useful principles.

# LOGARITHMS.

A logarithm is the index of the power of any number which that index denotes.

Thus $9=3^2$, where the index 2 is the logarithm of 9. Taking 3 as a common root, we have

$$\begin{array}{rl} 3=3^1 & 1=3^0 \\ 9=3^2 & \frac{1}{3}=3^{-1} \\ 27=3^3 & \frac{1}{9}=3^{-2} \\ 81=3^4 & \frac{1}{27}=3^{-3} \end{array}$$

These indices may, however, be the logarithms of any root. Thus:

$$\begin{array}{rl} 4=4^1 & 1=4^0 \\ 16=4^2 & \frac{1}{4}=4^{-1} \\ 64=4^3 & \frac{1}{16}=4^{-2} \end{array}$$

Also, $8=4^{\frac{3}{2}}$, which means that 8 is the second root of the third power of 4. The third power of 4, or $4\times4\times4=64$, and 8 is the second root of 64, for $8\times8=64$. Again, since $8=2^3$, and $16=2^4$, it follows that the index of any number between 8 and 16, when 2 is the root, will be 3, *plus* some fraction, that is, the index will be some number greater than 3, and less than 4.

If 4 be the root, then, as above, $8=4^{\frac{3}{2}}$, where the index is $\frac{3}{2}$, or 1.5, greater than 1, but less than 2. In this case, the index expressed decimally is a terminator, but even when the decimal does not terminate, it may be found approximately with sufficient accuracy. From this it is obvious that logarithms may be calculated for all numbers. The root from which logarithms are calculated is called the *Base*. The bases we have taken for illustration are 2, 3, and 4, but any number may be assumed as base. The base of the original system, invented by Baron Napier, and hence called Napierian logarithms, was 2.718+, but the base that has been found most convenient is 10, as it falls in so exactly with the decimal notation. These, by way of distinction, are called *Common Logarithms.* The common logarithm, then, of a number, *is the index of the power of 10, which is equal to that number.*

Hence starting from the base 10, we have the following ascending and descending scales:

| Power. | No. | Log. | Power. | No. | Log. |
|---|---|---|---|---|---|
| $10^0=$ | 1 | 0 | $10^{-1}=$ | $\frac{1}{10}$ | .1 |
| $10^1=$ | 10 | 1 | $10^{-2}=$ | $\frac{1}{100}$ | .01 |
| $10^2=$ | 100 | 2 | $10^{3}$ $=$ | $\frac{1}{1000}$ | .001 |
| $10^3=$ | 1000 | 3 | $10^{-4}=$ | $\frac{1}{10000}$ | .0001 |
| $10^4=$ | 10000 | 4 | | | |

Now we must observe that, since it is proved by Algebra that any quantity with an index zero is always equal to unity, the Logarithm of 1 is 0, for $x^0=1$, $a^0=1$, $m^0=1$, and $10^0=1$, as exhibited in the above scheme. Again, since log. 1=0, and log. 10=1, it follows that the logarithms of all the numbers between 1 and 10 will be less than 1, but above zero, and also that the log. of any base will be 1. Also, since log. 10=1, and log. 100=2, it follows that the logarithms of all numbers between 10 and 100 will be greater than 1, but less than 2, and that the logarithms of all numbers between 100 and 1000 will be between 2 and 3, and so on.

Logarithms are always expressed in the decimal form.

The integral part is called the *Characteristic.*

The decimal part is called the *Mantissa.*

It is not usual in constructing tables to notice the characteristic, because, as the above explanations will show, it will always be denoted by *as many units as there are digits in the given number, less one.* Thus, the characteristic of 24796 will be 4, and of 879 it will be 2, and therefore it can always be found by counting the digits. It is also obvious that the characteristics of numbers less than unity must be negative, for log. $\frac{1}{10}$=log. $10^{-1}=-1$, and log. $\frac{1}{100}$=log. $10^{-2}$ =log. .01=—2. Since the characteristic is always 1 less than the number of digits, we have the same characteristic for 100 as for 199, but for 100 we have simply 2, while for 199 we have 2 *plus* some fraction, and since for 4.297, where we have only one digit, the characteristic is zero, ONE less than the number of digits, it follows that for .297, where there is no whole number, the characteristic must be —1 *plus* some fraction. Hence the mantissa is always positive, and is the same for decimals as for whole numbers. This fraction is .472756, and therefore we have $-1+.472756$, which, for convenience is written $\overline{1}.472756$, the negative sign being written *above* the characteristic, an artifice which is extremely convenient in the arranging of tables, and also in operations.

Since $100\times100=10000$, and $100=10^2$, it follows that $10^2\times 10^2=10000=10^4$. Now the last index 4 is the sum of the two preceding ones, $2+2=4$. Again, $10^2\times10^3=100\times1000=100000 =10^5$. Hence we see that quantities can be multiplied by adding their indices, and therefore by adding their logarithms. In the same manner it may be shown that one quantity may be divided by another, by subtracting the logarithm of the divisor from the logarithm of the dividend. Also, that a quantity can be raised to any power by multiplying its logarithm by the index of the power, and that any root may be extracted by dividing the logarithm of the number by the index.

## EXPLANATION OF THE TABLES.

In the first column are marked the natural numbers for three digits, and along the top of the page the ten digits. Opposite the three digits, on the margin and under the zero column, will be found the mantissa for the number represented by those digits, or those digits with a cipher annexed, for the annexing of a cipher makes no difference in the mantissa, but adds a unit to the index. If the given number consists of four digits, the required logarithm will be found under the fourth figure on the top of the page, and opposite the other three on the margin. The numbers in the column marked D denote the closely approximate difference between any two contiguous numbers, or the increment of the logarithms as the natural numbers increase by the successive additions of a unit.

To find by the tables the logarithm of any number.

If the number does not exceed 100 look for it at the beginning of the table in the vertical column marked N. or No., and both characteristic and mantissa will be found opposite to it in the column usually marked log. or L.

Thus the logarithm of 98 will be found by simple inspection to be 1.991126.

If the number consist of four digits, find the first three in the vertical column marked N. or No., and from opposite that, in the horizontal column, and under the fourth along the top, take the mantissa, and before it write the characteristic, found by taking a unit for every digit in the given number, less one.

### EXAMPLES.

To find the logarithm of 4389. In the vertical column marked No. will be found 438, and opposite this, in the column headed 9, will be found the mantissa .642366, before which must be placed the characteristic 3, a unit less than the number of digits, giving 3.642366.

It is usual to record only four digits in tables of logarithms, but if it be required to find the logarithm of a number expressed by more than four digits, it can readily be found in the following manner. If it is required to find the logarithm of 43896.24. We first find, as in the last example, the mantissa of 4389, which is .642366, then we find the number opposite to it, in the column marked diff. or D., which in this case is 99, the difference between this and the succeeding logarithm. We then multiply the remaining figures of the given number by 99, which gives 617.76, and as .76 approaches so nearly to another unit, we write 618, and add this to the logarithm for 4389, and place the characteristic before the result thus:

$$\begin{array}{r} .642366 \\ 618 \\ \hline 4.642984 \end{array}$$

### EXERCISES.

1. Find the logarithm of 41. Ans. 1.612784.
2. Find the logarithm of 75. Ans. 1.875061.
3. Find the logarithm of 100. Ans. 1.000000.
4. Find the logarithm of 1. Ans. 0.000000.
5. Find the logarithm of 117. Ans. 2.068186.
6. Find the logarithm of 786111. Ans. 5.895484.
7. Find the logarithm of 7861.11. Ans. 3.895484.
8. Find the logarithm of 46782.79. Ans. 4.670086.
9. Find the logarithm of .9876. Ans. $\bar{1}$.994581.
10. Find the logarithm of .08734. Ans. $\bar{2}$.941213.

To find the natural number corresponding to any logarithm:

Let it be required to find the natural number corresponding to 5.890197. By inspecting the table, we find in the first column of logarithms the figures .890, but the succeeding figures, 421, are too great; but by running the eye along the horizontal column above this, we find the exact logarithm, and opposite to it, under No., we find 776, and above it, at the top of the column, we find 6, so that 7766 are the first four figures of the number; but the index, 5, shows that the number must consist of six digits, and therefore we take the next lower logarithm and subtract it from the given one, getting a remainder of 56. We now annex two ciphers, and divide by the tabular difference, 99, which gives the other two figures, 5 and 6, and the whole, 776656, and if we annex more ciphers to the remainder we obtain decimals. This operation will be, when expressed as a

### RULE.

*Find the next less logarithm, and the three figures in the column marked No. will be the three first figures, and the one at the top of the page, above the logarithm, will be the fourth figure.*

*Subtract this logarithm from the given one, and divide the remainder with ciphers by the annexed tabular difference, until one digit more than the number of units in the index be obtained. This will give the whole number, and all figures following that will be decimals.*

NOTE.—The logarithms of all numbers consisting of not more than four figures will be found in the tables.

### EXERCISES.

11. What is the natural number corresponding to the logarithm 2.371068? Ans. 235.

12. What is the natural number corresponding to the logarithm 1.776992? Ans. 59.84.

13. What is the natural number corresponding to the logarithm 5.895484? Ans. 786111.

14. Multiply by logarithms 104777.7 by .045. The product is 8765.

15. Divide .00987 by .082. Ans. .12036.

16. Find the second power of 818. Ans. 663124.

17. Find the fifth power of 19. Ans. 2276099.

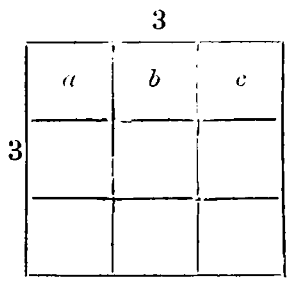
18. Find the second root of 7569. Ans. 87.5.

19. Find the third root of 92.61. Ans. 21.

20. Find the fifth root of 759375. Ans. 15.

# MENSURATION.[*]

We have already observed that no solid body can have more than *three* dimensions, viz.: length, breadth, and thickness, or depth, and that a line is length, or breadth, or depth, or it is a line or unit repeated a certain number of times. A foot in length is a line measured by repeating the linear unit called an inch 12 times, and a yard is the linear unit called a foot, repeated 3 times, and so on. Thus, 1 ft. 1 ft. 1 ft.=3 feet. But there may be two such lines drawn at right angles to each other, and each three feet long, and if the figure be completed it is a square. Also, if lines be drawn, each an inch apart from the other, and parallel to the two first-mentioned lines, we shall find that there are three small figures, each an inch square, between the two upper horizontal lines, and 3 of the same extent between the two intermediate lines, and 3 between the two lower lines, making 9 in all, or 3 times 3. This is the origin of the expression that 9 is the square of 3. Let the learner mark the difference between 3 square feet and 3 feet square. *a*, *b* and *c* are 3 *square feet*, but the whole figure is 3 *feet square*, and therefore three feet square must be equal to 9 square feet. Three feet square, then, is a square, each of whose sides measures 3 *linear feet;* but 3 square feet would denote 3 squares, each side of each measuring one linear foot. The space thus inclosed is called the *area*.



This is the principle on which surfaces are measured.

### PROBLEM I.

To find the area of a paralellogram:

### RULE.

*Multiply the length by the perpendicular breadth. If the figure be rectangular, one of the sides will be the perpendicular breadth.*

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[*] We have taken for granted that those studying mensuration have learned, at least, the elementary principles of geometry. We have, therefore, only given the rules, as our space would not admit of our giving demonstrations as this would require a separate treatise

*If the figure be not rectangular, either the perpendicular breadth must be given or data from which to find it.*

### EXERCISES.

1. How many acres are there in a square, each side of which is 24 rods? Ans. 3 acres, 2 roods, 16 rods.

2. What is the area of a square picture frame, each side of which is 5 ft. 9 in.? Ans. 33 ft. 9 in.

3. How many acres are there in a rectangular field, the length of which is 13¾ chains, and the breadth 9½? Ans. 130.625 square chains, or 13 acres, 0 roods, 10 rods.

4. What is the area of a rectangle, whose sides are 14 ft. 6 in. and 4 ft. 9 in.? Ans. 68 ft., 126 sq. in.

5. What does the surface of a plank measure, which is 12 ft. 6 in. long and 9 in. broad? Ans. 9 sq. ft. 54 sq. in.

6. What is the area of a rhomboidal field, the length of which is 10.52 chains and the perpendicular breath 7.63 chains? Ans. 8 acres, 0 roods, 4.2816 rods.

7. What is the area of a rhomboidal field, the length of which is 24 rods and the perpendicular breadth 24 rods? Ans. 3 acres, 2 roods, 16 rods.

8. What is the length of each side of a square field, the area of which is 788544 square yards? Ans. 888 yards.

9. The area of a rectangular garden is 1848 square yards, and one side is 56 yards; what is the other? Ans. 33 yards.

10. The area of a rhomboidal pavement is 205, and the length is 20 feet; what is the perpendicular breadth? Ans. 10¼ feet.

### PROBLEM II.

To find the area of a triangle.

1. If the base and perpendicular, or data to find them, be given, we have the

### RULE.

*Multiply the base by the perpendicular, and take half the product; or, multiply half the one by the other.*

2. If the three sides are given

### RULE.

*From half the sum of the sides subtract each side successively, and the square root of the continual product of the half sum, and these three remainders will be the area.*

Expressed algebraically this area$=\sqrt{s(s-a)(s-b)(s-c)}$.

EXERCISES.

11. What is the area of a triangle, the base of which is 17 inches, and the altitude 12 inches? Ans. 102 square inches.

12. What is the area of a triangular garden, the length of which is 46 rods, and the breadth 19 rods? Ans. 437 square rods.

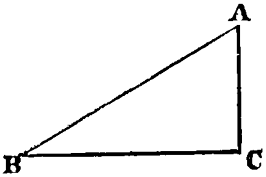
13. Find how many acres, &c., are in a triangular field, the length of which is 49.75 rods, and the breadth $34\frac{1}{2}$ rods.

Ans. 5 acres, 1 rood, $18\frac{3}{16}$ rods.

14. The area of a triangular inclosure is 150 square rods, and the base is 30 linear rods; what is the altitude? Ans. 10 rods.

15. The area of a triangle is 400 rods, and the altitude 40 rods, what is the base? Ans. 20 rods.

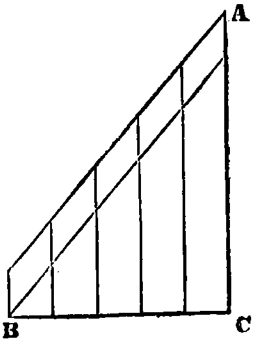
16. Three trees are so planted that the lines joining them form a right angled triangle; the two sides containing the right angle are 33 and 56 yards; what is the area in square yards? Ans. 924.



17. Let the position of the trees, as in the last example, be represented by the triangle A B C, and let the distance from A to B be 50 rods, and from B to C 30 rods. Required the area.—(See Euclid I. 47.)

Ans. 600 square rods.

18. In the figure annexed to 17, suppose A B to represent the pitch of a gallery in a church, inclined to the ground at an angle of 45°; how many more persons will the gallery contain than if the seats were made on the flat B C, supposing B C to be 20 feet and the frontage 60 feet in length? Ans. None.



We have introduced this question and the next to correct a common misapprehension on this point. Because the distance from B to A is greater than the distance from B to C, it is commonly supposed that more persons can be accommodated on the slant A B, than on the flat B C. By inspecting the annexed diagram it will be seen that the seats are not perpendicular to A B, but to B C, and that precisely the same number of seats can be made, and the same number of persons accommodated on B C as on A B.

19. If B C be half the base of a hill, and A B one of its sloping sides, and B C=30 yards, and A B=50 yards; how many more rows of trees can be planted on A B, than on B C, at 1 yard apart?

Ans. None, because the trees being all perpendicular to the horizon, are parallel to each other as represented by the vertical lines in the last figure.

20. How many acres, &c., are there in a triangular field of which the perpendicular length and breadth are 12 chains, 76 links and 9 chains, 43 links? Ans. 6 acres, 0 roods, 2½ rods.

21. A ship was stranded at a distance of 40 yards from the base of a cliff 30 yards high; what was the length of a cable which reached from the top of the cliff to the ship? Ans. 50 yds.

22. A cable 100 yards long was passed from the bow to the stern of a ship through the cradle of a mast placed in midships at the height of 30 yards; what was the length of the ship?

Ans. 80 yards.

23. A man attempts to row a boat directly across a river 200 yards broad, but is carried 80 yards down the stream by the current; through how many yards was he carried? Ans. 215.4+yards.

24. Let the three sides. of a triangle be 30, 40, 20; to find the area in square feet. Ans. 290.4737 square feet.

25. What is the area of an isosceles triangle, each of the equal sides being 15 feet, and the base 20 feet?* Ans. 111.803 sq. feet.

26. What is the area of a triangular space, of which the base is 56, and the hypotermse 65 yards? Ans. 924 square yards.

27. What is the area of a triangular clearing, each side of which is 25 chains? Ans. 27.0632 acres.

28. What is the area of a triangular clearing, of which the three sides are 380, 420 and 765? Ans. 9 acres, 37½ perches.

29. A lot of ground is represented by the three sides of a right angled triangle, of which the hypotenuse is 100 rods, and the base 60 rods; what is the area? Ans. 15 acres.

30. What is the area of a triangular field, of which the sides are 49, 34 and 27 rods respectively? Ans. 2 acres, 3 roods+.

31. What is the area of a triangular orchard, the sides of which are 13, 14 and 15 yards? Ans. 84 square yards.

32. Three divisions of an army are placed so as to be represented

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* This question, and some others may be solved by either rule, and it will be found a good exercise to solve by both.

by three sides of a triangle, 12, 18 and 24; how many square miles do they guard within their lines?

Ans. Between 104 and 105 square miles.

33. A ladder, 50 feet long, was placed in a street, and reached to a parapet 28 feet high, and on being turned over reached a parapet on the other side 30 feet high; what was the breadth of the street? Ans. 76.123+feet.

### PROBLEM III.

To find the area of a regular Polygon.

1. When one of the equal sides, and the perpendicular on it from the centre, are given.

*Multiply the perimeter by the perpendicular on it from its centre, and take half the product; or, multiply either by half the other.*

2. When a side only is given.

*Multiply the square of the side by the number found opposite the number of sides in the subjoined table.*

Note.—This table shows the area when the side is unity; or, which is the same thing, the square is the unit.

| SIDES. | REGULAR FIGURES. | |
|---|---|---|
| 3 | Triangle | 0.4330127. |
| 4 | Square | 1.0000000. |
| 5 | Pentagon | 1.7204774. |
| 6 | Pexagon | 2.5980762. |
| 7 | Heptagon | 3.6339125. |
| 8 | Octagon | 4.8284272. |
| 9 | Nonagon | 6.1818241. |
| 10 | Decagon | 7.6942088. |
| 11 | Heredecagon | 9.3656395. |
| 12 | Dodecagon | 11.1961524. |

34. If the side of a pentagon is 6 feet and the perpendicular 3 feet, what is the area? Ans. 45 feet.

35. What is the area of a regular polygon, each side of which is 15 yards? Ans. 387.107325 sq. yds.

36. If each side of a hexagon be 6 feet, and a line drawn from the centre to any angle be 5 feet, what is the area?

Ans. 72 sq. feet.

37. The side of a decagon is 20.5 rods; what is the area?

Ans. 20 acres, 0 roods, 33.5 rods, nearly.

38. A hexagonal table has each side 60 inches, and a line from the centre to any corner is 50 inches; how many square feet in the surface of the table? Ans. 38 feet, 128 inches.

39. What is the area of a regular heptagon, the side being $19\frac{19}{50}$ and the perpendicular 10? Ans. 678.3.

40. An octagonal enclosure has each side 6 yards, what is its area? Ans. 3 acres, 2 roods, 14 rods. 19 yards.

41. Five divisions of an army guard a certain tract of country—each line is 20 miles; how many square miles are guarded?

Ans. 688.2, nearly.

42. Find the same if there are 6 divisions, and each line extends 5 miles? Ans. 64.95+ miles.

43. The area of a hexagonal table is $73\frac{3}{4}$ feet; what is each side? Ans. $5\frac{1}{3}$ feet.

### PROBLEM IV.

To find the area of an irregular polygon.

*Divide it into triangles by a perpendicular on each diagonal from the opposite angle.*

*Find the area of each triangle separately, and the sum of these areas will be the area of the trapezium.*

NOTE.—Either the diagonals and perpendiculars must be given, or data from which to find them.

44. The diagonal extent of a four-sided field is 65 rods, and the perpendiculars on it from the opposite corners are 28 and 33.5 rods; what is the area? Ans. 1 acre, 1 rood, 22.083 rods.

45. A quadrangle having two sides parallel, and the one is 20.5 feet long and the other 12.25 feet, and the perpendicular distance between them is 10.75 feet; what is the area?

Ans. 176.03125 sq. feet.

46. Required the area of a six-sided figure, the diagonals of which are as follows: the two extreme ones, 20.75 yards and 18.5, and the intermediate 27.48; the perpendicular on the first is 8.6, on the second 12.8, and those on the intermediate one 14.25 and 9.35?

Ans. 531.889 yards.

47. If the two sides of a hexagon be parallel, and the diagonal parallel to them be 30.15 feet, and the perpendiculars on it from

the opposite angles are, on the left, 10.56, and on the right 12.24, and the part of the diagonal cut off to the left by the first perpendicular, 8.26, and to the right by the second, 10.14; on the other side, the perpendicular and segment of the diagonal to the left are 8.56 and 4.54, and on the right 9.26 and 3.93; what is the area?
Ans. 470.4155 sq. feet.

### PROBLEM V.

To find the area of a figure, the boundaries of which are partly right lines and partly curves or salients.

*Find the average breadth by taking several perpendiculars from the nearest and most remote points, from a fixed base, axd dividing the sum of these by their number, the quotient, multiplied by the length, will be a close approximation to the area.*

Let the perpendiculars 9.2, 10.5, 8.3, 9.4, 10.7, their sum is 48.1, then 48.1÷5=9.62, and if the base is 20, we have 9.62×20= 192.4, the area.

When practicable, as large a portion of the space as possible should be laid off, so as to form a regular figure, and the rest found as above.

A field is to be measured, and the greater part of it can be laid off in the form of a rectangle, the sides of which are 20.5 and 10.5, and therefore its area is 215.25, and the offsets of the irregular part are 10.2, 8.7, 10.9, and 8.5, the sum of which, divided by their number, is 7.66, and 7.66×20.5=157.03, the area of the irregular part, and this, added so the area of the rectangles, gives 372.28, the whole area.

48. The length of an irregular clearing is 47 rods, and the breadths at 6 equal distances are 5.7, 4.8, 7.5, 5.1, 8.4 and 6.5; what is the area? Ans. 1 acre, 1 rood, 29.86 rods.

### PROBLEM VI.

To find the circumference of a circle when the diameter is known, or the diameter when the circumference is known.*

The most accurate rule is the well-known theorem that the diameter is to the circumference in the ratio of 113 to 355, and

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* In strictness the circumference and diameter are not like quantities, but we may suppose that a cord is stretched round the circumference, and then drawn out into a straight line, and its linear units compared with those of the diameter.

consequently the circumference to the diameter as 355 to 113. Now, 355÷113=3.1416 nearly, and for general purposes, sufficient accuracy will be attained by this

### RULE.

To find the circumference from a given diameter, *multiply the diameter by* 3.1416; and to find the diameter from a given circumference, *divide by* 3.1416.

49. What is the length round the equator of a 15-inch globe?
Ans. 47.124 inches.

50. If a round log has a circumference of 6 feet, 10 inches; what is its diameter? Ans. 2 feet, $2\frac{1}{10}$ inches nearly.

51. If we take the distance from the centre of the earth to the equator to be 3979; what is the number of miles round the equator?
Ans. 25001 nearly.

### PROBLEM VII.

To find the area of a circle.

1. If the circumference and diameter are known,—

*Multiply the circumference by the diameter, and take one-fourth of the product.*

2. If the diameter alone is given,—

*Multiply the square of the diameter by* .7854.

3. If the circumference alone be given,—

*Multiply the square of the number denoting the circumference by* .07958.

52. If the diameter of a circle is 7, and the circumference 22; what is the area? Ans. $38\frac{1}{2}$.

53. What is the area of a circle, the radius of which is $3\frac{5}{6}$ yds?
Ans. $3\frac{5}{6}$ square yards.

54. If a semicircular arc be denoted by 10.05; what is the area of the circle? Ans. 289.36.

55. If the diameter of a grinding stone be 20 inches; what superficial area is left when it is ground down to 15 inches diameter, and what superficial area has been worn away?
Ans. 176.715 sqr. inches left, and 137.445 worn away.

56. If the chord of an arc be 24 inches, and the perpendicular on it from the centre 11.9; what is the area of the circle?
Ans. 2.689804.

## MENSURATION OF SOLIDS.

To find the solid contents of a parallelopiped, or any regularly box-shaped body:

Let it be required to find the number of cubic feet in a box 8 feet long, $4\frac{1}{2}$ feet broad, and $6\frac{3}{4}$ feet deep.

In the first place, the length being 8 feet and the breadth $4\frac{1}{2}$, the area of the base is $8\times4\frac{1}{2}=36$ square feet, and therefore every foot of altitude, or depth, or thickness, will give 36 cubic feet, and as there are $6\frac{3}{4}$ feet of depth, the whole solid content will be 36 times $6\frac{3}{4}$, or 243 cubic feet. Hence the

### RULE.

*Take the continual product of the length, breadth, and depth.*

NOTE.—Let it be carefully observed that the unit of measure in the case of solids is to be taken as a cube, the base of which is a superficial unit used in the measurement of surfaces. The solid content is indicated by the repetition of this unit a certain number of times. If the body is of uniform breadth the rule needs no modification, but if it is rounded or tapering, as a globe, cone, or pyramid, the calculation becomes virtually to find how much the rounded or tapering body differs from the one of uniform breadth. Suppose, for example, we take a piece of wood 6 feet high, in the form of a pyramid, and having the length and breadth of the base each 6 feet, then the area of the base is 36; but if, at the height of 1 foot, the dimensions have each diminished by 1 foot, the area is 25; at another foot higher it is 16; at the next 9; at the next 4; at the next 1; and at the 6th 0, *i. e.*, it has come to a point, and the calculation is, how much remains from the solid cube after so much has been cut off each side as to give it this form.

This gives rise to the following varieties:

I. To find the solid contents of a cone or pyramid:

*Multiply the area of the base by the perpendicular height, and take one-third of the product.*

II. To find the solid contents of a cylinder or prism:

*Multiply the area of the base by the perpendicular height.*

III. To find the surface of a sphere:

*Multiply the square of the diameter by* 3.1416.

IV. To find the solid contents of a globe or sphere:

*Multiply the third power of the diameter by* .5236.

V. To find the volume of a spheroid, the axes being given :

*Multiply the square of the axis of revolution by the fixed axis, and the product by* .5236.

### EXERCISES.

57. If the diameter of the base of a cylinder be 2 feet, and its height 5 feet, what is the solid content ? Ans. 25.708 feet:

58. If the diameter of the base of a cone be 1 foot 6 inches, and the altitude 15 feet, what are the solid contents ?

Ans. 8 feet, 120 inches.

59. If the diameter of the base of a cylinder be 7 feet, and the height 5 feet, what is the solid content ? Ans. 245 cubic feet.

60. What are the solid contents of a hexagonal prism, each side of the base being 16 inches and the height 15 feet?

Ans. 69.282 cubic feet.

61. A triangular pyramid is 30 feet high, and each side of the base is 3 feet ; required the solid contents. Ans. 39.98 cubic feet.

62. What are the solid contents of the earth, the diameter being taken as 7918.7 miles ? Ans. 259992732079.87.

63. In a spheroid the less axis is 70 and the greater 90 ; what are the solid contents ? Ans. 230907.6.

## PILING OF BALLS AND SHELLS.

Balls are usually piled on a base which is either a triangle, or square, or rectangle, each side of each course containing one ball less than the one below it.

If the base is an equilateral figure, the vertex of a complete pile will be a single ball ; but if one side of the base be greater than the contiguous one, the vertex will be a *row* of balls. Hence, if the base be an equilateral figure, the pile will be a pyramid, and as the side of each layer contains one layer less than the one below it, the number of balls in height will be the same as the number of balls in one side of the lowest layer. If the pile be rectangular, each layer must also be rectangular, and the number of balls in height will be the same as the number in the less side of the base. If the base be triangular, we have the

### RULE.

*Multiply the number on one side of the bottom row by itself* PLUS *one, and the product by the same base row* PLUS *two, and divide the result by six.*

For a complete square pile we have the

### RULE.

*Multiply the number of balls in one side of the lowest course by itself* PLUS *one, and this product by double the first multiplier* PLUS *one, and take one-sixth of the result.*

If the pile be rectangular, we have the

### RULE.

*From three times the number of balls in the length of the lowest course subtract one less than the number in the breadth of the same course; multiply the remainder by the breadth, and this product by one-sixth the breadth* PLUS *one.*

*If the pile be incomplete, find what it would be if complete; find also what the incomplete one would be as a separate pile, and subtract the latter from the former.*

### EXERCISES.

64. In a complete triangular pile each side of the base is 40; how many balls are there? Ans. 11480.

65. In each side of the base of a square pile there are 20 shells; how many in the whole pile? Ans. 2870.

66. In a rectangular pile there are 59 balls in the length, and 20 in the breadth of the base; how many are in all? Ans. 11060.

67. In an incomplete triangular pile, each side of the lowest layer consists of 40 balls, and the side of the upper course of 20; what is the number of balls? Ans. 10150.

NOTE.—Since the upper course is 20, the first row in the wanting part would be 19.

## MEASUREMENT OF TIMBER.

Timber is measured sometimes by the square foot, and sometimes by the cubic foot.

Cleared timber, such as planks, beams, &c., are usually measured by the square foot.

What is called board measure is a certain length and breadth, and a uniform thickness of *one inch.*

Large quantities of round timber are often estimated by the ton.

To find either the superficial extent or board measure of a plank, &c.

### RULE.

*Multiply the length in feet by the breadth in inches, and divide by* 12.

NOTE.—The thickness being taken uniformly as *one inch*, the rule for finding the contents in square feet becomes the same as that for finding surface. If the thickness be not an inch,—

*Multiply the board measure by the thickness.*

If the board be a tapering one, take half the sum of the two extreme widths for the average width.

If a one-inch plank be 24 feet long, and 8 inches thick, then we have 8 inches equal $\frac{2}{3}$ of a foot, and $\frac{2}{3}$ of 24 feet=16 feet.

A board 30 feet long is 26 inches wide at the one end, and 14 inches at the other, hence 20 is the mean width, *i. e.*, $1\frac{2}{3}$ feet, and $30\times1\frac{2}{3}=50$; or, $30\times20=600$, and $600\div12=50$.

To find the solid contents of a round log when the girt is known.

### RULE.

*Multiply the square of the quarter girt in inches by the length in feet, and divide the product by* 144.

If a log is 40 inches in girt, and 30 feet long, the solid contents will be found by taking the square of 10, the quarter girt in inches, which is 100, and $100\times30=3000$, and $3000\div144=20\frac{5}{6}$.

To find the number of square feet in round timber, when the mean diameter is given.

### RULE.

*Multiplg the diameter in inches by half the diameter in inches, and the product by the length in feet, and divide the result by* 12.

If a log is 30 feet long, and 56 inches mean diameter, the number of square feet is $56\times28\times30\div12=1960$ feet.

To find the solid contents of a log when the length and mean diameter are given.

### RULE.

*Multiply the square of half the diameter in inches by* 3.1416, *and this product by the length in feet, and divide by* 144.

68. How many cubic feet are there in a piece of timber $14\times18$, and 28 feet long? Ans. 49+cubic feet.

69. How many cubic feet are there in a round log 21 inches in diameter, and 40 feet in length? Ans. $96\frac{19}{90}$.

70. What are the solid contents of a log 24 inches in diameter, and 34 feet in length? Ans. 106.81+cubic feet.

71. How many feet, board measure, are there in a log 23 inches in diameter, and 12 feet long? Ans. $264\frac{1}{2}$.

72. How many feet, board measure, are there in a log, the diameter of which is 27 inches, and the length 16 feet. Ans. 486.

73. What are the solid contents of a round log 36 feet long, 18 inches diameter at one end, and 9 at the other?

Ans. 30.63+cubic feet.

74. How many feet of square timber will a round log 36 inches in diameter and 10 feet long yield? Ans. 540 solid feet.

75. How many solid feet are there in a board 15 feet long, 5 inches wide, and 3 inches thick? Ans. $1\frac{9}{16}$ cubic feet.

76. What are the solid contents of a board 20 feet long, 20 inches broad, and 10 inches thick? Ans. $27\frac{7}{9}$ feet.

77. What is the solid content of a piece of timber 12 feet long, 16 inches broad, and 12 inches thick? Ans. 16 feet.

78. How many cubic feet are there in a log that is 25 inches in diameter, and 32 feet long?

79. How many feet, board measure, does a log 28 inches in diameter, and 14 feet in length contain? Ans. $457\frac{1}{3}$.

80. How many cubic feet are contained in a piece of squared timber that is 12 by 16 inches, and 47 feet in length? Ans. $62\frac{2}{3}$.

81. How many feet, board measure, are there in 22 one-inch boards, each being 13 inches in width, and 16 feet in length? Ans. $381\frac{1}{3}$.

### BALES, BINS, &C.

As bales are usually of the same form as boxes, the same rule applies.

82. Hence, a bale measuring $4\frac{1}{2}$ inches in length, 33 in width, and $3\frac{1}{3}$ in depth, is, in solid content, $37\frac{1}{2}$ feet.

83. A crate is 5 feet long, $4\frac{3}{4}$ broad, and $3\frac{7}{12}$ deep, what is the solid content? Ans. $85\frac{5}{48}$.

To find how many bushels are in a *bin* of grain:

### RULE.

*Find the product of the length, breadth and depth, and divide by* 5150.4.

84. A bin consists of 12 compartments; each measures 6 feet 3 inches in length, 4 feet 8 inches in width, and 3 feet 9 inches in depth; how many bushels of grain will it hold? Ans. 1055, nearly.

To find how many bushels of grain are in a conical heap in the middle of a floor:

### RULE.

*Multiply the area of the base by one-third the height.*

The base of such a pile is 8 feet diameter and 4 feet high; what is the content?

The area of the base is $64 \times .7854 = 83.777$, and $83.777 \times \frac{4}{3} =$ 67.02, the number of bushels.

If it be heaped against a wall take half the above result.

If it be heaped in a corner, take one-fourth the above result.

## MISCELLANEOUS EXERCISES.

1. What number is that $\frac{2}{3}$ and $\frac{3}{5}$ of which make 255?
Ans. $201\frac{6}{19}$.

2. What must be added to $217\frac{1}{2}$, that the sum may be $17\frac{1}{4}$ times $19\frac{1}{2}$? Ans. $118\frac{7}{8}$.

3. What sum of money must be lent, at 7 per cent., to accumulate to $455 interest in 3 months? Ans. $26000.

4. Divide $1000 among A, B and C, so that A may have $156 more than B, and B $62 less than C.
Ans. A. $$416\frac{2}{3}$; B, $$260\frac{2}{3}$; C, $$322\frac{2}{3}$.

5. Where shall a pole 60 feet high be broken, that the top may rest on the ground 20 feet from the stump? Ans. $26\frac{2}{3}$ feet.

6. A man bought a horse for $68, which was $\frac{3}{7}$ as much again as he sold it for, lacking $1; how much did he gain by the bargain?
Ans. $12.50.

7. A fox is 120 leaps before a hound, and takes 5 leaps to the hound's 2; but 4 of the hound's leaps equal 12 of the fox's; how many leaps must the hound take to catch the fox? Ans. 240.

8. A, B and C can do a certain piece of work in 10 days; how long will it take each to do it separately, if A does $1\frac{1}{2}$ times as much as B, and B does $\frac{1}{2}$ as much as C?
Ans. A, 30 days; B, 45; C, $22\frac{1}{2}$.

9. At what time between five and six o'clock, are the hour and minute hands of a clock exactly together?
Ans. 27 min., $16\frac{4}{11}$ sec. past 5.

10. A courier has advanced 35 miles with despatches, when a second starts with additional instructions, and hurries to overtake the first, travelling 25 miles for 18 that the first travels; how far will both have travelled when the second overtakes the first?
Ans. 125 miles.

11. What is the sum of the series $\frac{2}{5}-\frac{4}{15}+\frac{8}{45}-\frac{16}{135}+\frac{32}{405}-$&c.?
Ans. $\frac{6}{25}$.

12. If a man earn $2 more each month than he did the month before, and finds at the end of 18 months that the rate of increase will enable him to earn the same sum in 14 months; how much did he earn in the whole time? Ans. $4032.

13. How long would it take a body, moving at the rate of 50

miles an hour, to pass over a space equal to the distance of the earth from the sun, *i. e.*, 95 millions of miles, a year being 365 days?

Ans. 216 years, 326 days, 16 hours.

14. Two soldiers start together for a certain fort, and one travels 18 miles a day, and after travelling 9 days, turns back as far as the second had travelled during those 9 days, he then turns, and in $22\frac{1}{2}$ days from the time they started, arrives at the fort at the same time as his comrade; at what rate did the second travel?

Ans. 18 miles a day.

15. What quantity must be subtracted from the square of 48, so that the remainder may be the product of 54 by 16? Ans. 1440.

16. A father gave $\frac{3}{8}$ of his farm to his son, the son sold $\frac{2}{3}$ of his share for $1260; what was the value of the whole farm?

Ans. $5040.

17. There were $\frac{5}{8}$ of a flock of sheep stolen, and 672 were left; how many were there in all? Ans. 1792.

18. A boy gave 2 cents each for a number of pears, and had 42 cents left, but if he had given 5 cents for each, he would have had nothing left. Required the number of pears. Ans. 14.

19. Simplify $\dfrac{1}{1+\dfrac{1}{2+\frac{1}{2}}}$. Ans. $\frac{5}{7}$.

20. A man contracted to perform a piece of work in 60 days, he employed 30 men, and at the end of 48 days it was only half finished; how many additional hands had to be employed to finish it in the stipulated time?

21. A gentleman gave his eldest daughter twice as much as his second, and the second three times as much as the third, and the third got $1573; how much did he give to all? Ans. $15730.

22. The sum of two numbers is 5643, and their difference 125; what are the numbers? Ans. 2884 and 2759.

23. How often will all the four wheels of a carriage turn round in going 7 miles, 1 furlong, and 8 rods, the hind wheels being each 7 feet 6 inches in circumference, and the fore wheels 5 feet $7\frac{1}{2}$ inches?

Ans. 23716.

24. What is the area of a right angled triangular field, of which the hypotenuse is 100 rods and the base 60? Ans. 2400 sq. rds.

25. Simplify $\dfrac{5\frac{4}{5}-2\frac{1}{6}}{3\frac{3}{4}+\frac{9}{20}}$ of $\dfrac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}}$ of $\dfrac{2\frac{3}{5}+1\frac{2}{3}}{7\frac{19}{24}-2\frac{1}{4}}$. Ans. $1\frac{53}{75}$.

26. Find the value of $\frac{1}{1+\frac{1}{1+\frac{1}{2}}}$. Ans. $\frac{3}{5}$.

27. If $\frac{2}{3}$ of A's age is $\frac{5}{8}$ of Bs', and A is $37\frac{1}{2}$, what age is B? Ans. 40.

28. What is the excess of $\frac{1}{99} \div \frac{1}{101}$ above $\frac{1}{999} \div \frac{1}{1001}$? Ans. $\frac{200}{10989}$.

29. The sum of two numbers is 5330 and their difference 1999; what are the numbers? Ans. $3664\frac{1}{2}$ and $1665\frac{1}{2}$.

30. A person being asked the hour of the day, replied that the time past noon was equal to one-fifth of the time past midnight; what was the time? Ans. 3 P.M.

31. A snail, in getting up a pole 20 feet high, climbed up 8 feet every day, but slipped back 4 feet every night; in what time did he reach the top? Ans 4 days.

32. What number is that whose $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ parts make 48? Ans. $44\frac{4}{13}$.

33. A merchant sold goods to a certain amount, on a commission of 4 per cent., and, having remitted the net proceeds to the owner, received $\frac{1}{4}$ per cent. for immediate payment, which amounted to $15.60; what was the amount of his commission? Ans. $260.

34. A criminal has 40 miles the start of the detective, but the detective makes 7 miles for 5 that the fugitive makes; how far will the detective have travelled before he overtakes the criminal? Ans. 140 miles.

35. A man sold 17 stoves for $153; for the largest size he received $19, for the middle size $7, and for the small size $6; how many did he sell of each size?
Ans. 3 of the large size, 12 of the middle, 2 of the small.

36. A merchant bought goods to the amount of $12400; $4060 of which was on a credit of 3 months, $4160 on a credit of 8 months and the remainder on a credit of 9 months; how much ready money would discharge the debt, money being worth 6 per cent.? Ans. $12000.

37. If a regiment of soldiers, consisting of 1000 men, are to be clothed, each suit to contain $3\frac{3}{4}$ yards of cloth that is $1\frac{7}{8}$ yards wide, and to be lined with flannel $1\frac{1}{4}$ yards wide; how many yards will it take to line the whole? Ans. 5625.

38. Taking the moon's diameter at 2180 miles, what are the solid contents? Ans. 5424617475+ sq. miles.

39. A certain island is 73 miles in circumference, and if two men start out from the same point, in the same direction, the one walking at the rate of 5 and the other at the rate of 3 miles an hour; in what time will they come together? Ans. 36 hours, 30 minutes.

40. A circular pond measures half an acre; what length of cord will be required to reach from the edge of the pond to the centre?
Ans. 83263+ feet.

41. A gentleman has deposited $450 for the benefit of his son, in a Savings' Bank, at compound interest at a half-yearly rate of 3½ per cent. He is to receive the amount as soon as it becomes $1781.66½. Allowing that the deposit was made when the son was 1 year old, what will be his age when he can come in possession of the money? Ans. 21 years.

42. The select men of a certain town appointed a liquor agent, and furnished him with liquor to the amount of $825.60, and cash, $215. The agent received cash for liquor sold, $1323.40. He paid for liquor bought, $937; to the town treasurer, $300; sundry expenses, $29; his own salary, $265; he delivered to indigent persons, by order of the town, liquor to the amount of $13.50. Upon taking stock at the end of the year, the liquor on hand amounted to $616.50. Did the town gain or lose by the agency, and how much; has the agent any money in his hands belonging to the town; or does the town owe the agent, and how much in either case?
Ans. The town lost $103.20; the agent owes the town $7.40.

43. A holds a note for $575 against B, dated July 13th, payable in 4 months from date. On the 9th August, A received in advance $62; and on the 5th September, $45 more. According to the terms of agreement it will be due, adding 3 days of grace, on the 16th November, but on the 3rd of October B proposes to pay a sum which, in addition to the sums previously paid, shall extend the pay day to forty days beyond the 16th of November; how much must B pay on the 3rd of October? Ans. $111.43.

44. A accepted an agency from B to buy and sell grain for him. A received from B grain in store, valued at $135.60, and cash, $222.10; he bought grain to the value of $1346.40, and sold grain to the amount of $1171.97. At the end of four months B wished to close the agency, and A returned him grain unsold, valued at $437.95; A was to receive for services, $48.12. Did A owe B, or B owe A, and how much? Ans. B owed A 45 cents.

45. A general ranging his men in the form of a square, had 59 men over, but having increased the side of the square by one man, he lacked 84 of completing the square; how many men had he?
Ans. 5100.

46. What portion, expressed as a common fraction, is a pound and a half troy weight of three pounds avoirdupois? Ans. $\frac{72}{175}$.

47. What would the last fraction be if we reckoned by the ounces instead of grains according to the standards? Ans. $\frac{3}{8}$.

48. If 4 men can reap $6\frac{2}{3}$ acres of wheat in $2\frac{1}{2}$ days, by working $8\frac{1}{4}$ hours per day, how many acres will 15 men, working equally, reap in $3\frac{3}{4}$ days, working 9 hours per day? Ans. $40\frac{10}{11}$ days.

49. Out of a certain quantity of wheat, $\frac{1}{3}$ was sold at a certain gain per cent., $\frac{1}{4}$ at twice that gain, and the remainder at three times the gain on the first lot; what was the gain on each, the gain on the whole being 20 per cent.? Ans. $9\frac{3}{5}$, $19\frac{1}{5}$ and $28\frac{4}{5}$ per cent.

50. If a man by travelling 6 hours a day, and at the rate of $4\frac{1}{2}$ miles an hour, can accomplish a journey of 540 miles in 20 days; how many days, at the rate of $4\frac{2}{3}$ miles an hour, will he require to accomplish a journey of 600 miles? Ans. $21\frac{3}{7}$.

51. Smith in Montreal, and Jones in Toronto, agree to exchange operations, Jones chiefly making the purchases, and Smith the sales, the profits to be equally divided; Smith remitted to Jones a draft for $8000 after Jones had made purchases to the amount of $13682.24;—Jones had sent merchandise to Smith, of which the latter had made sales to the value of $9241.18; Jones had also made sales to the worth of $2836.24; Smith has paid $364.16 and Jones $239.14 for expenses. At the end of the year Jones has on hands goods worth $2327.34 and Smith goods worth $3123.42. The term of the agreement having now expired, a settlement is made, what has been the gain or loss? What is each partner's share of gain or loss? What is the cash balance, and in favor of which partner?

52. In a certain factory a number of men, boys and girls are employed, the men work 12 hours a day, the boys 9 hours and the girls 8 hours; for the same number of hours each man receives a half more than each boy, and each boy a third more than each girl; the sum paid each day to all the boys is double the sum paid to all the girls, and for every five shillings earned by all the boys each day, twelve shillings are earned by all the men; it

is required to find the number of men, the number of boys and the number of girls, the whole number being 59.

Ans. 24 men, 20 boys and 15 girls.

53. A holds B's note for $575, payable at the end of 4 months from the 13th July; on the 9th August, A received $62 in advance, as part payment, and on the 5th September $45 more; according to agreement the note will not be due till 16th November, three days of grace being added to the term; but on the 3rd October B tenders such a sum as will, together with the payments already made, extend time of payment forty days forward; how much must B pay on the 3rd of October? Ans. $111.43.

54. If a man commence business with a capital of $5000 and realises, above expenses, so much as to increase his capital each year by *one tenth* of itself less $100, what will his capital amount to in twenty years? Ans. $27910.

55. A note for $100 was to come due on the 1st October, but on the 11th of August, the acceptor proposes to pay as much in advance as will allow him 60 days after the 1st of October to pay the balance; how much must he pay on the 11th of August? Ans. $54.

56. A person contributed a certain sum in dollars to four charities;—to one he gave one half of the whole and half a dollar; to a second half the remainder and half a dollar; to a third half the remainder and half a dollar; and also to the fourth half the remainder and half a dollar, together with one dollar that was left; how much did he give to each?

Ans. To the first, $16; to the second, $8; to the third, $4; to the fourth, $3.

57. A farmer being asked how many sheep he had, replied that he had them in four different fields, and that two-thirds of the number in the first field was equal to three-fourths of the number in the second field; and that two-thirds of the number in the second field was equal to three-fourths of the number in the third field; and that two-thirds of the number in the third field was equal to four-fifths of the number in the fourth field; also that there were thirty-two sheep more in the third field than in the fourth; how many sheep were in each field and how many altogether?

Ans. First field, 243; second field, 216; third field, 192; fourth field, 160. Total. 811.

58. How many hours per day must 217 men work for $5\frac{1}{2}$ days to dig a trench $23\frac{1}{4}$ yards long, $3\frac{2}{3}$ yards wide, and $2\frac{1}{3}$ deep, if 24 men working equally can dig one $33\frac{3}{4}$ yards long, $5\frac{3}{5}$ wide, and $3\frac{1}{2}$ deep, in 189 days of 14 hours each. Ans. 16 hours.

59. A man bequeathed one-fourth of his property to his eldest son;—to the second son one-fourth of the remainder, and $350 besides; to the third one-fourth of the remainder, together with $975; to the youngest one-fourth of the remainder, and $1400; he gives his wife a life interest in the remainder, and her share is found to be one-fifth of the whole; what was the amount of the property?

Ans. $20000.

60. Five men formed a partnership which was dissolved after four years' continuance; the first contributed $60 at first and $800 more at the end of five months, and again $1500 at the end of a year and eight months; the second contributed $600 and $1800 more at the end of six months; the third gave at first $400 and $500 every six months; the fourth did not contribute till the end of eight months, he then gave $900, and the same sum every six months; the fifth having no capital, contributed by his labor in keeping the books at a salary of $1.25 per day; at the expiration of the partnership what was the share of each, the whole gain having been $20000?

Ans. 1st, $2019.65 nearly; 2nd, $4871.81 nearly; 3rd, $4815.81 nearly; 4th, $6467.74 nearly; 5th, $1825.00.

# LOGARITHMS OF NUMBERS

Numbers from 1 to 100.

| No. | Log. | No. | Log. | No. | Log. | No. | Log. | No. | Log. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0·000000 | 21 | 1·322219 | 41 | 1·612784 | 61 | 1·785330 | 81 | 1·908485 |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| 28 | 3 | 4861 | 4953 | 5045 | 5137 | 5228 | 5320 | 5412 | 5503 | 5595 | 5687 | 92 |
| 37 | 4 | 5778 | 5870 | 5962 | 6053 | 6145 | 6236 | 6328 | 6419 | 6511 | 6602 | 92 |
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| 83 | 9 | 680336 | 680426 | 680517 | 680607 | 680698 | 680789 | 680879 | 0970 | 1060 | 1151 | 91 |
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| 54 | 6 | 6636 | 6726 | 6815 | 6904 | 6994 | 7083 | 7172 | 7261 | 7351 | 7440 | 89 |
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| 72 | 8 | 8420 | 8509 | 8598 | 8687 | 8776 | 8865 | 8953 | 9042 | 9131 | 9220 | 89 |
| 81 | 9 | 9309 | 9398 | 9486 | 9575 | 9664 | 9753 | 9841 | 9930 | 690019 | 690107 | 89 |
|  | 490 | 690196 | 690285 | 690373 | 690462 | 690550 | 690639 | 690728 | 690816 | 690905 | 690993 | 89 |
| 9 | 1 | 1081 | 1170 | 1258 | 1347 | 1435 | 1524 | 1612 | 1700 | 1789 | 1877 | 88 |
| 18 | 2 | 1965 | 2053 | 2142 | 2230 | 2318 | 2406 | 2494 | 2583 | 2671 | 2759 | 88 |
| 26 | 3 | 2847 | 2935 | 3023 | 3111 | 3199 | 3287 | 3375 | 3463 | 3551 | 3639 | 88 |
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| 44 | 5 | 4605 | 4693 | 4781 | 4868 | 4956 | 5044 | 5131 | 5219 | 5307 | 5394 | 88 |
| 53 | 6 | 5482 | 5560 | 5657 | 5744 | 5832 | 5919 | 6007 | 6094 | 6182 | 6269 | 87 |
| 62 | 7 | 6356 | 6444 | 6531 | 6618 | 6706 | 6793 | 6880 | 6968 | 7055 | 7142 | 87 |
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| 9 | 1 | 9838 | 9924 | 700011 | 700098 | 700184 | 700271 | 700358 | 700444 | 700531 | 700617 | 87 |
| 17 | 2 | 700704 | 700790 | 0877 | 0963 | 1050 | 1136 | 1222 | 1309 | 1395 | 1482 | 86 |
| 26 | 3 | 1568 | 1654 | 1741 | 1827 | 1913 | 1999 | 2086 | 2172 | 2258 | 2344 | 86 |
| 34 | 4 | 2431 | 2517 | 2603 | 2689 | 2775 | 2861 | 2947 | 3033 | 3119 | 3205 | 86 |
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| 60 | 7 | 5008 | 5094 | 5179 | 5265 | 5350 | 5436 | 5522 | 5607 | 5693 | 5778 | 86 |
| 69 | 8 | 5864 | 5949 | 6035 | 6120 | 6206 | 6291 | 6376 | 6462 | 6547 | 6632 | 85 |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| 8 | 1 | 6838 | 6921 | 7004 | 7088 | 7171 | 7254 | 7338 | 7421 | 7504 | 7587 | 83 |
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| 56 | 7 | 7987 | 8067 | 8146 | 8225 | 8305 | 8384 | 8463 | 8543 | 8622 | 8701 | 79 |
| 64 | 8 | 8781 | 8860 | 8939 | 9018 | 9097 | 9177 | 9256 | 9335 | 9414 | 9493 | 79 |
| 72 | 9 | 9572 | 9651 | 9731 | 9810 | 9889 | 9968 | 740047 | 740126 | 740205 | 740284 | 79 |
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| 8 | 1 | 1152 | 1230 | 1309 | 1388 | 1467 | 1545 | 1624 | 1703 | 1782 | 1860 | 79 |
| 16 | 2 | 1939 | 2018 | 2096 | 2175 | 2254 | 2332 | 2411 | 2489 | 2568 | 2647 | 79 |
| 23 | 3 | 2725 | 2804 | 2882 | 2961 | 3039 | 3118 | 3196 | 3275 | 3353 | 3431 | 78 |
| 31 | 4 | 3510 | 3588 | 3667 | 3745 | 3823 | 3902 | 3980 | 4058 | 4136 | 4215 | 78 |
| 39 | 5 | 4293 | 4371 | 4449 | 4528 | 4606 | 4684 | 4762 | 4840 | 4919 | 4997 | 78 |
| 47 | 6 | 5075 | 5153 | 5231 | 5309 | 5387 | 5465 | 5543 | 5621 | 5699 | 5777 | 78 |
| 55 | 7 | 5855 | 5933 | 6011 | 6089 | 6167 | 6245 | 6323 | 6401 | 6479 | 6556 | 78 |
| 62 | 8 | 6634 | 6712 | 6790 | 6868 | 6945 | 7023 | 7101 | 7179 | 7256 | 7334 | 78 |
| 70 | 9 | 7412 | 7489 | 7567 | 7645 | 7722 | 7800 | 7878 | 7955 | 8033 | 8110 | 78 |
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| 8 | 1 | 8963 | 9040 | 9118 | 9195 | 9272 | 9350 | 9427 | 9504 | 9582 | 9659 | 77 |
| 15 | 2 | 9736 | 9814 | 9891 | 9968 | 750045 | 750123 | 750200 | 750277 | 750354 | 750431 | 77 |
| 23 | 3 | 750508 | 750586 | 750663 | 750740 | 0817 | 0894 | 0971 | 1048 | 1125 | 1202 | 77 |
| 31 | 4 | 1279 | 1356 | 1433 | 1510 | 1587 | 1664 | 1741 | 1818 | 1895 | 1972 | 77 |
| 39 | 5 | 2048 | 2125 | 2202 | 2279 | 2356 | 2433 | 2509 | 2586 | 2663 | 2740 | 77 |
| 46 | 6 | 2816 | 2893 | 2970 | 3047 | 3123 | 3200 | 3277 | 3353 | 3430 | 3506 | 77 |
| 54 | 7 | 3583 | 3660 | 3736 | 3813 | 3889 | 3966 | 4042 | 4119 | 4195 | 4272 | 77 |
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| 8 | 1 | 6636 | 6712 | 6788 | 6864 | 6940 | 7016 | 7092 | 7168 | 7244 | 7320 | 76 |
| 15 | 2 | 7396 | 7472 | 7548 | 7624 | 7700 | 7775 | 7851 | 7927 | 8003 | 8079 | 76 |
| 23 | 3 | 8155 | 8230 | 8306 | 8382 | 8458 | 8533 | 8609 | 8685 | 8761 | 8836 | 76 |
| 30 | 4 | 8912 | 8988 | 9063 | 9139 | 9214 | 9290 | 9366 | 9441 | 9517 | 9592 | 76 |
| 38 | 5 | 9668 | 9743 | 9819 | 9894 | 9970 | 760045 | 760121 | 760196 | 760272 | 760347 | 75 |
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| 53 | 7 | 1176 | 1251 | 1326 | 1402 | 1477 | 1552 | 1627 | 1702 | 1778 | 1853 | 75 |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| 15 | 2 | 4923 | 4998 | 5072 | 5147 | 5221 | 5296 | 5370 | 5445 | 5520 | 5594 | 75 |
| 22 | 3 | 5669 | 5743 | 5818 | 5892 | 5966 | 6041 | 6115 | 6190 | 6264 | 6338 | 74 |
| 30 | 4 | 6413 | 6487 | 6562 | 6636 | 6710 | 6785 | 6859 | 6933 | 7007 | 7082 | 74 |
| 37 | 5 | 7156 | 7230 | 7304 | 7379 | 7453 | 7527 | 7601 | 7675 | 7749 | 7823 | 74 |
| 44 | 6 | 7898 | 7972 | 8046 | 8120 | 8194 | 8268 | 8342 | 8416 | 8490 | 8564 | 74 |
| 52 | 7 | 8638 | 8712 | 8786 | 8860 | 8934 | 9008 | 9082 | 9156 | 9230 | 9303 | 74 |
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| 67 | 9 | 770115 | 770189 | 770263 | 770336 | 770410 | 770484 | 770557 | 770631 | 770705 | 0778 | 74 |
|  | 590 | 770852 | 770926 | 770999 | 771073 | 771146 | 771220 | 771293 | 771367 | 771440 | 771514 | 74 |
| 7 | 1 | 1587 | 1661 | 1734 | 1808 | 1881 | 1955 | 2028 | 2102 | 2175 | 2248 | 73 |
| 15 | 2 | 2322 | 2395 | 2468 | 2542 | 2615 | 2688 | 2762 | 2835 | 2908 | 2981 | 73 |
| 22 | 3 | 3055 | 3128 | 3201 | 3274 | 3348 | 3421 | 3494 | 3567 | 3640 | 3713 | 73 |
| 29 | 4 | 3786 | 3860 | 3933 | 4006 | 4079 | 4152 | 4225 | 4298 | 4371 | 4444 | 73 |
| 37 | 5 | 4517 | 4590 | 4663 | 4736 | 4809 | 4882 | 4955 | 5028 | 5100 | 5173 | 73 |
| 44 | 6 | 5246 | 5319 | 5392 | 5465 | 5538 | 5610 | 5683 | 5756 | 5829 | 5902 | 73 |
| 51 | 7 | 5974 | 6047 | 6120 | 6193 | 6265 | 6338 | 6411 | 6483 | 6556 | 6629 | 73 |
| 58 | 8 | 6701 | 6774 | 6846 | 6919 | 6992 | 7064 | 7137 | 7209 | 7282 | 7354 | 73 |
| 66 | 9 | 7427 | 7499 | 7572 | 7644 | 7717 | 7789 | 7862 | 7934 | 8006 | 8079 | 72 |
|  | 600 | 778151 | 778224 | 778296 | 778368 | 778441 | 778513 | 778585 | 778658 | 778730 | 778802 | 72 |
| 7 | 1 | 8874 | 8947 | 9019 | 9091 | 9163 | 9236 | 9308 | 9380 | 9452 | 9524 | 72 |
| 14 | 2 | 9596 | 9669 | 9741 | 9813 | 9885 | 9957 | 780029 | 780101 | 780173 | 780245 | 72 |
| 22 | 3 | 780317 | 780389 | 780461 | 780533 | 780605 | 780677 | 0749 | 0821 | 0893 | 0965 | 72 |
| 29 | 4 | 1037 | 1109 | 1181 | 1253 | 1324 | 1396 | 1468 | 1540 | 1612 | 1684 | 72 |
| 36 | 5 | 1755 | 1827 | 1899 | 1971 | 2042 | 2114 | 2186 | 2258 | 2329 | 2401 | 72 |
| 43 | 6 | 2473 | 2544 | 2616 | 2688 | 2759 | 2831 | 2902 | 2974 | 3046 | 3117 | 72 |
| 50 | 7 | 3189 | 3260 | 3332 | 3403 | 3475 | 3546 | 3618 | 3689 | 3761 | 3832 | 71 |
| 58 | 8 | 3904 | 3975 | 4046 | 4118 | 4189 | 4261 | 4332 | 4403 | 4475 | 4546 | 71 |
| 65 | 9 | 4617 | 4689 | 4760 | 4831 | 4902 | 4974 | 5045 | 5116 | 5187 | 5259 | 71 |
|  | 610 | 785330 | 785401 | 785472 | 785543 | 785615 | 785686 | 785757 | 785828 | 785899 | 785970 | 71 |
| 7 | 1 | 6041 | 6112 | 6183 | 6254 | 6325 | 6396 | 6467 | 6538 | 6609 | 6680 | 71 |
| 14 | 2 | 6751 | 6822 | 6893 | 6964 | 7035 | 7106 | 7177 | 7248 | 7319 | 7390 | 71 |
| 21 | 3 | 7460 | 7531 | 7602 | 7673 | 7744 | 7815 | 7885 | 7956 | 8027 | 8098 | 71 |
| 28 | 4 | 8168 | 8239 | 8310 | 8381 | 8451 | 8522 | 8593 | 8663 | 8734 | 8804 | 71 |
| 36 | 5 | 8875 | 8946 | 9016 | 9087 | 9157 | 9228 | 9299 | 9369 | 9440 | 9510 | 71 |
| 43 | 6 | 9581 | 9651 | 9722 | 9792 | 9863 | 9933 | 790004 | 790074 | 790144 | 790215 | 70 |
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| 7 | 1 | 3092 | 3162 | 3231 | 3301 | 3371 | 3441 | 3511 | 3581 | 3651 | 3721 | 70 |
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| 35 | 5 | 5880 | 5949 | 6019 | 6088 | 6158 | 6227 | 6297 | 6366 | 6436 | 6505 | 69 |
| 42 | 6 | 6574 | 6644 | 6713 | 6782 | 6852 | 6921 | 6990 | 7060 | 7129 | 7198 | 69 |
| 49 | 7 | 7268 | 7337 | 7406 | 7475 | 7545 | 7614 | 7683 | 7752 | 7821 | 7890 | 69 |
| 56 | 8 | 7960 | 8029 | 8098 | 8167 | 8236 | 8305 | 8374 | 8443 | 8513 | 8582 | 69 |
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|  | 630 | 799341 | 799409 | 799478 | 799547 | 799616 | 799685 | 799754 | 799823 | 799892 | 799961 | 69 |
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| 14 | 2 | 0717 | 0786 | 0854 | 0923 | 0992 | 1061 | 1129 | 1198 | 1266 | 1335 | 69 |
| 21 | 3 | 1404 | 1472 | 1541 | 1609 | 1678 | 1747 | 1815 | 1884 | 1952 | 2021 | 69 |
| 28 | 4 | 2089 | 2158 | 2226 | 2295 | 2363 | 2432 | 2500 | 2568 | 2637 | 2705 | 69 |
| 35 | 5 | 2774 | 2842 | 2910 | 2979 | 3047 | 3116 | 3184 | 3252 | 3321 | 3389 | 68 |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| 26 | 4 | 2168 | 2233 | 2299 | 2364 | 2430 | 2495 | 2560 | 2626 | 2691 | 2756 | 65 |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| 15 | 3 | 5961 | 6010 | 6059 | 6108 | 6157 | 6207 | 6256 | 6305 | 6354 | 6403 | 49 |
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| 25 | 5 | 6943 | 6992 | 7041 | 7090 | 7140 | 7189 | 7238 | 7287 | 7336 | 7385 | 49 |
| 29 | 6 | 7434 | 7483 | 7532 | 7581 | 7630 | 7679 | 7728 | 7777 | 7826 | 7875 | 49 |
| 34 | 7 | 7924 | 7973 | 8022 | 8070 | 8119 | 8168 | 8217 | 8266 | 8315 | 8364 | 49 |
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| 28 | 6 | 1895 | 1943 | 1990 | 2038 | 2085 | 2132 | 2180 | 2227 | 2275 | 2322 | 47 |
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| 42 | 9 | 3316 | 3363 | 3410 | 3457 | 3504 | 3552 | 3599 | 3646 | 3693 | 3741 | 47 |
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| 9 | 2 | 4731 | 4778 | 4825 | 4872 | 4919 | 4966 | 5013 | 5061 | 5108 | 5155 | 47 |
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| 19 | 4 | 5672 | 5719 | 5766 | 5813 | 5860 | 5907 | 5954 | 6001 | 6048 | 6095 | 47 |
| 23 | 5 | 6142 | 6189 | 6236 | 6283 | 6329 | 6376 | 6423 | 6470 | 6517 | 6564 | 47 |
| 28 | 6 | 6611 | 6658 | 6705 | 6752 | 6799 | 6845 | 6892 | 6939 | 6986 | 7033 | 47 |
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| 9 | 2 | 9416 | 9463 | 9509 | 9556 | 9602 | 9649 | 9695 | 9742 | 9789 | 9835 | 47 |
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| PP | N. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | D. |
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| 36 | 8 | 1366 | 1411 | 1456 | 1501 | 1547 | 1592 | 1637 | 1683 | 1728 | 1773 | 45 |
| 41 | 9 | 1819 | 1864 | 1909 | 1954 | 2000 | 2045 | 2090 | 2135 | 2181 | 2226 | 45 |
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| 5 | 1 | 2723 | 2769 | 2814 | 2859 | 2904 | 2949 | 2994 | 3040 | 3085 | 3130 | 45 |
| 9 | 2 | 3175 | 3220 | 3265 | 3310 | 3356 | 3401 | 3446 | 3491 | 3536 | 3581 | 45 |
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| 18 | 4 | 4077 | 4122 | 4167 | 4212 | 4257 | 4302 | 4347 | 4392 | 4437 | 4482 | 45 |
| 23 | 5 | 4527 | 4572 | 4617 | 4662 | 4707 | 4752 | 4797 | 4842 | 4887 | 4932 | 45 |
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| 5 | 1 | 7219 | 7264 | 7309 | 7353 | 7398 | 7443 | 7488 | 7532 | 7577 | 7622 | 45 |
| 9 | 2 | 7666 | 7711 | 7756 | 7800 | 7845 | 7890 | 7934 | 7979 | 8024 | 8068 | 45 |
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| 18 | 4 | 8559 | 8604 | 8648 | 8693 | 8737 | 8782 | 8826 | 8871 | 8916 | 8960 | 45 |
| 23 | 5 | 9005 | 9049 | 9094 | 9138 | 9183 | 9227 | 9272 | 9316 | 9361 | 9405 | 45 |
| 27 | 6 | 9450 | 9494 | 9539 | 9583 | 9628 | 9672 | 9717 | 9761 | 9806 | 9850 | 44 |
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| 36 | 8 | 990339 | 990383 | 990428 | 0472 | 0516 | 0561 | 0605 | 0650 | 0694 | 0738 | 44 |
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